

...

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kontakt osobisty

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## **Wyłączenie odpowiedzialności**

Prezentowane materiały, będące dodatkiem pomocniczym do wykładów, z konieczności fragmentarycznym i niedopracowanym, należy wykorzystywać z pełną świadomością faktu, że mogą nie być pozbawione przypadkowych błędów, braków, wypaczeń i przeinaczeń :-)

Autor

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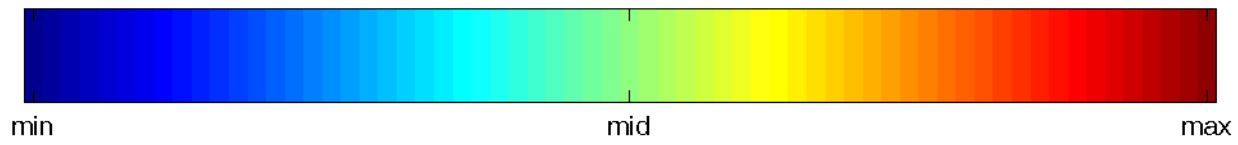
<b>a</b>	<b>c</b>
<b>b</b>	<b>d</b>

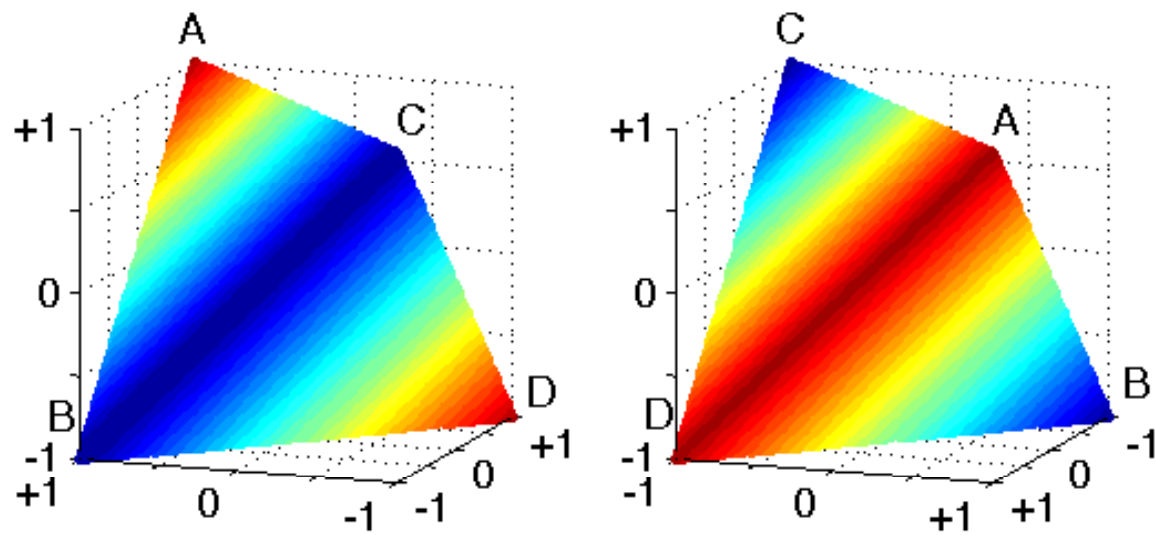
		predicted	
		1	0
original	1	TP	FN
	0	FP	TN

		predicted	
		1	0
original	1	TP <b>A</b> a	FN <b>C</b> c
	0	FP <b>B</b> b	TN <b>D</b> d

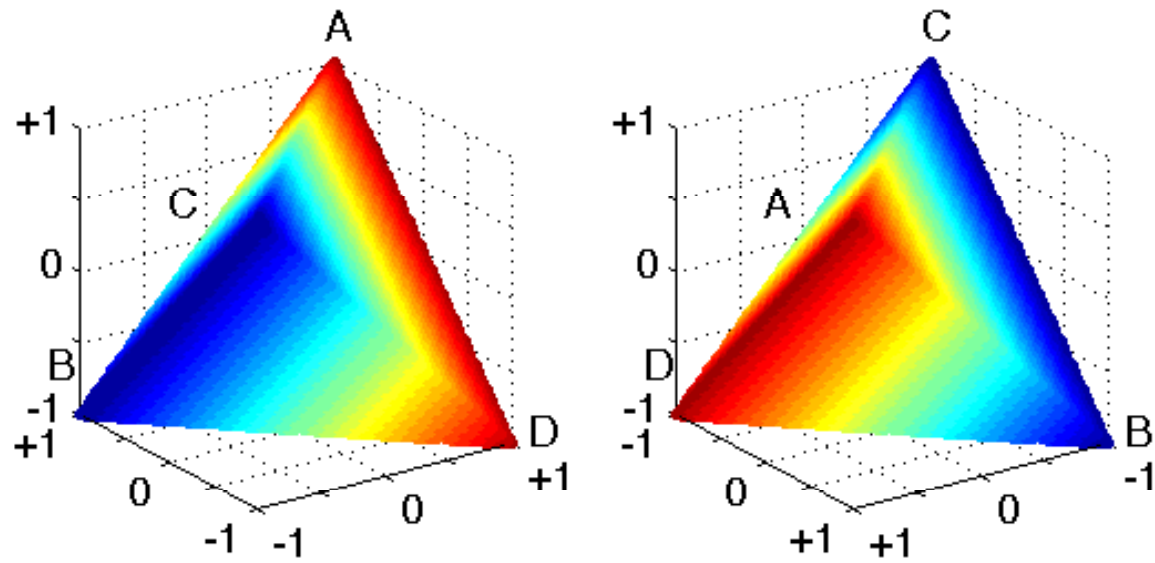
- Zakres wartości [min, max] = [0, 1]
  - (+ ewentualne NaN'y /fioletowe/)



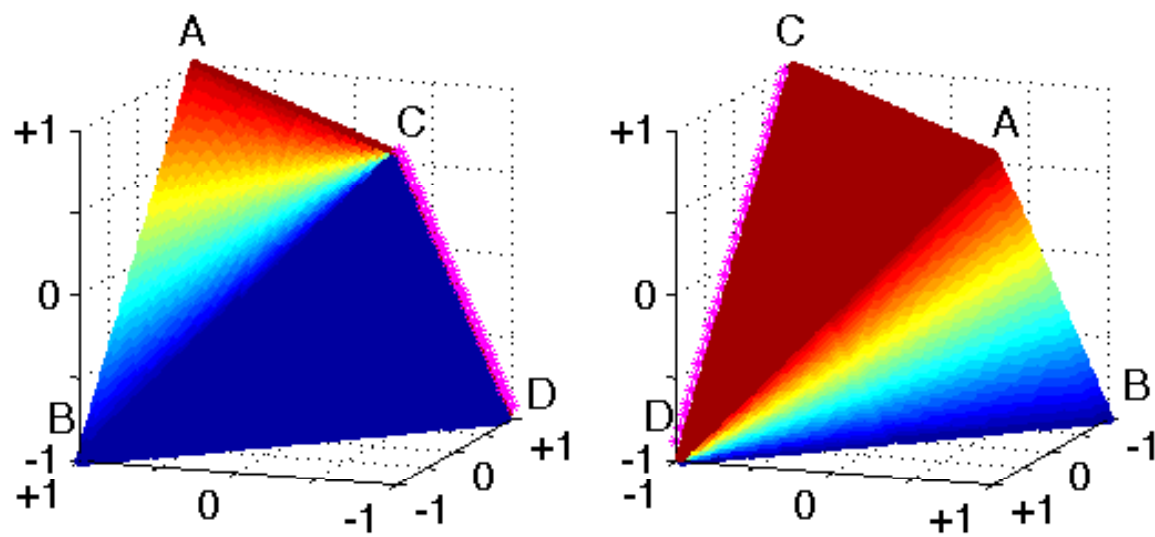




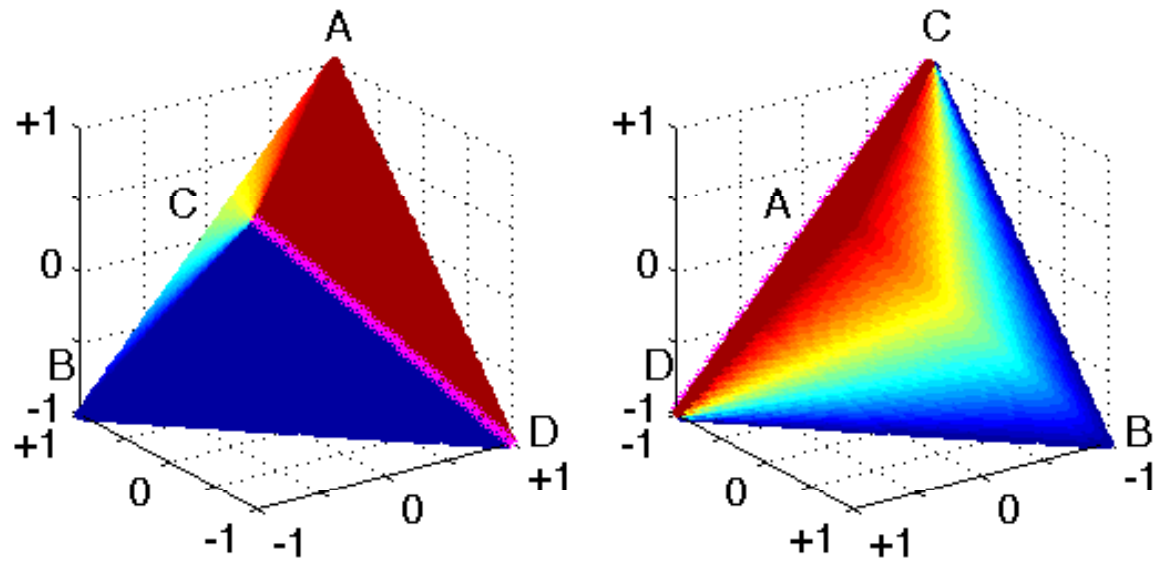
$$f(a,b,c,d) = (a+d)/(a+b+c+d) \text{ (accuracy)}$$



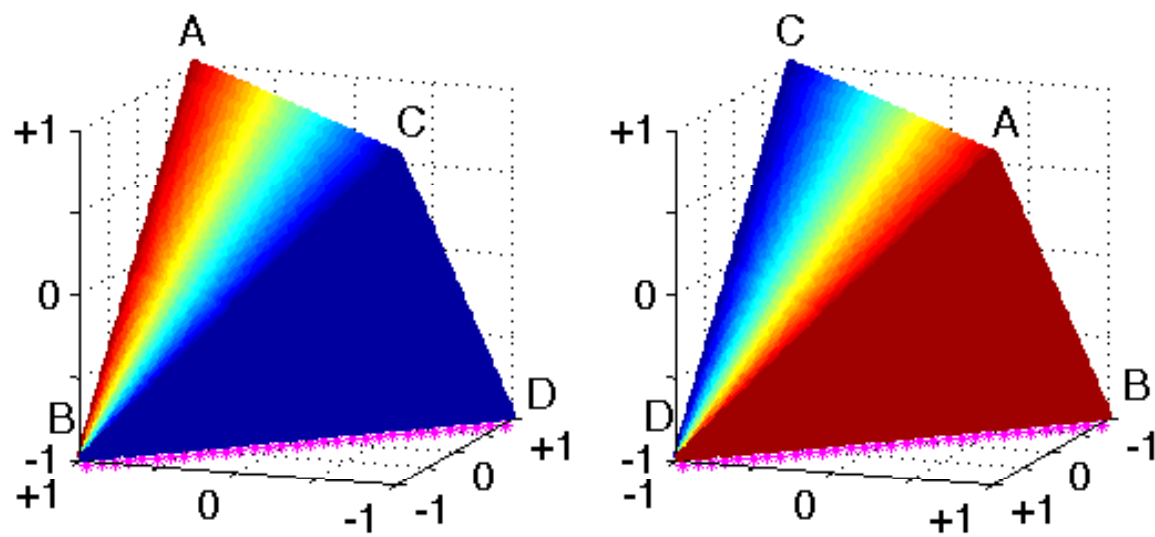
$$f(a,b,c,d) = \frac{a+d}{a+b+c+d} \text{ (accuracy)}$$



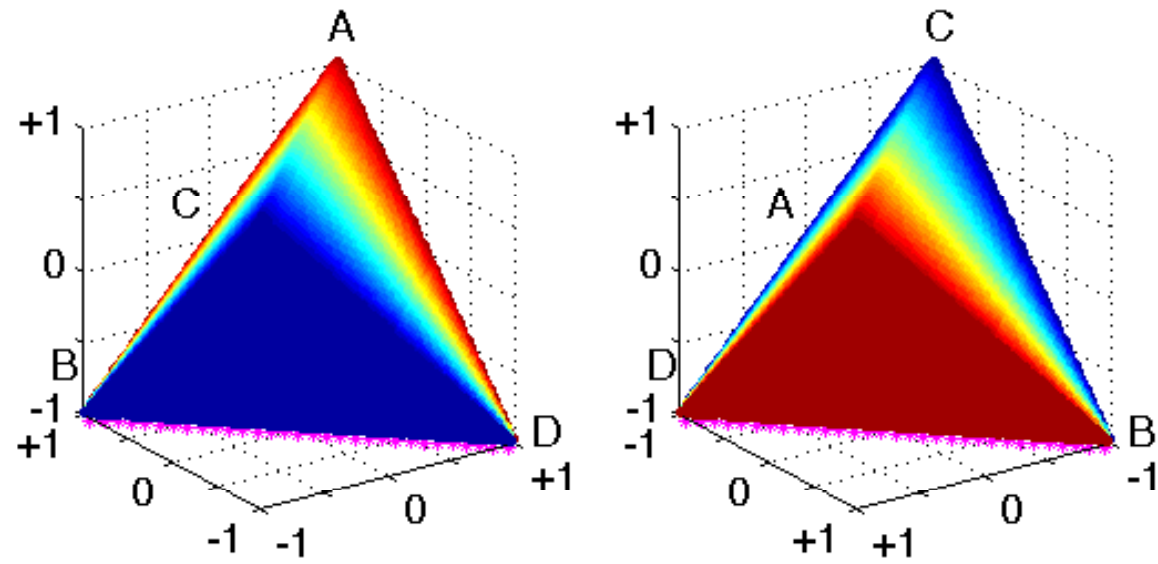
$$f(a,b,c,d) = a/(a+b) \text{ (precision)}$$



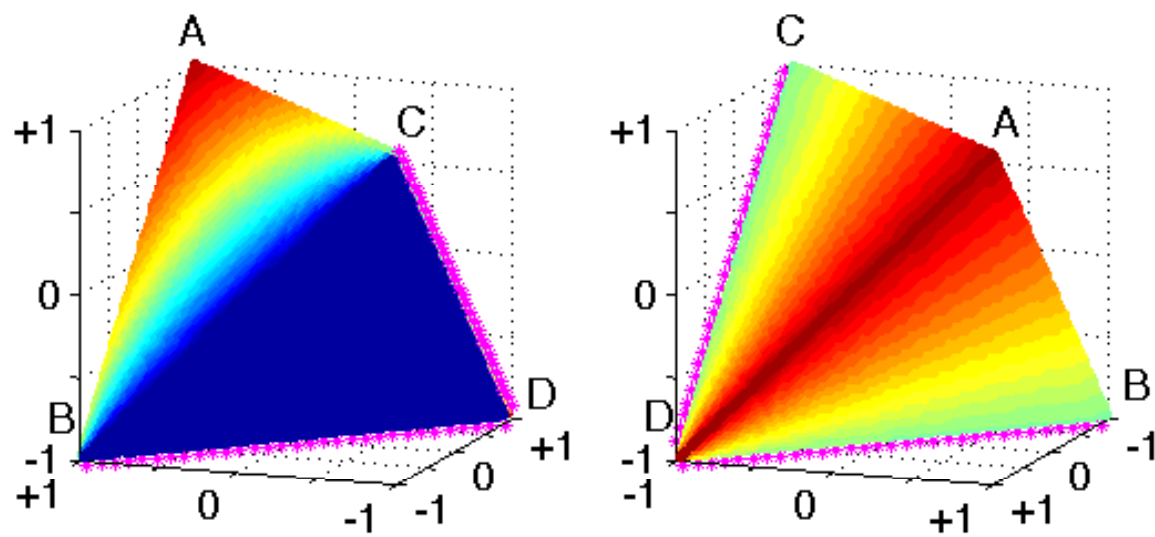
$$f(a,b,c,d) = a/(a+b) \text{ (precision)}$$



$$f(a,b,c,d) = a/(a+c) \text{ (recall)}$$

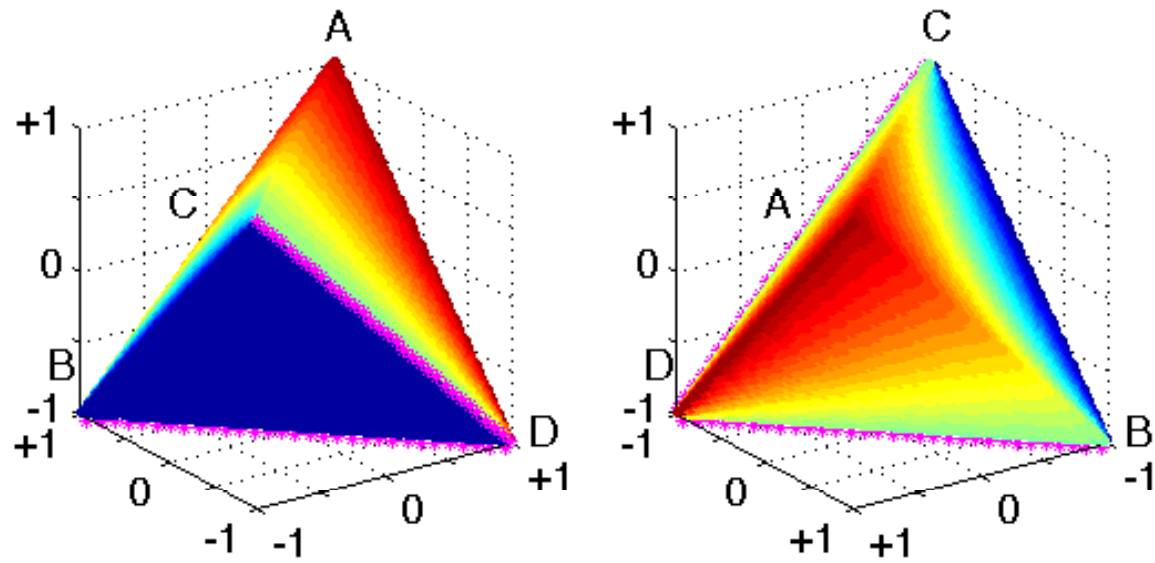


$$f(a,b,c,d) = a/(a+c) \text{ (recall)}$$

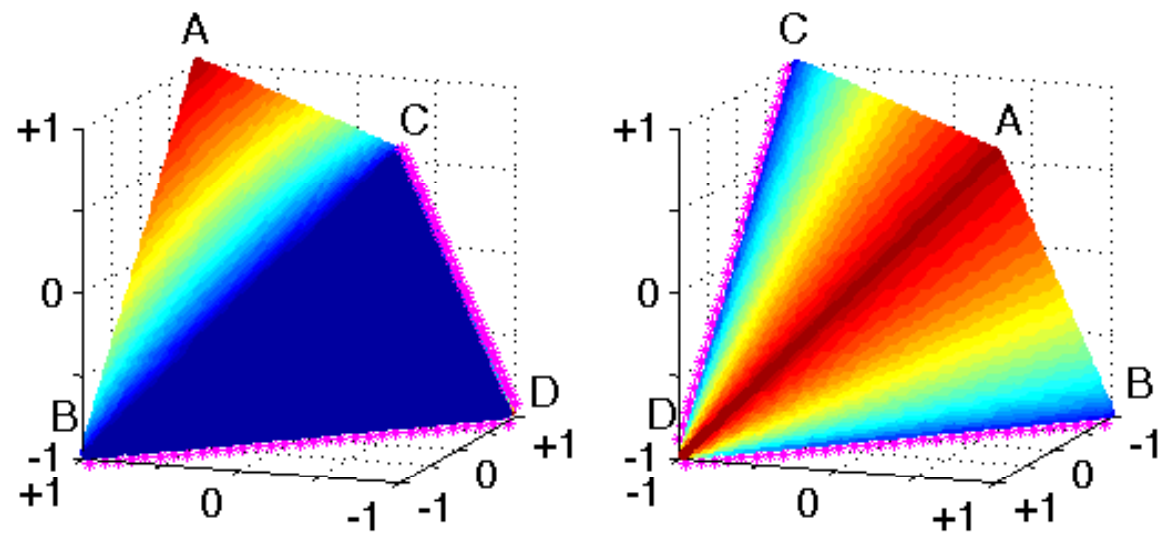


$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$

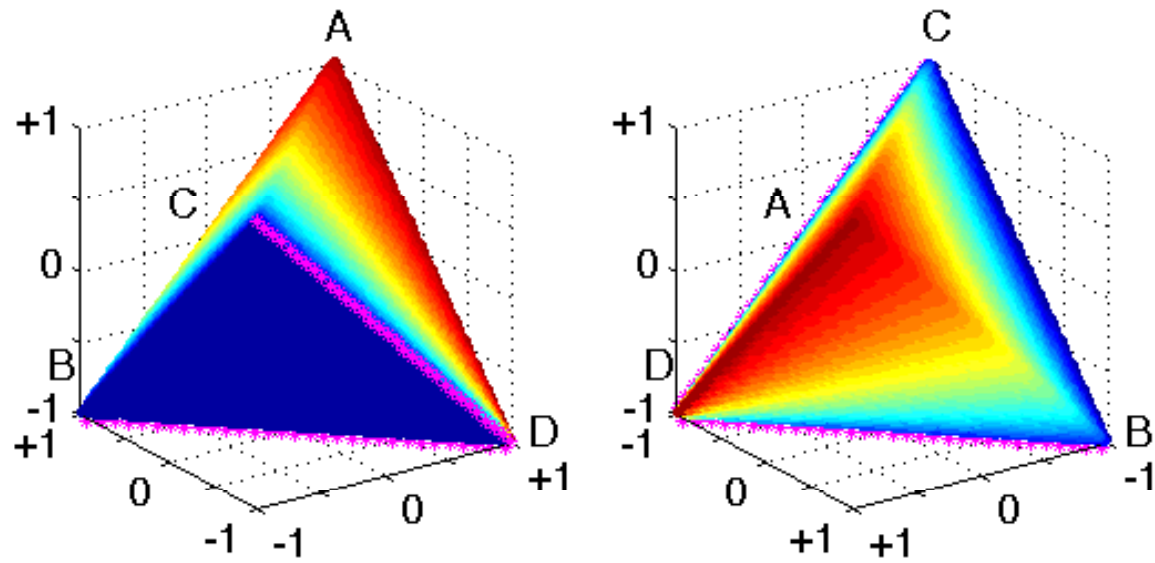




$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$



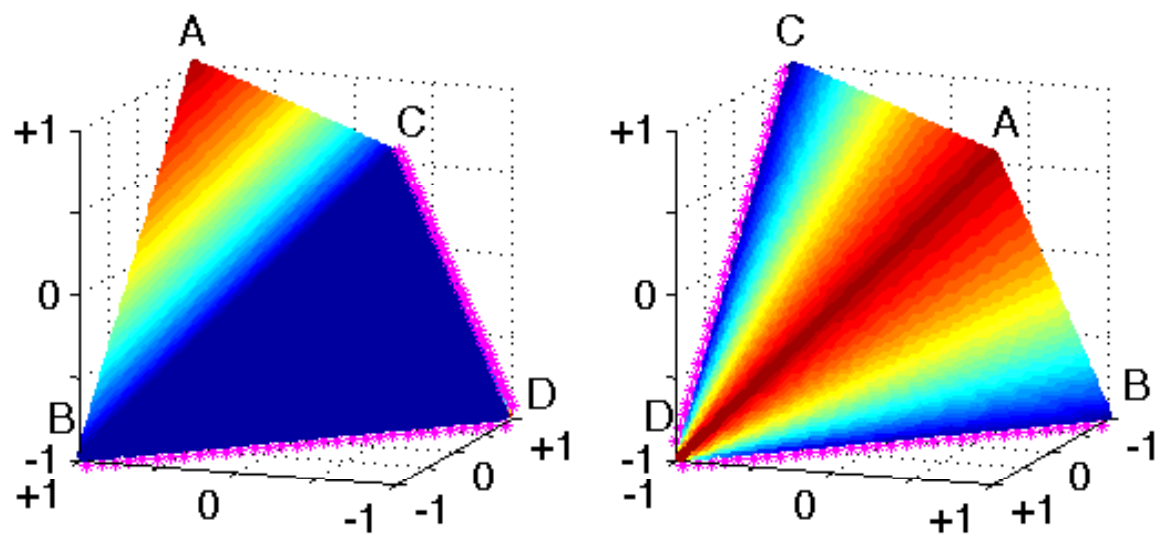
$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$



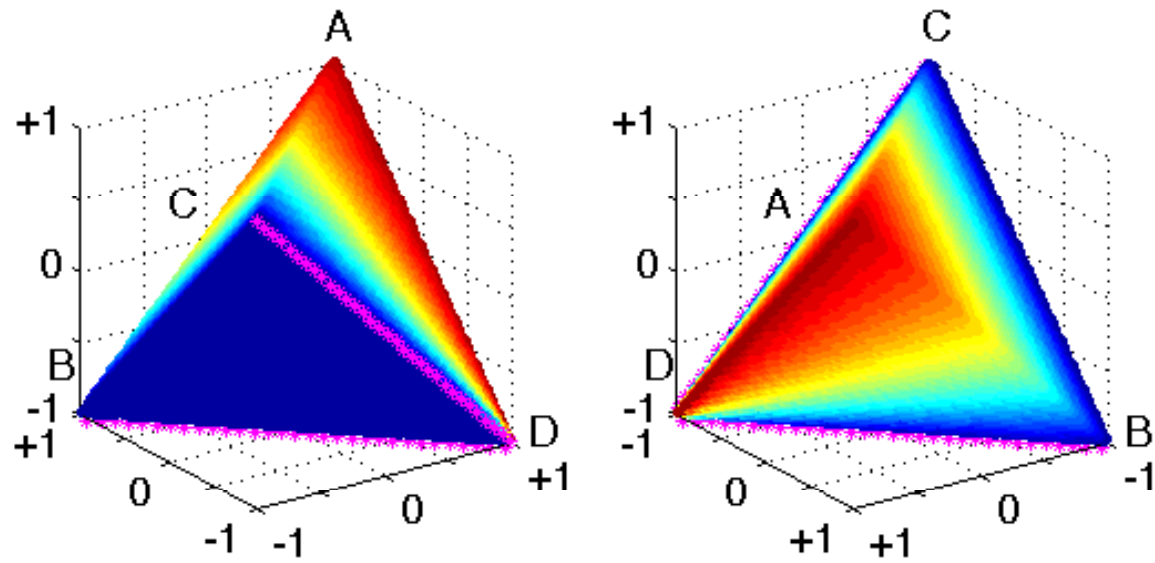
$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$

$\text{gmean}(\text{precision}, \text{recall}) = \text{G-measure}$

$$A \geq G \geq H$$



$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

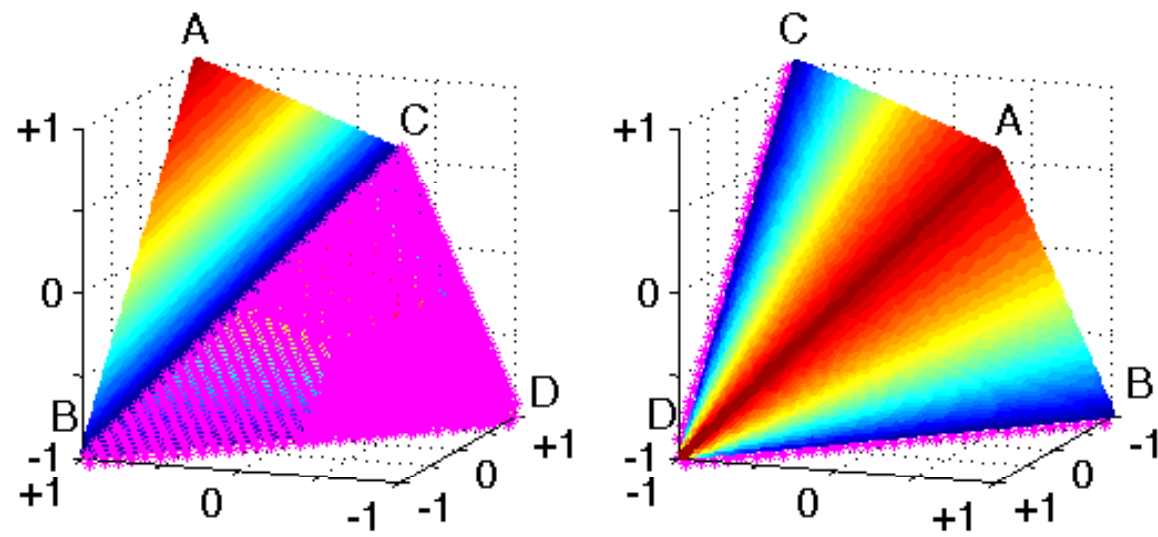


$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

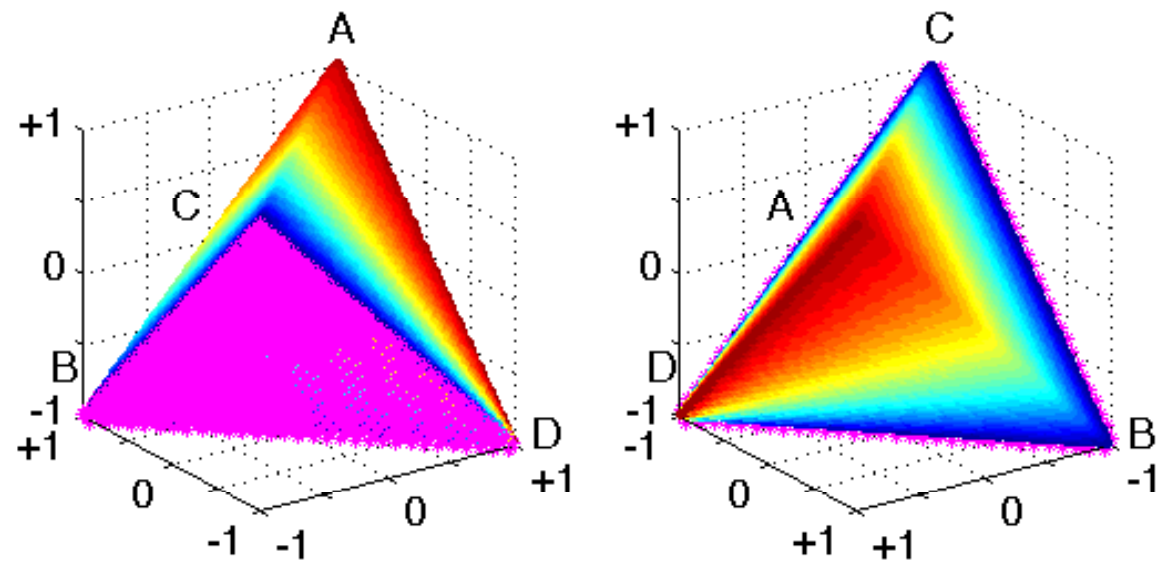
$\text{hmean}(\text{precision}, \text{recall}) = F_1$

– wyjątki: wartości specjalne



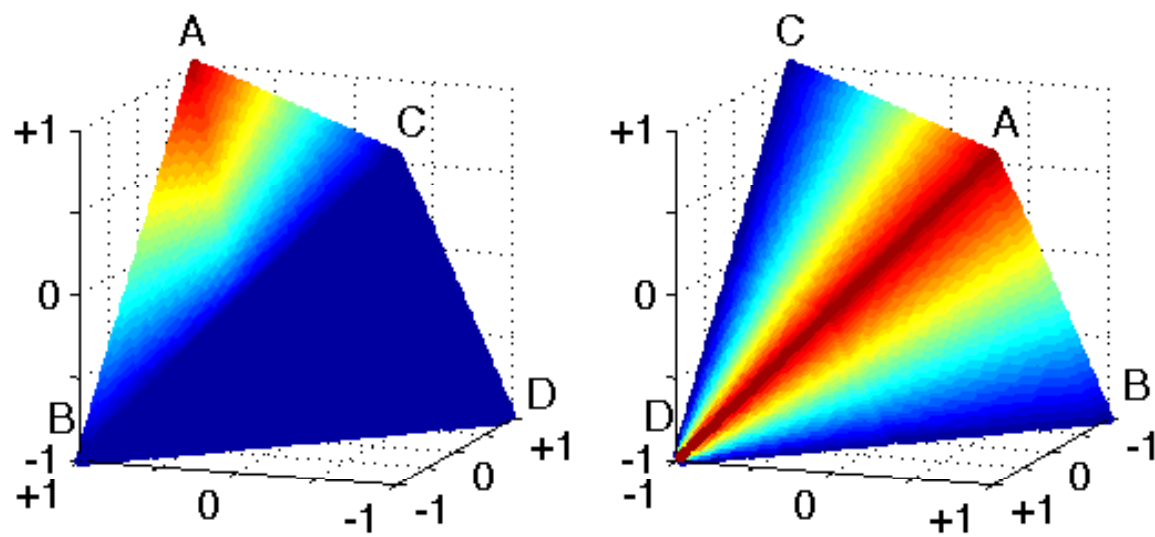


$$f(a,b,c,d) = F_1$$

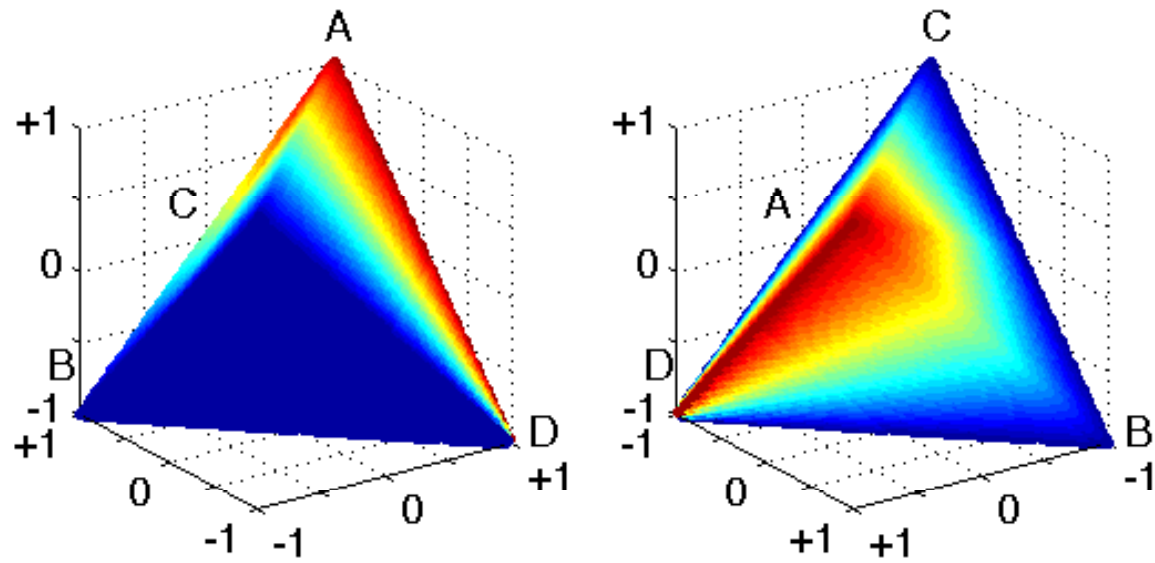


$$f(a,b,c,d) = F_1$$

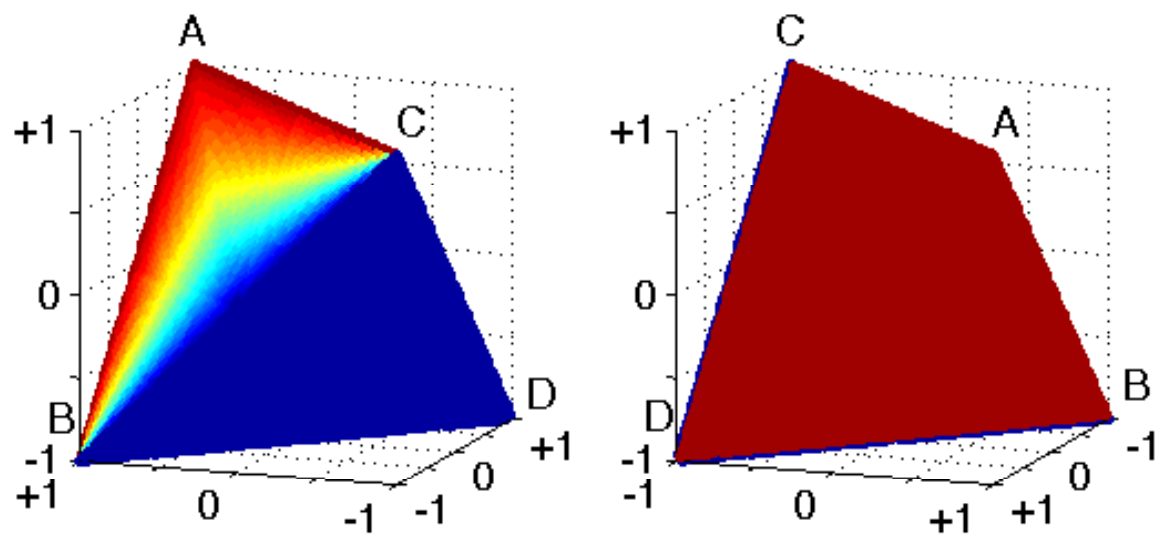
$\max \geq A \geq G \geq H \geq \min$



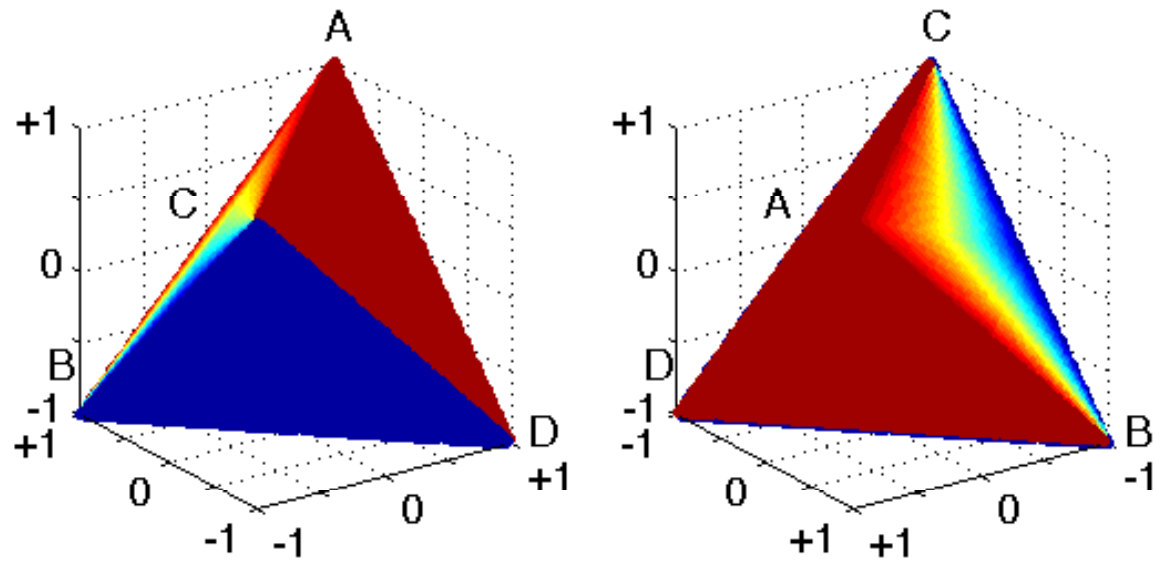
$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$



$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$



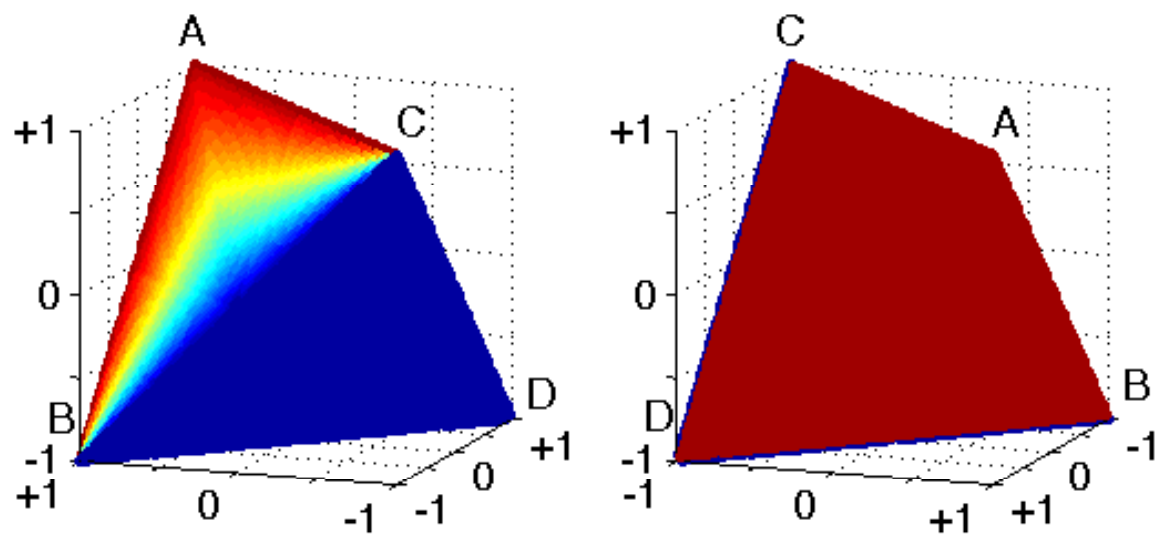
$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$



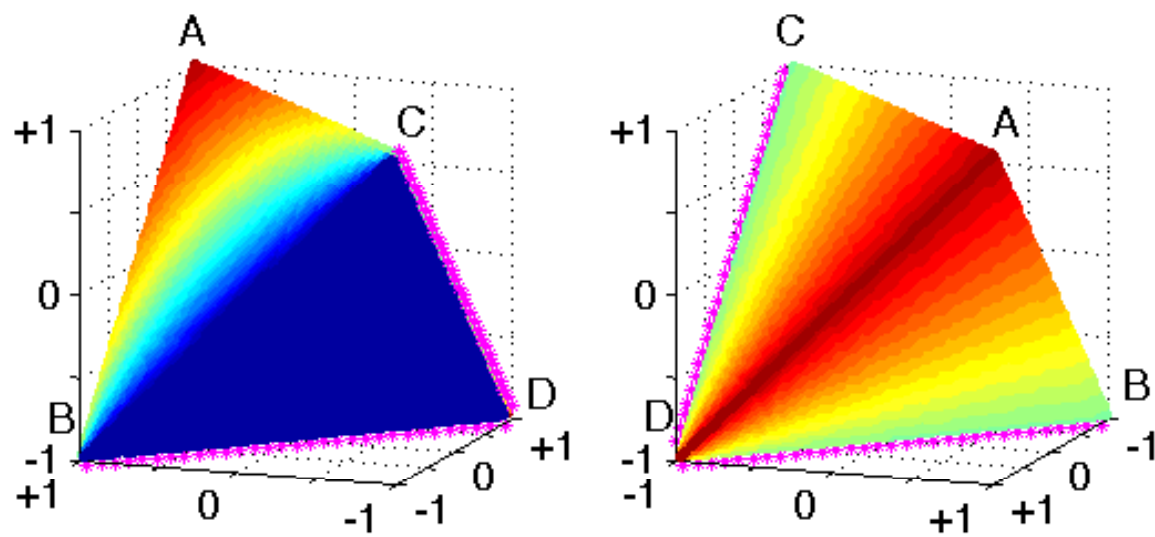
$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$

- sukcesja:  $\max \rightarrow A \rightarrow G \rightarrow H \rightarrow \min$ 
  - wizualizacja a c / b d (+ rewers)

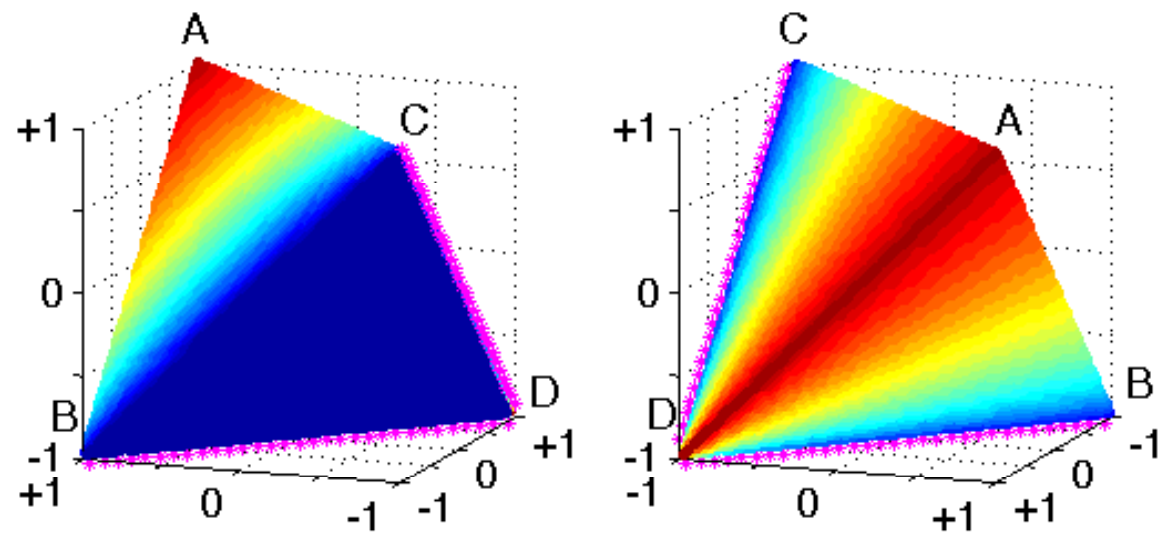




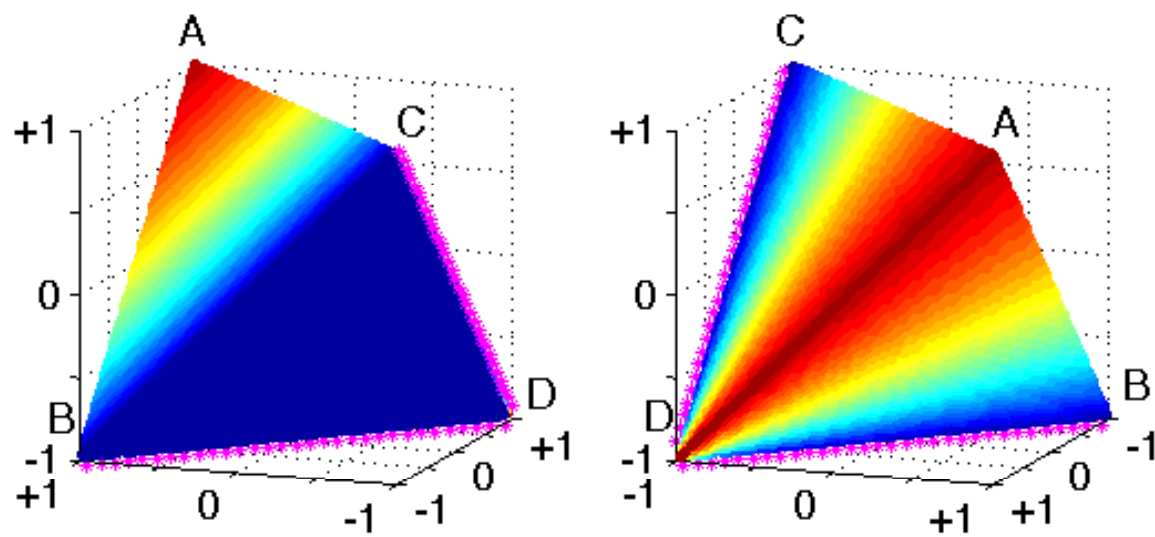
$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$



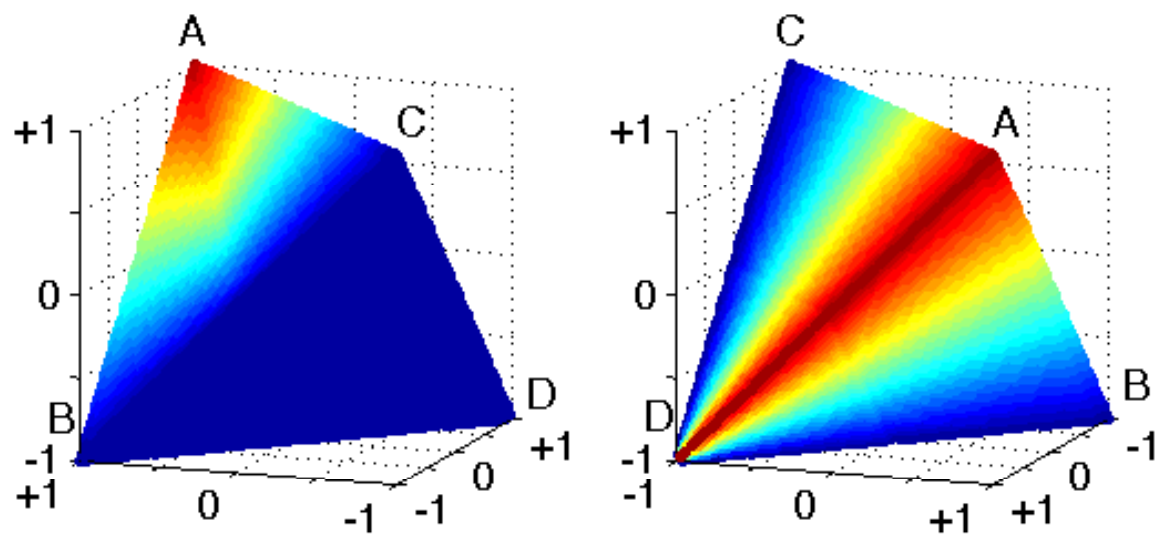
$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$



$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$

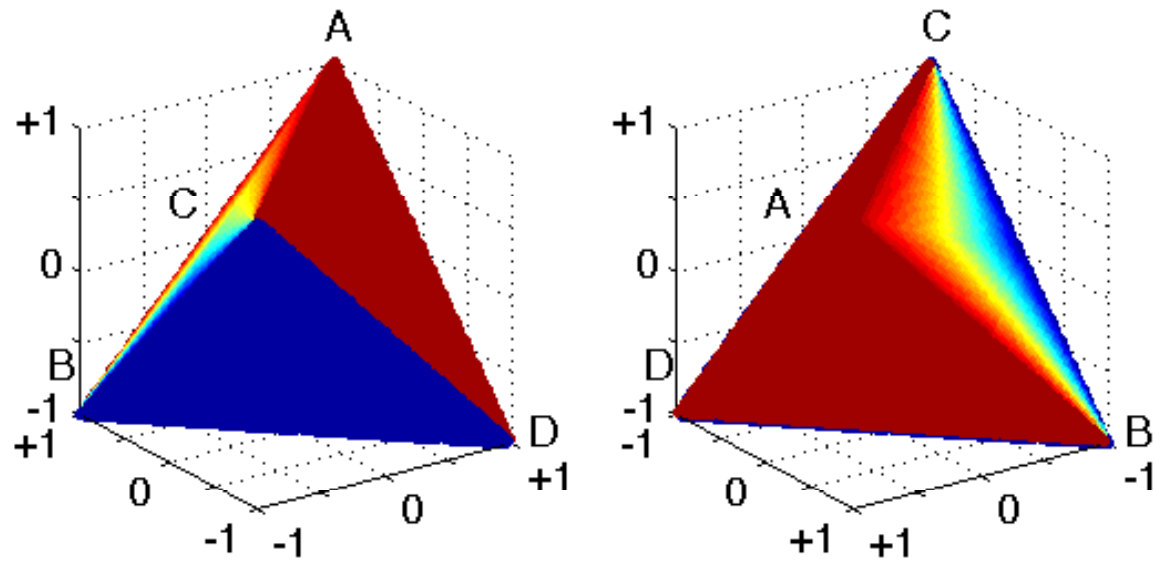


$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

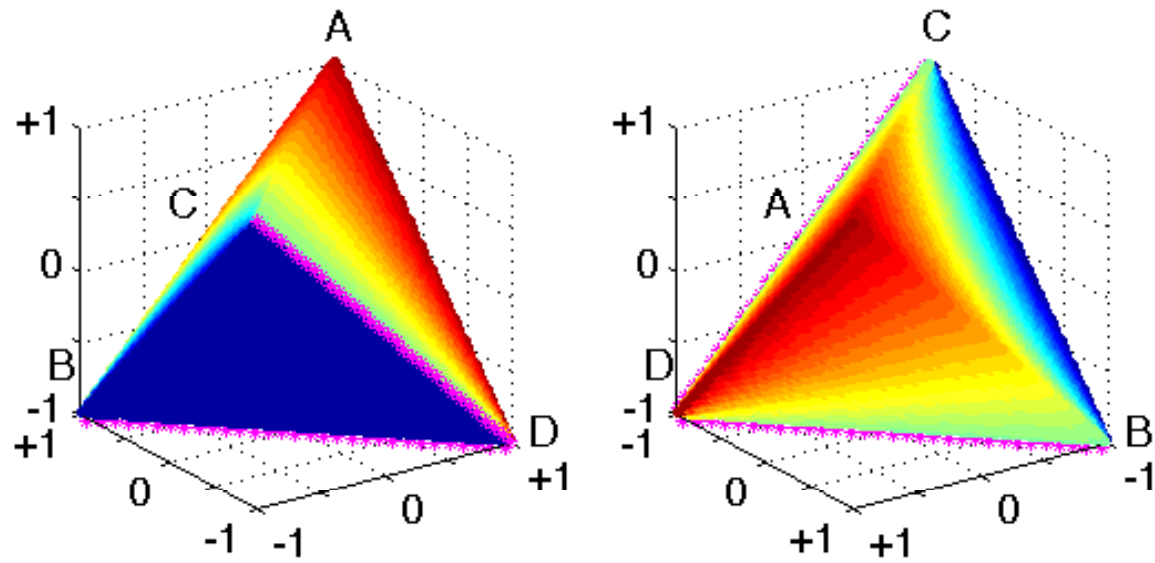


$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$

- sukcesja:  $\max \rightarrow A \rightarrow G \rightarrow H \rightarrow \min$ 
  - wizualizacja c a / b d (+ rewers)

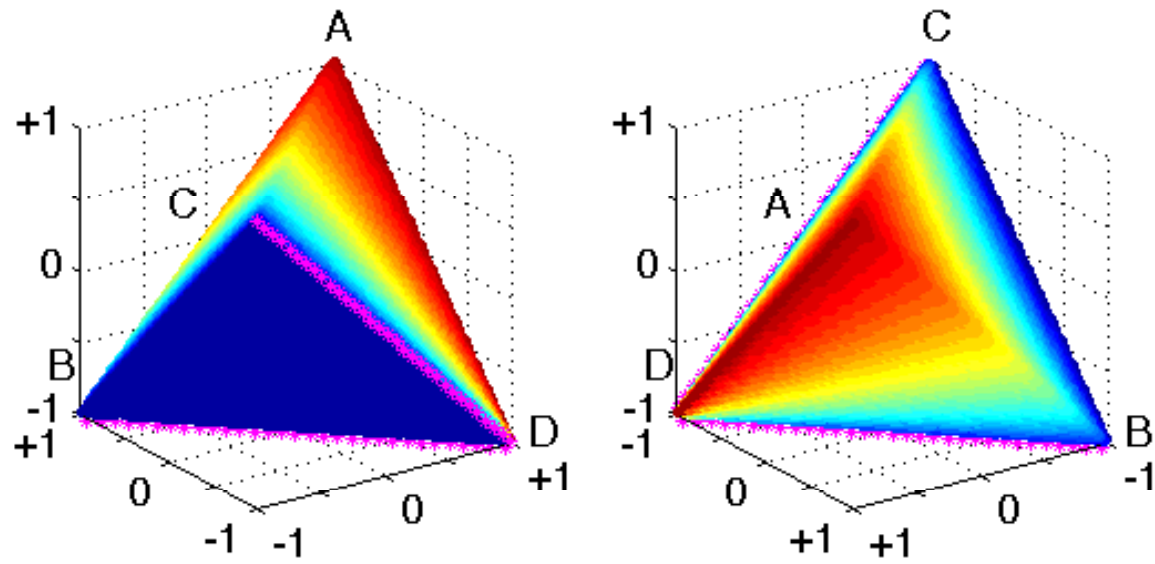


$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$

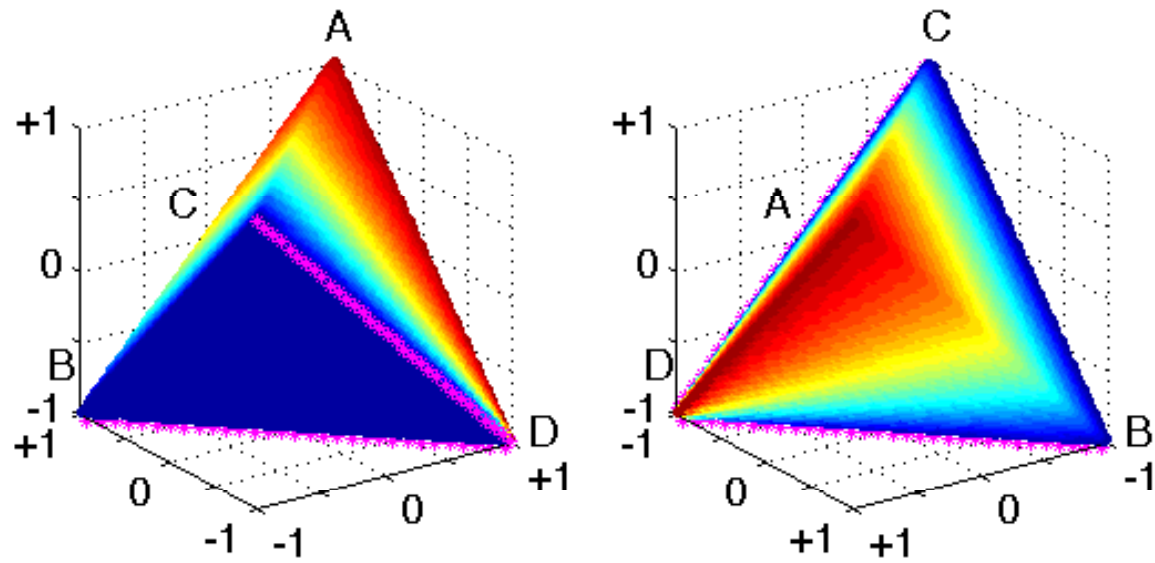


$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$

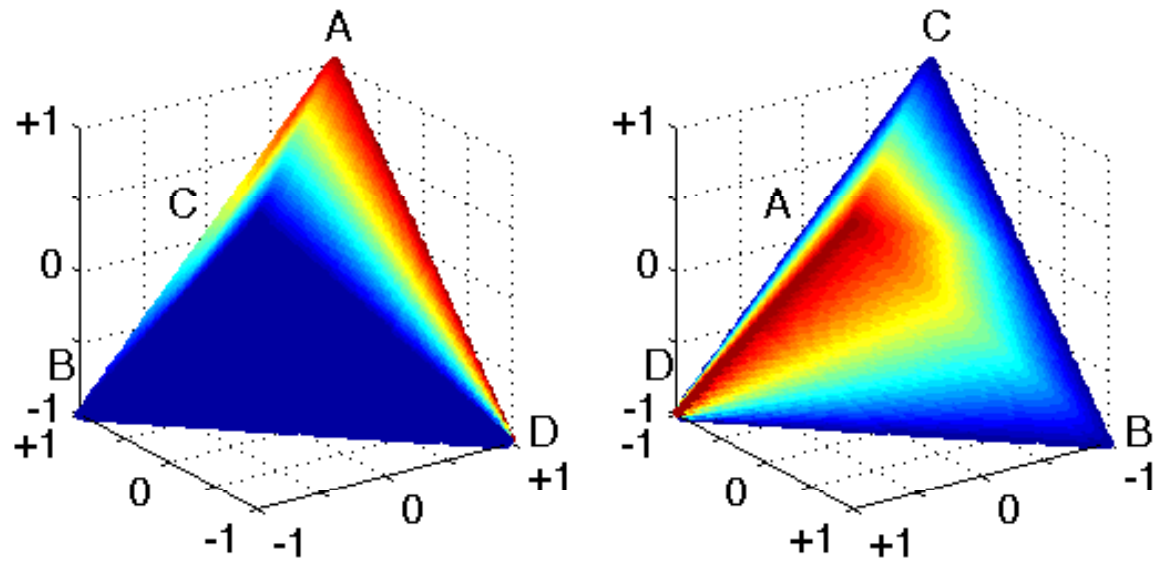




$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$

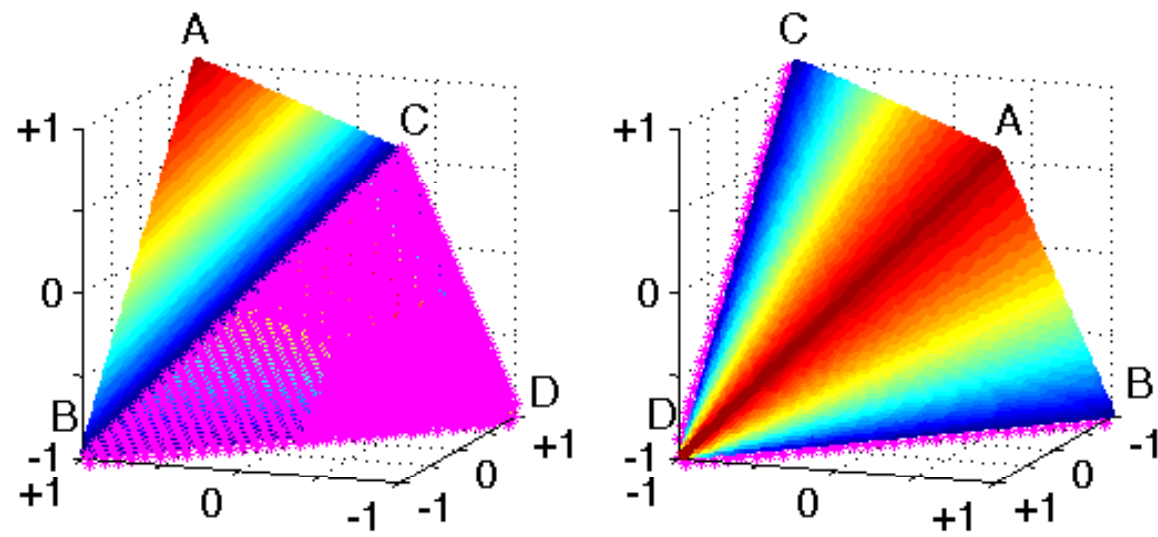


$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

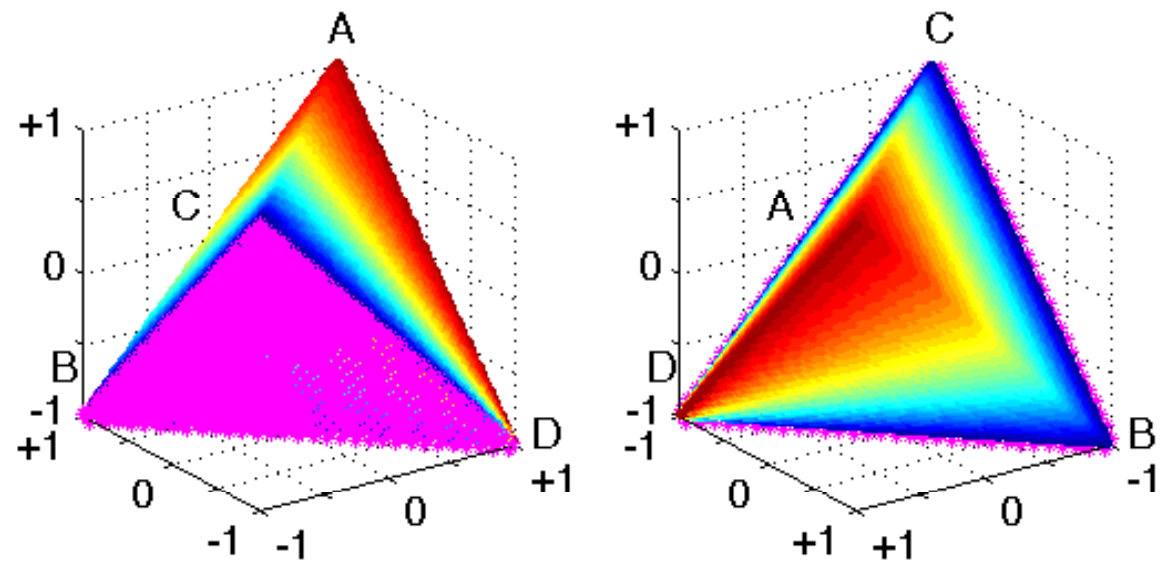


$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$

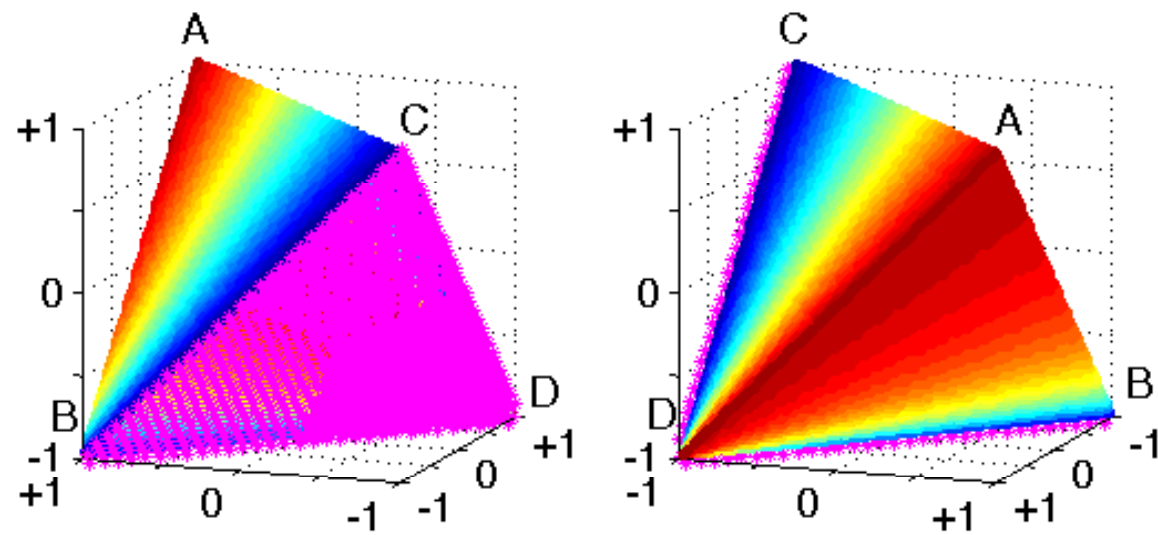
- Uogólnieniem  $F_1$  (hmean) jest  $F_\beta$  (dla  $\beta \neq 1$ )



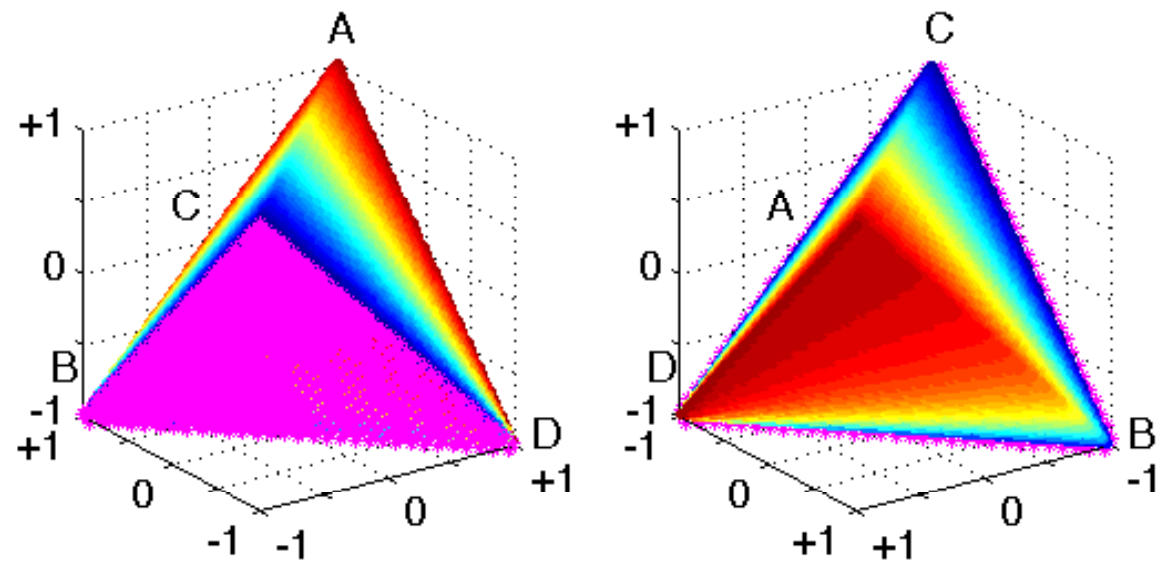
$$f(a,b,c,d) = F_1$$



$$f(a,b,c,d) = F_1$$

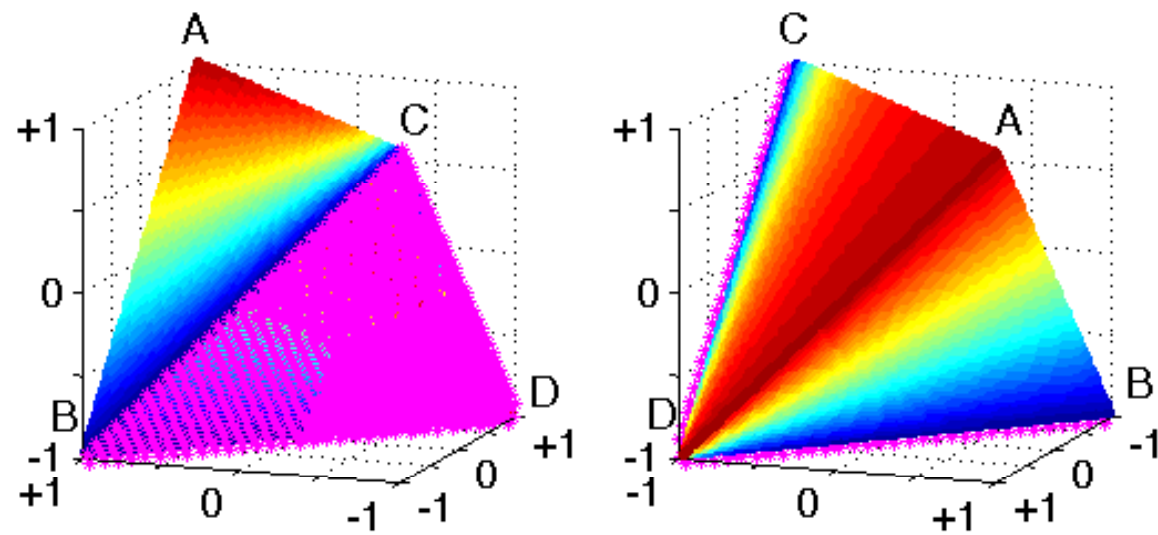


$$f(a,b,c,d) = F_3$$

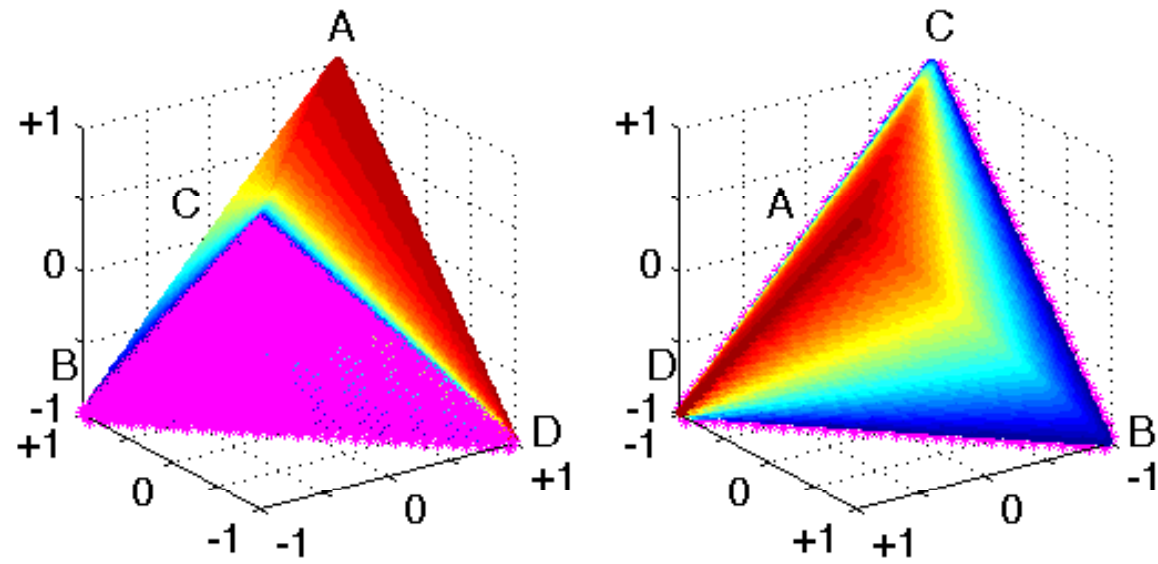


$$f(a,b,c,d) = F_3$$





$$f(a,b,c,d) = F_{1/3}$$

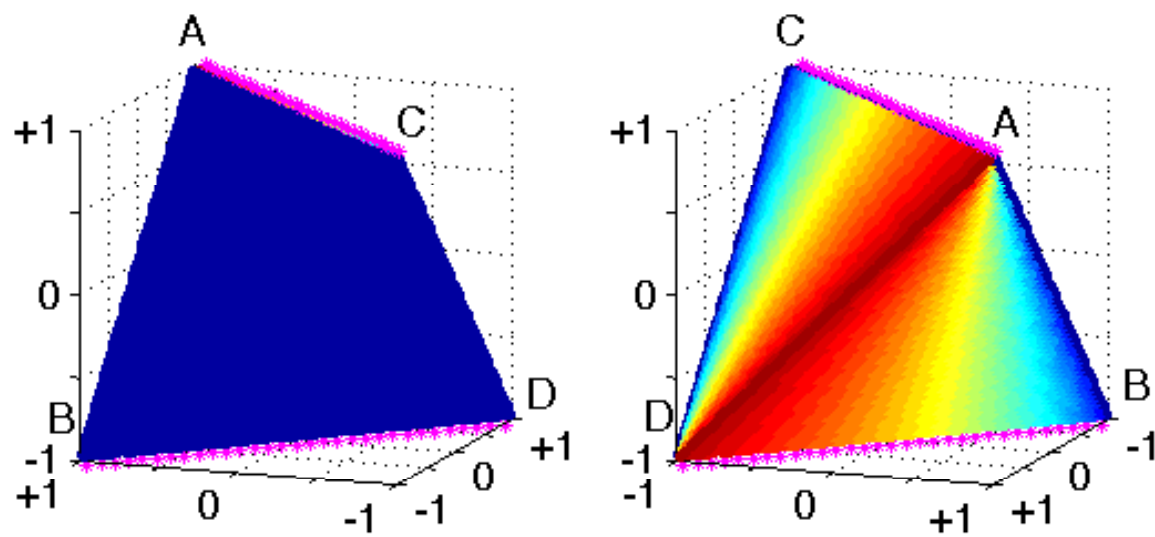


$$f(a,b,c,d) = F_{1/3}$$

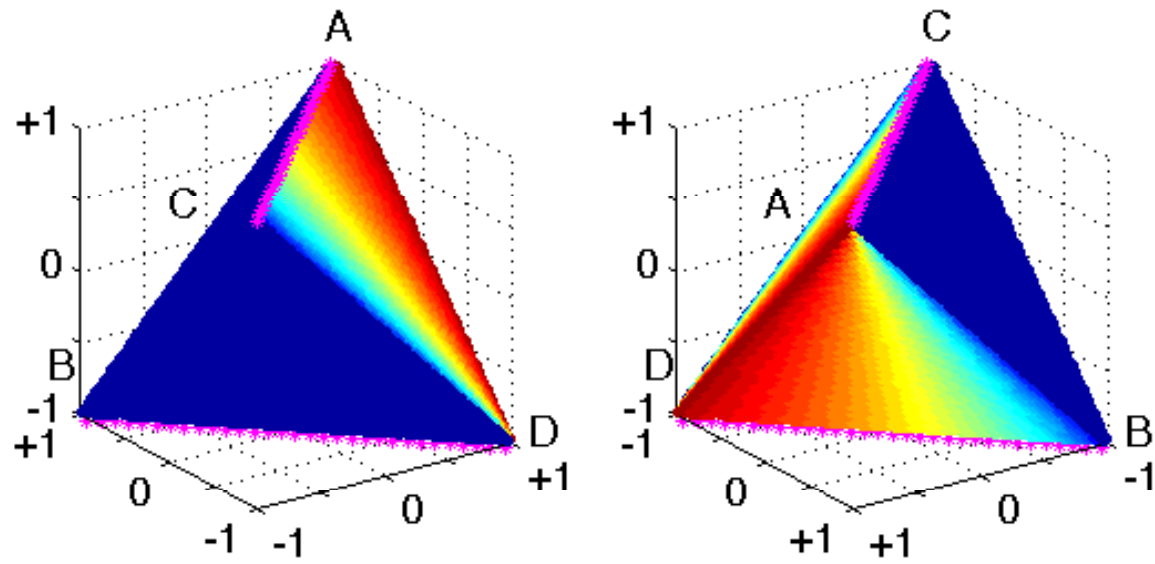
- G-mean:

- $\text{gmean}(\text{recall}, \text{specificity})$

- $\text{gmean}(TP/(TP+FN), TN/(TN+FP)) = \text{gmean}(a/(a+c), d/(d+b))$

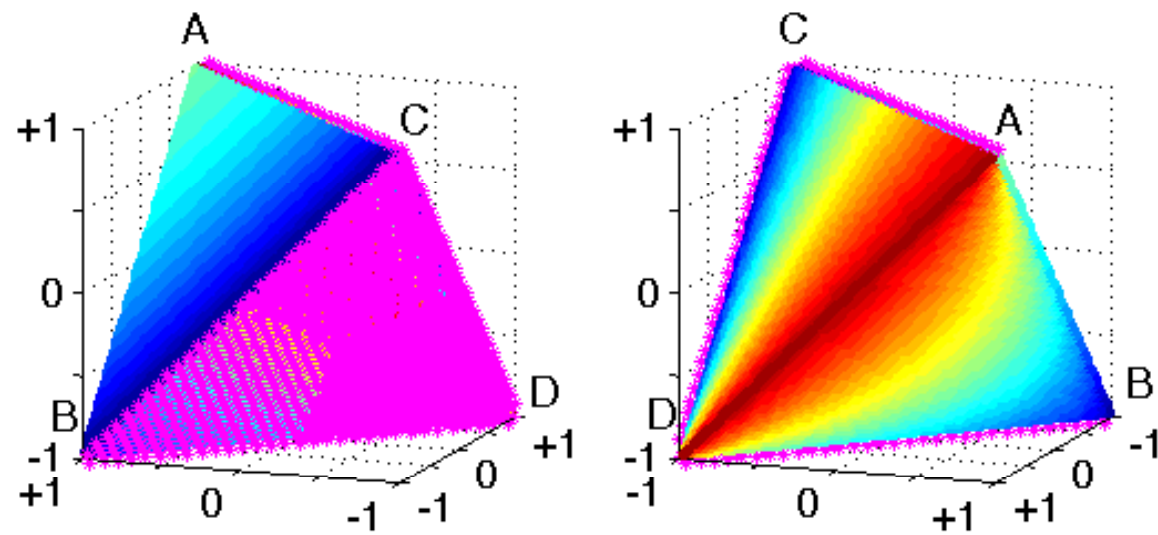


$$f(a,b,c,d) = \text{G-mean}$$

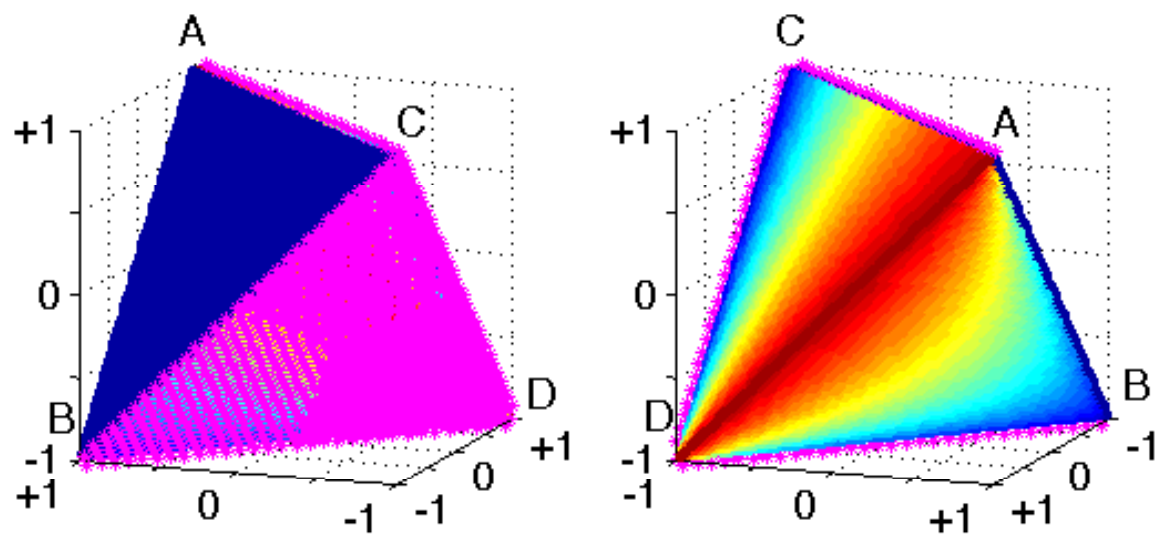


$$f(a,b,c,d) = \text{G-mean}$$

- kombinacije  $F_\beta$  i G-mean?

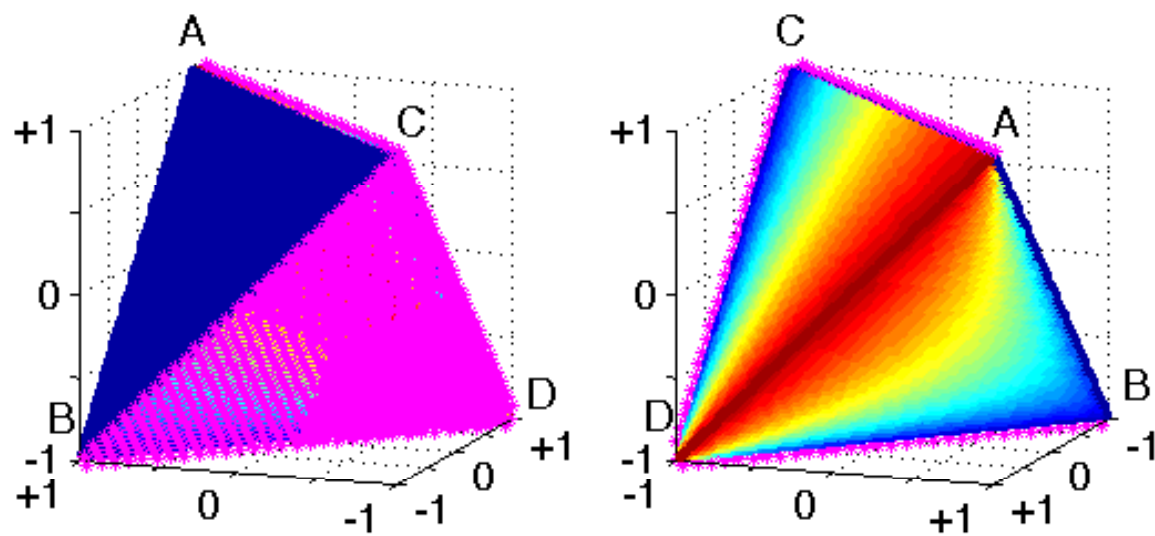


$$f(a,b,c,d) = \text{amean}(F_\beta, \text{G-mean})$$

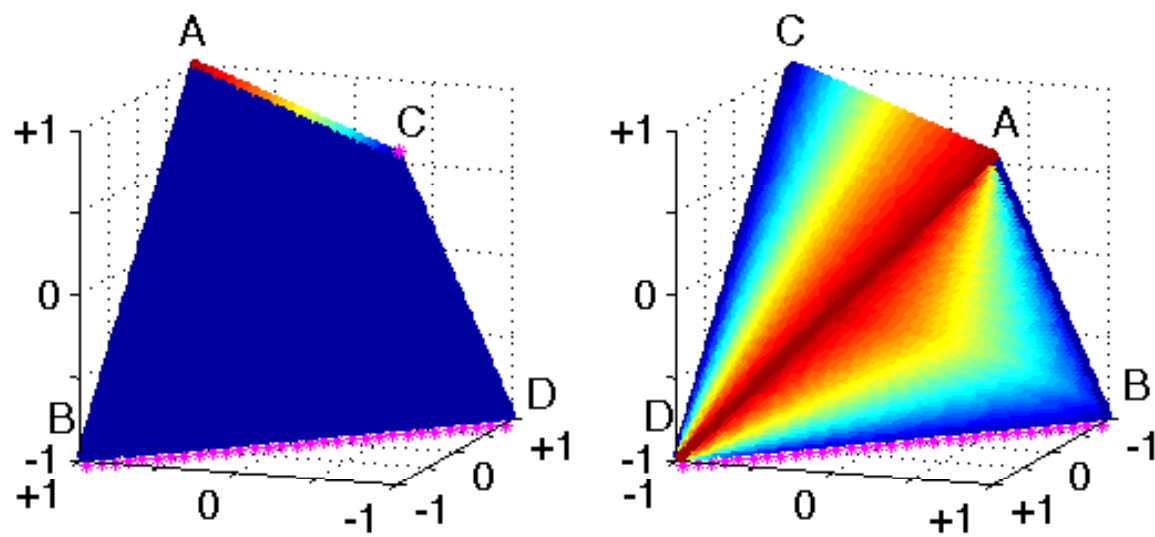


$$f(a,b,c,d) = \text{gmean}(F_{\beta}, \text{G-mean})$$

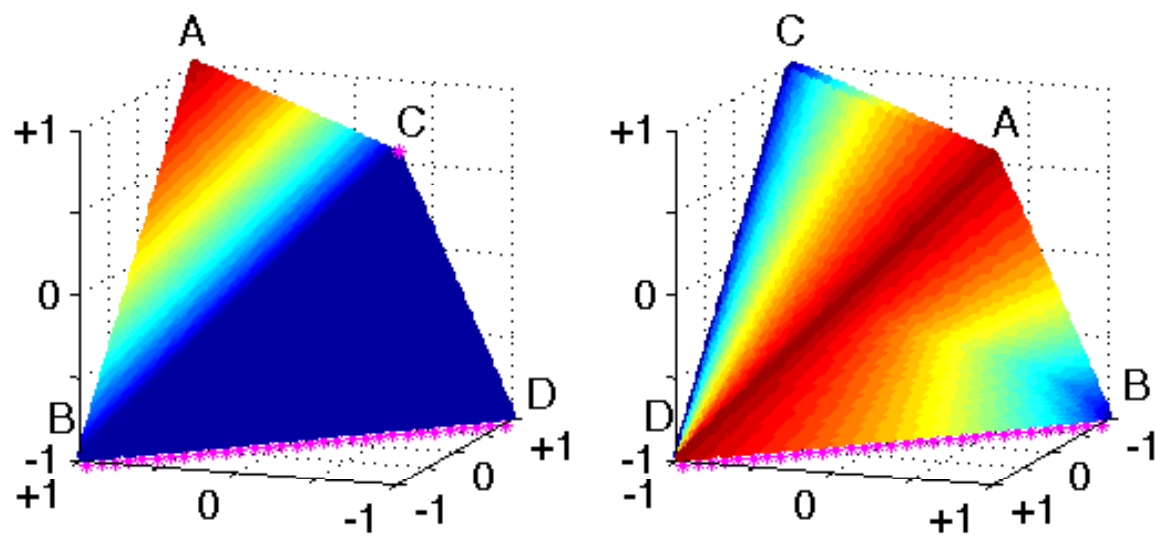




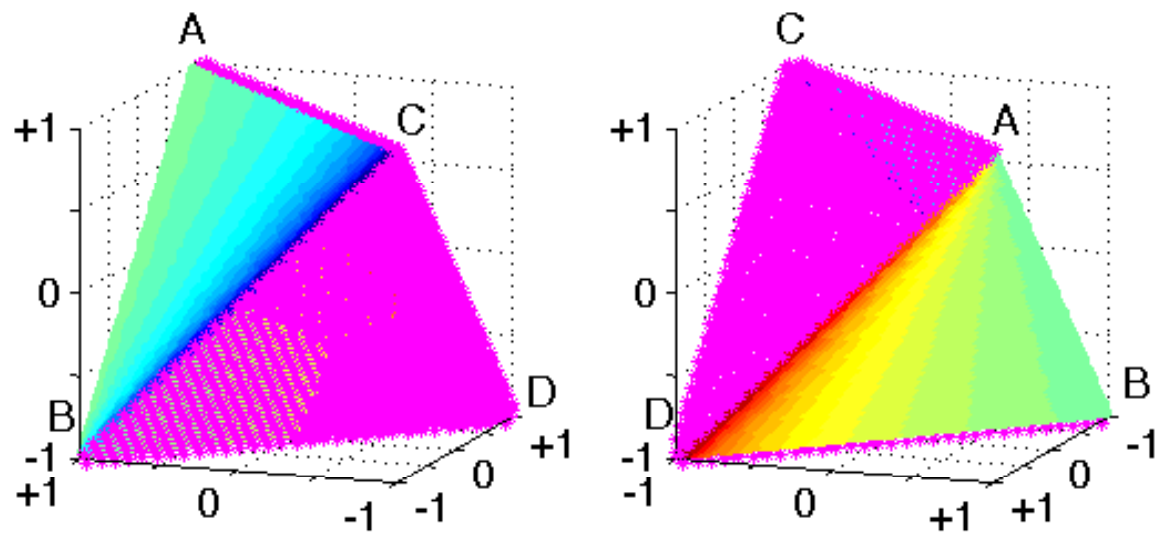
$$f(a,b,c,d) = \text{hmean}(F_\beta, \text{G-mean})$$



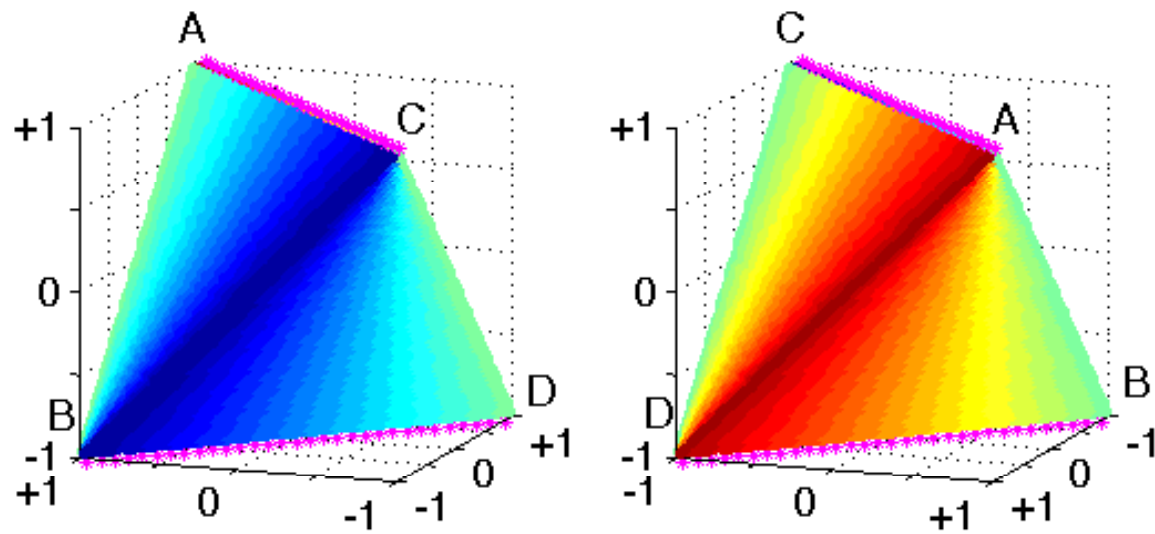
$$f(a,b,c,d) = \min(F_\beta, \text{G-mean})$$



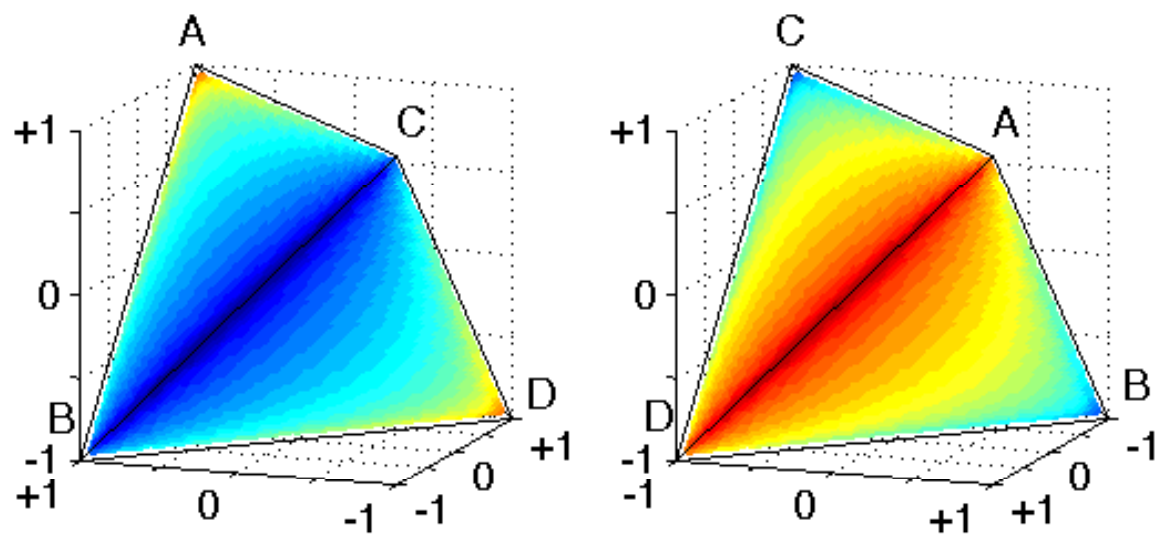
$$f(a,b,c,d) = \max(F_\beta, G\text{-mean})$$



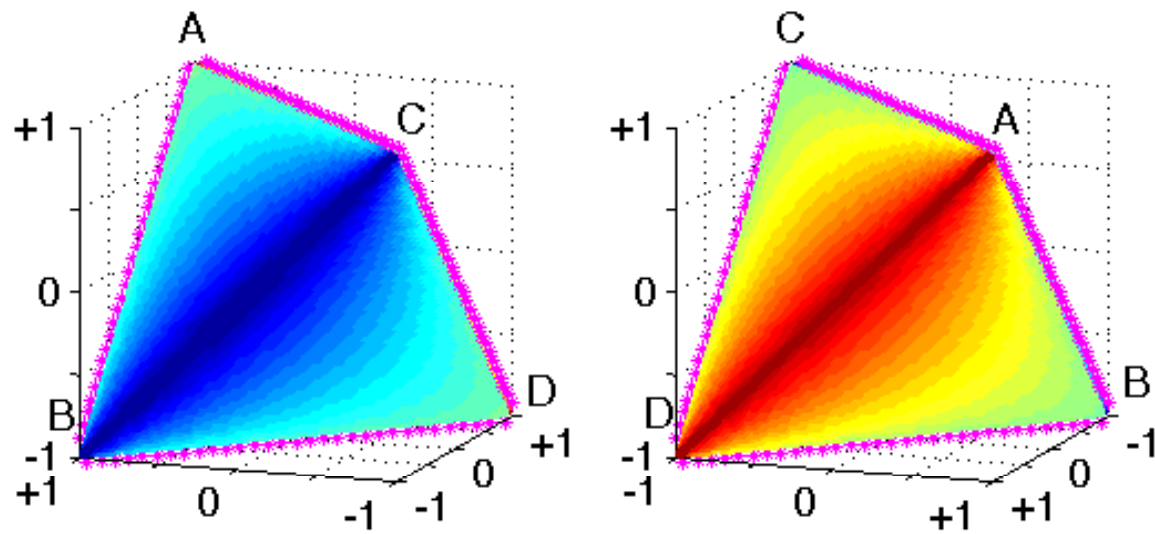
$f(a,b,c,d) = \text{Positive likelihood ratio}$



$f(a,b,c,d) = \text{Balanced accuracy}$



$f(a,b,c,d) = \text{Discriminant power}$



$$f(a,b,c,d) = \text{MCC}$$

...



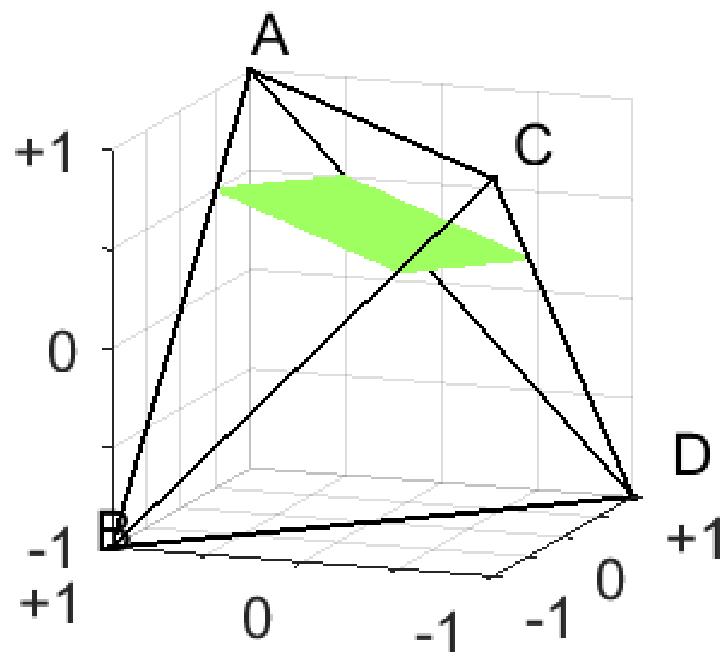
		predicted	
		1	0
original	1	TP <b>A</b> a	FN <b>C</b> c
	0	FP <b>B</b> b	TN <b>D</b> d

- Cel rozważań: relacje pomiędzy licznosciami klas

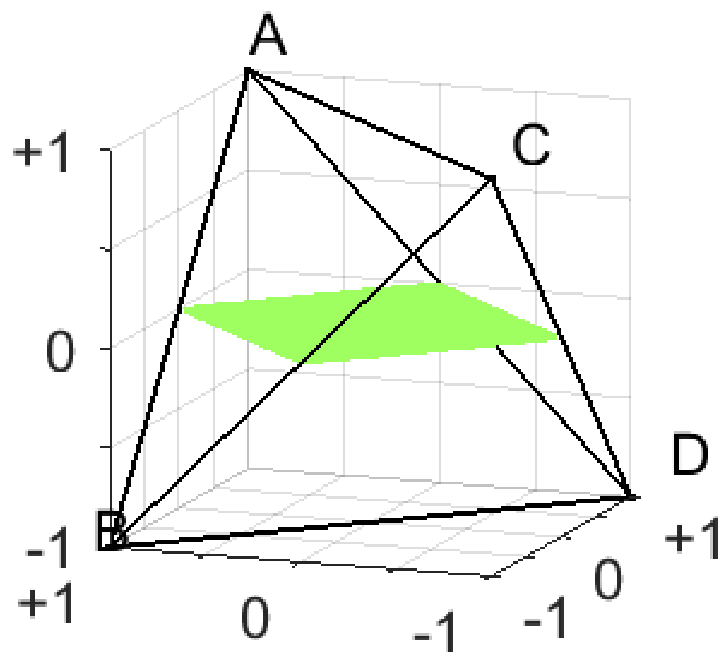
- liczność:
  - klasy 1:  $a+c$
  - klasy 0:  $b+d$

- wizualizacja ustalonej proporcji klas, np.  $p : q$   
(przy założeniu, że  $a+b+c+d = 1$ )
  - przecięcie czworościanu z poziomą płaszczyzną  
(przebiegającą na poziomie wyznaczonym przez  $p$  i  $q$ )

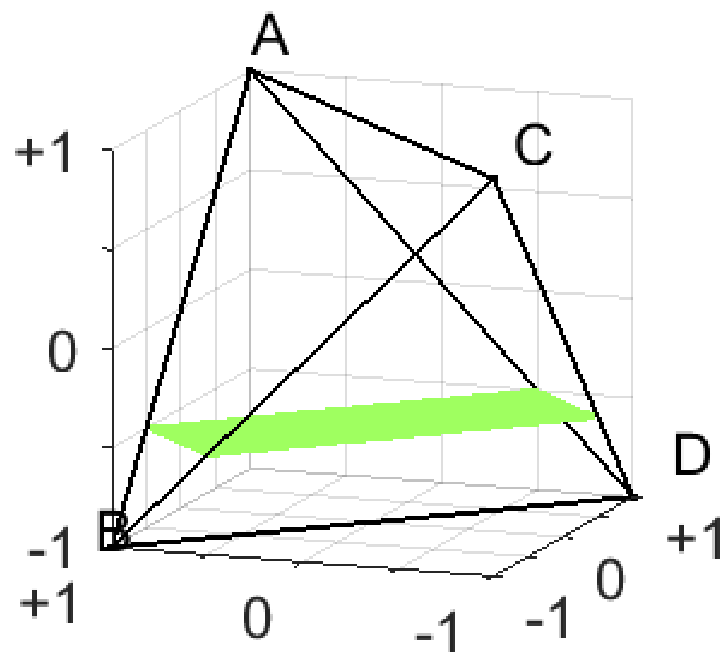
# przewaga klasy 1



# równowaga klas



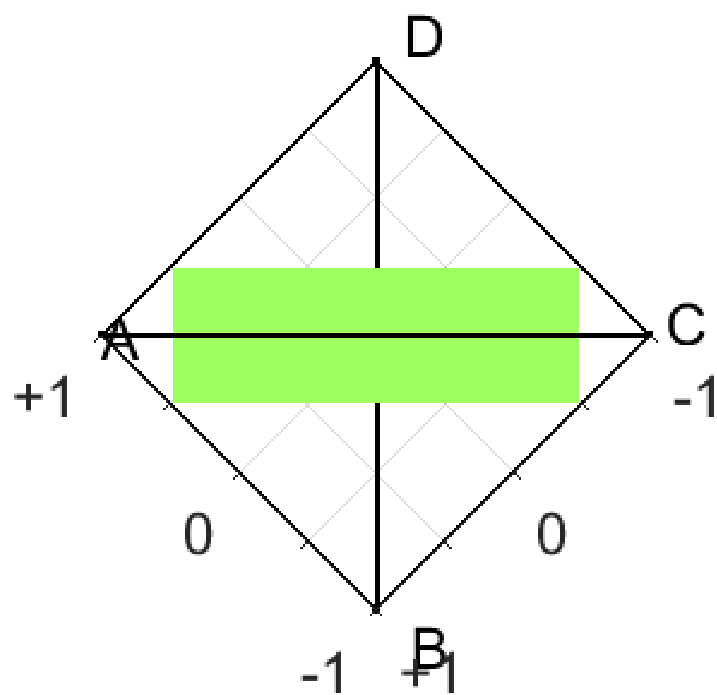
# przewaga klasy 0



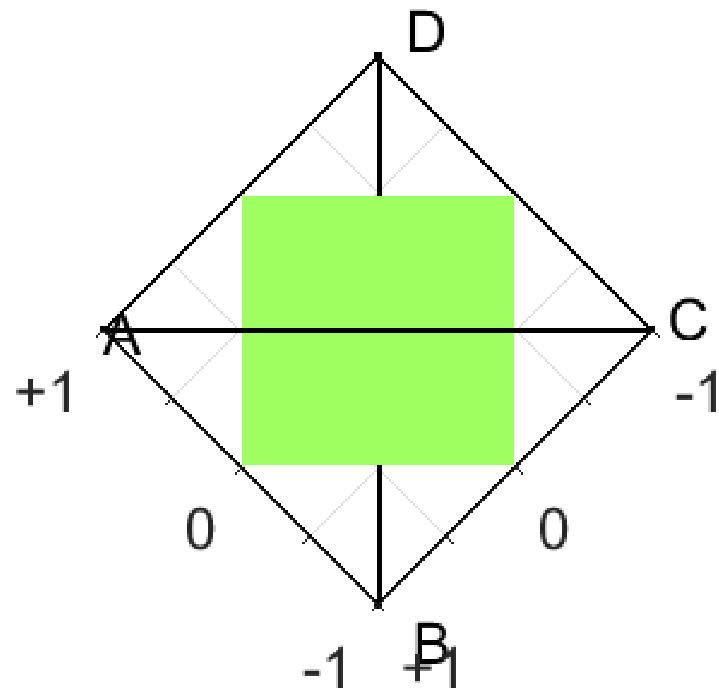
- dla ułatwienia: widok z góry  
(figura dodatkowo lekko obrócona w lewo)



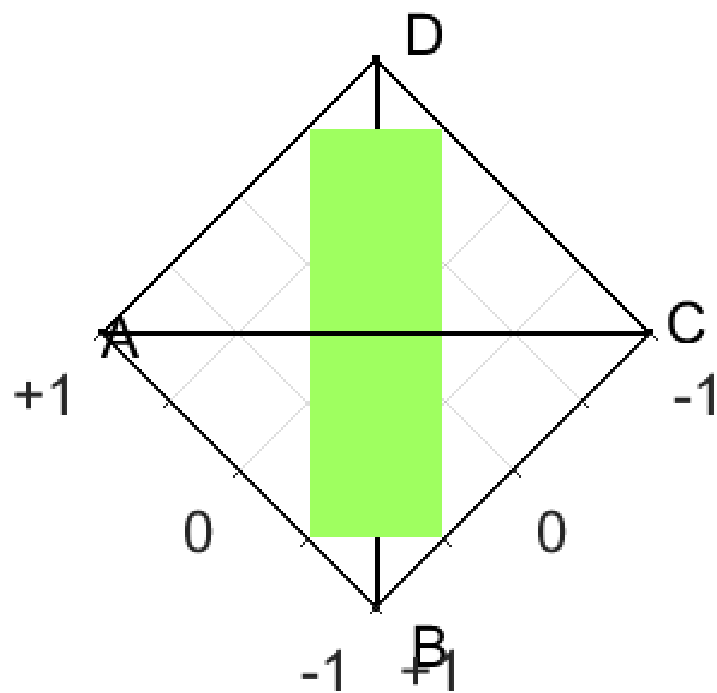
# przewaga klasy 1



# równowaga klas

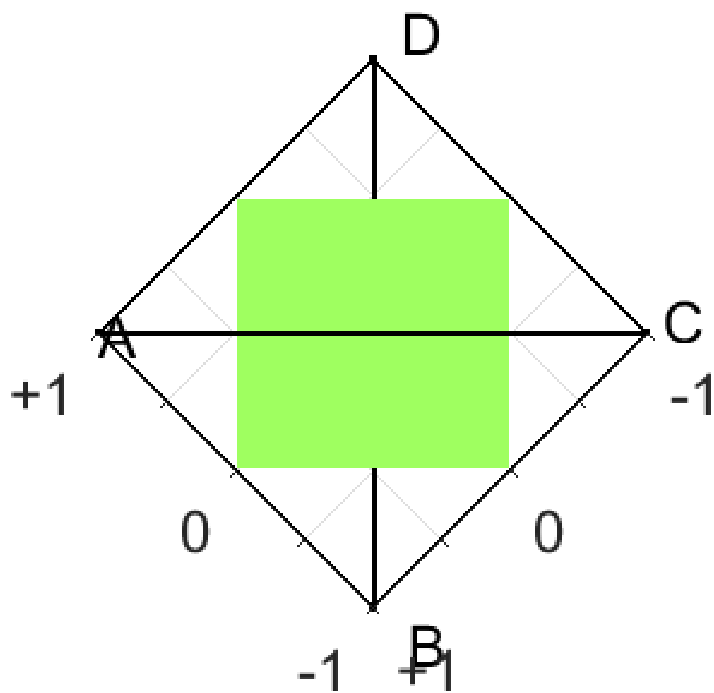


przewaga klasy 0



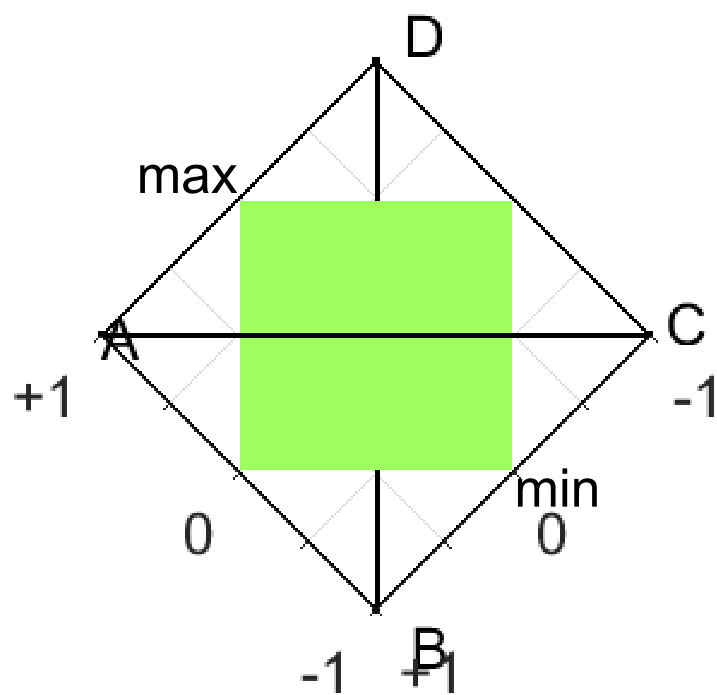
- na początek
  - równość klas:  $a+c = b+d$

# równowaga klas



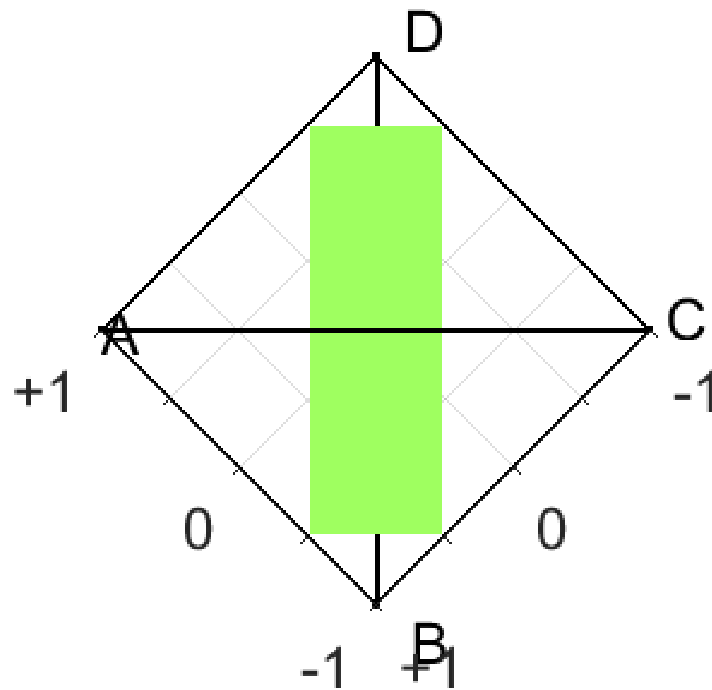
pożądany przebieg/wygląd miar?

# równowaga klas

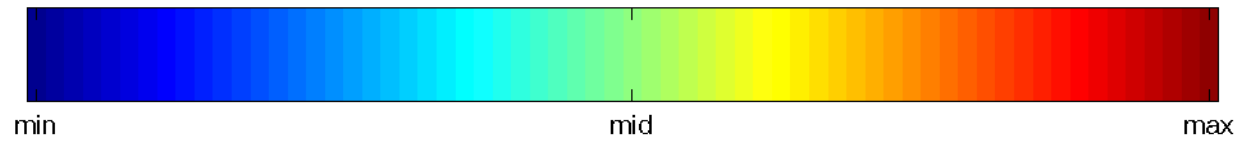


pożądany przebieg/wygląd miar

przewaga klasy 0



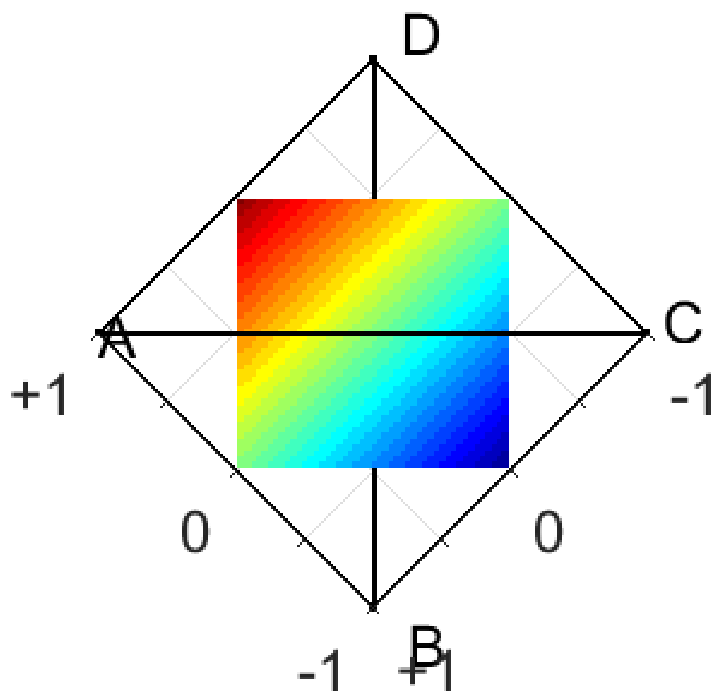
a teraz: pożądaný przebieg/wygląd miar???





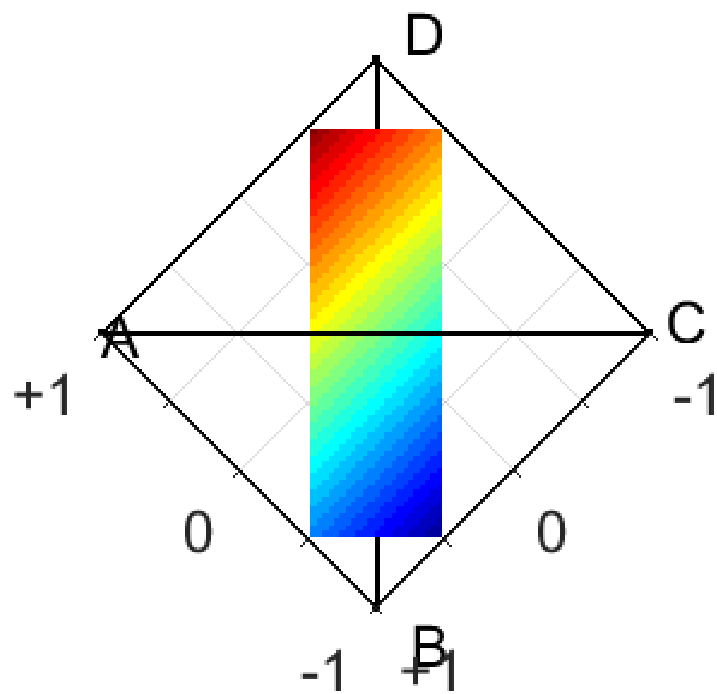
- miara na początek:
  - CA

# równowaga klas



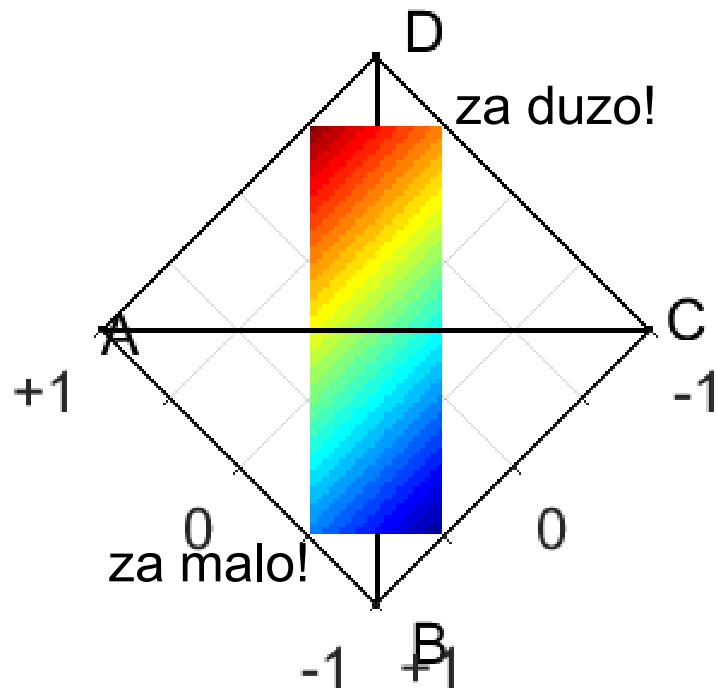
CA

# przewaga klasy 0



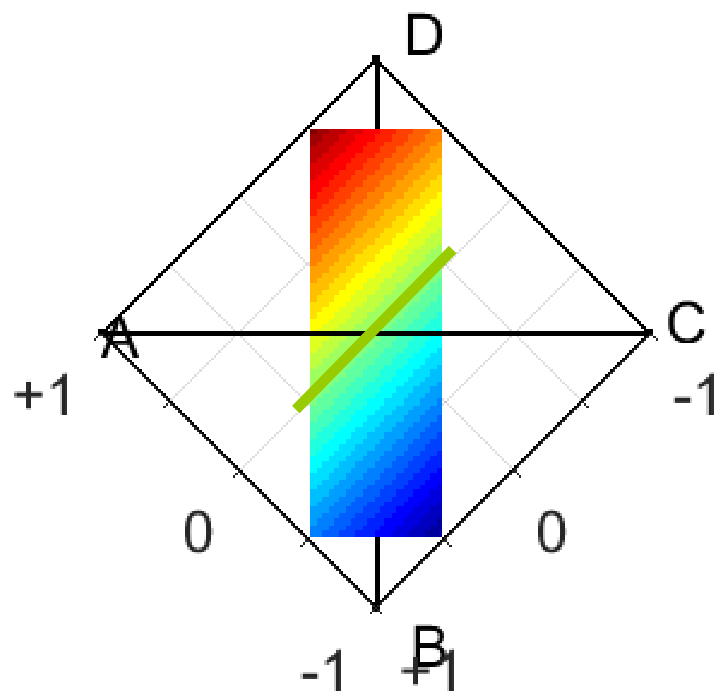
CA

# przewaga klasy 0



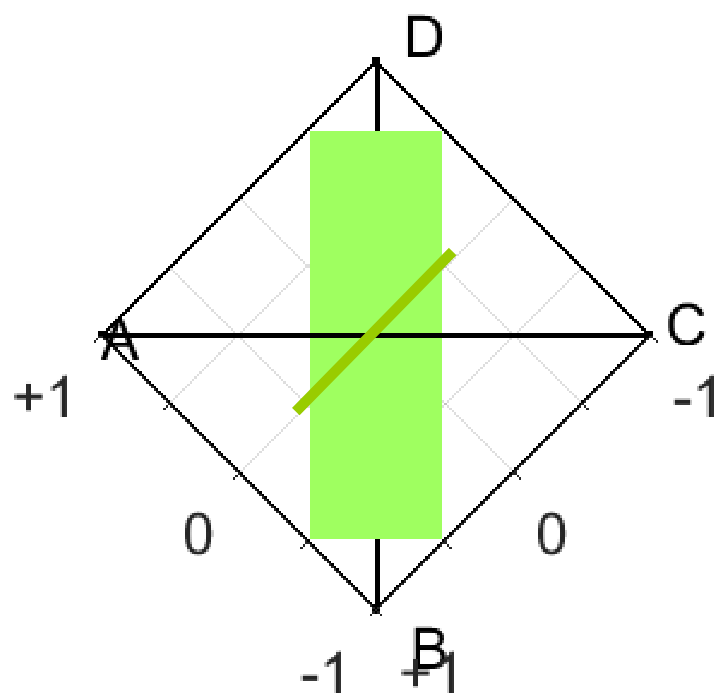
CA

przewaga klasy 0



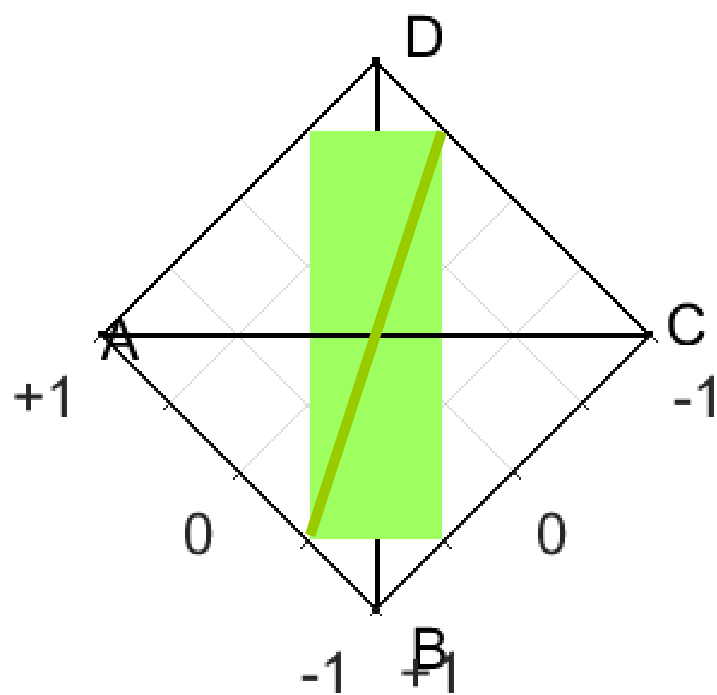
CA

przewaga klasy 0



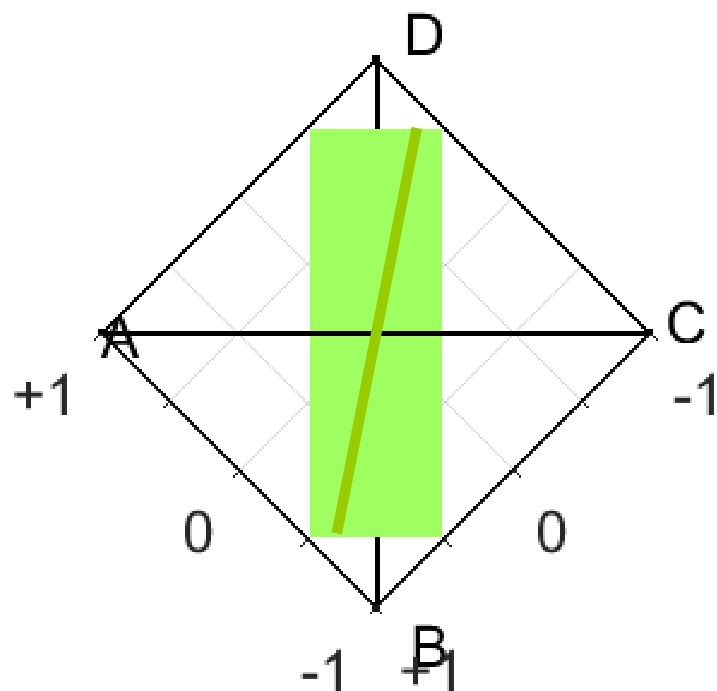
jak CA

przewaga klasy 0



lepiej? niż CA

przewaga klasy 0

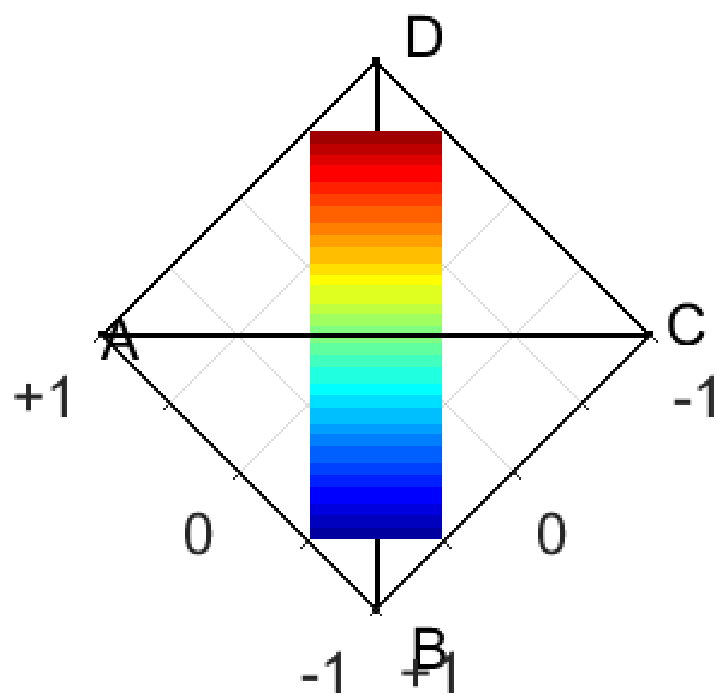


(przesadnie) lepiej? niż CA



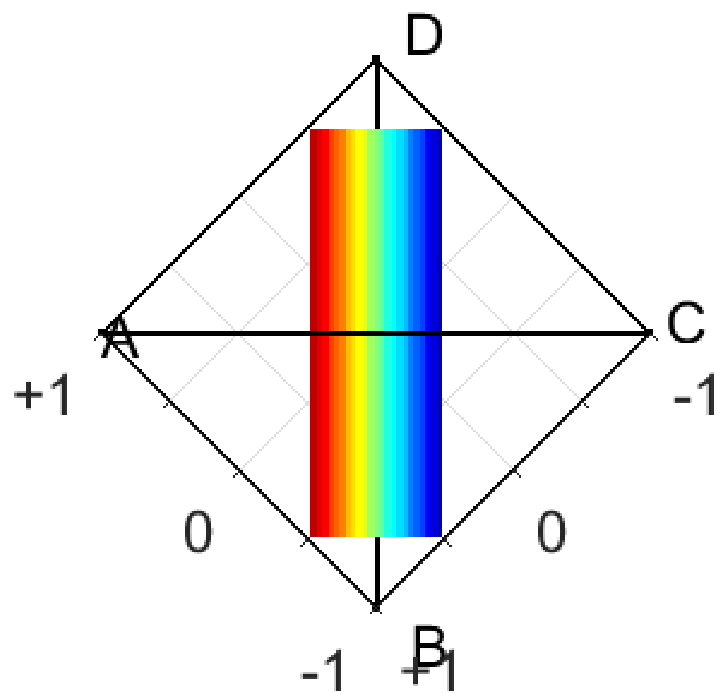
- inne miary:

przewaga klasy 0



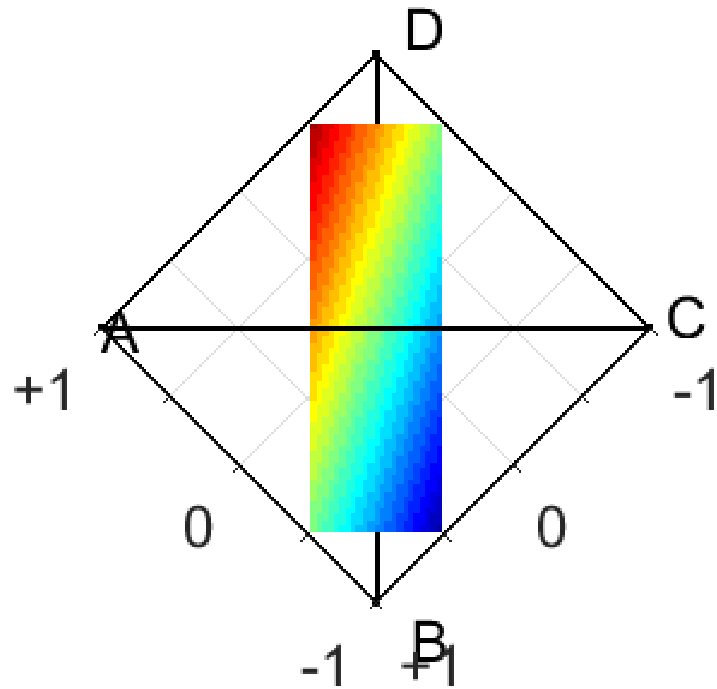
specificity

przewaga klasy 0



recall

przewaga klasy 0



`amean(specificity,recall)`

- kombinacje wypukłe

- $p, q \geq 0$

- $p+q = t$ , gdzie  $t > 0$

- wtedy

- $p/(p+q) \geq 0, q/(p+q) \geq 0$

- $p/(p+q) + q/(p+q) = 1$

- współczynniki kombinacji wypukłej

- ułamki  $p/(p+q)$  i  $q/(p+q)$  sterują proporcjami  $p$  i  $q$

- $p/(p+q) = 0$  oznacza, że  $p = 0, q = t$

- $p/(p+q) = 1/2$  oznacza, że  $p = t/2, q = t/2$

- $p/(p+q) = 1$  oznacza, że  $p = t, q = 0$

- (analogicznie dla  $q/(p+q)$ )

- warunki dotyczące  $a, b, c, d$ 
  - $a, b, c, d \geq 0$
  - $a+b+c+d = 1$

- przypadki dotyczące proporcji klas

- $a+c = p$

- $b+d = q$

gdzie

- $p, q \geq 0$

- $p+q = 1 > 0$

- wtedy

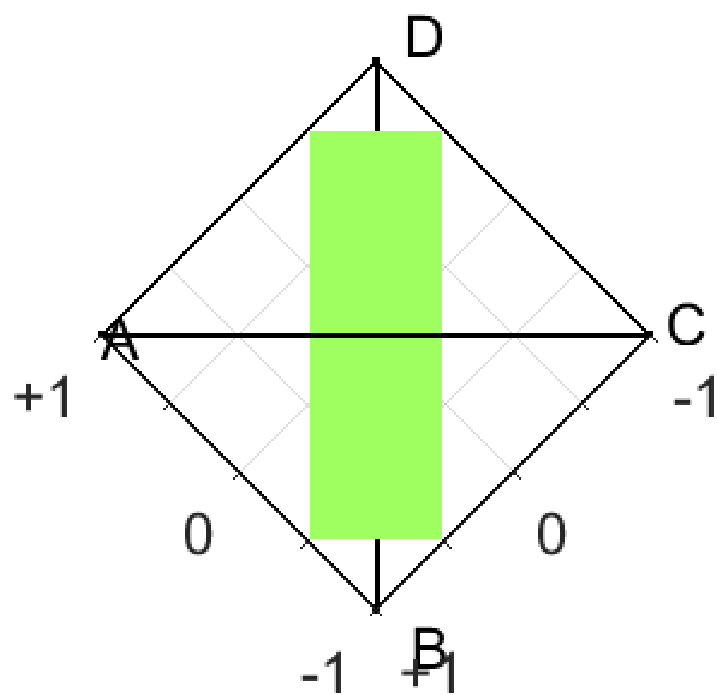
- ułamki  $(a+c)/(a+b+c+d)$  i  $(b+d)/(a+b+c+d)$  sterują proporcjami klas

- dla  $(a+c)/(a+b+c+d) = p$  mamy  $(b+d)/(a+b+c+d) = 1 - p = q$

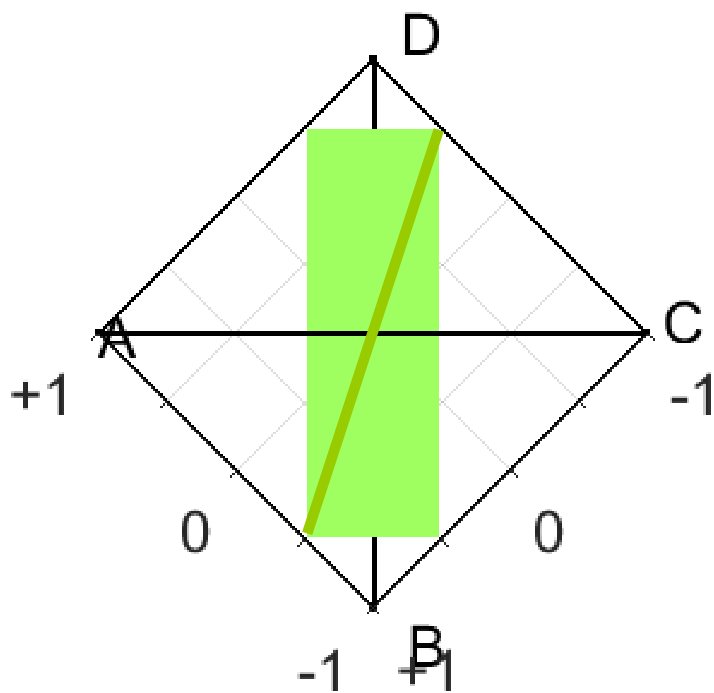
- dla ustalonych  $a+c = p$  i  $b+d = q$  zmianom mogą podlegać
  - a i c (w ramach sumy  $a+c = p$ )
    - współczynniki kombinacji wypukłej
    - ułamki  $a/(a+c)$  i  $c/(a+c)$  sterują proporcjami a i c
      - $a/(a+c) = \text{recall}$
      - $c/(a+c) = 1 - \text{recall}$
  - b i d (w ramach sumy  $b+d = q$ )
    - współczynniki kombinacji wypukłej
    - ułamki  $b/(b+d)$  i  $d/(b+d)$  sterują proporcjami b i d
      - $b/(b+d) = 1 - \text{specificity}$
      - $d/(b+d) = \text{specificity}$



przewaga klasy 0

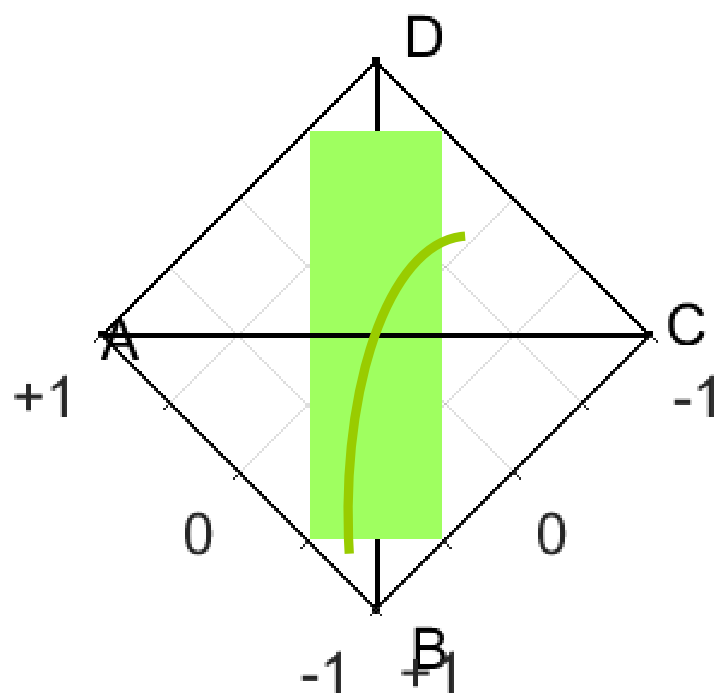


przewaga klasy 0



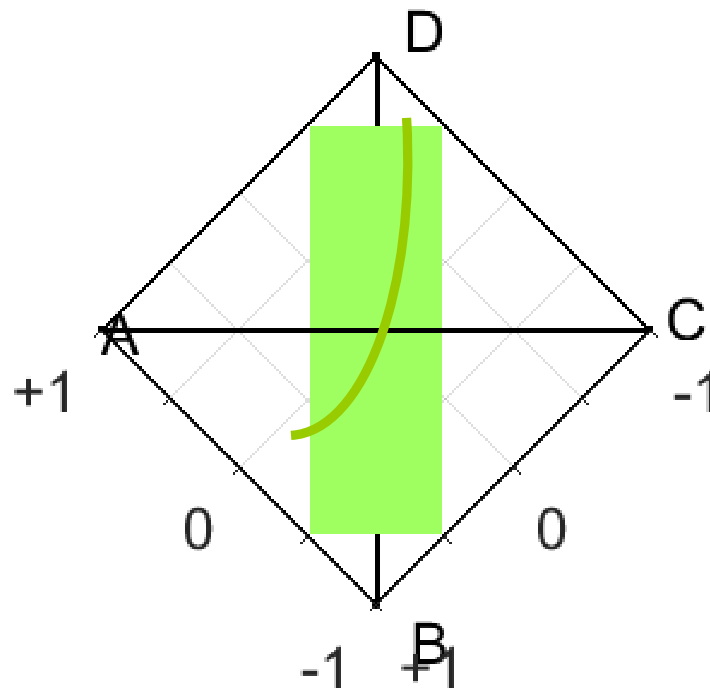
agregacja typu amean

przewaga klasy 0



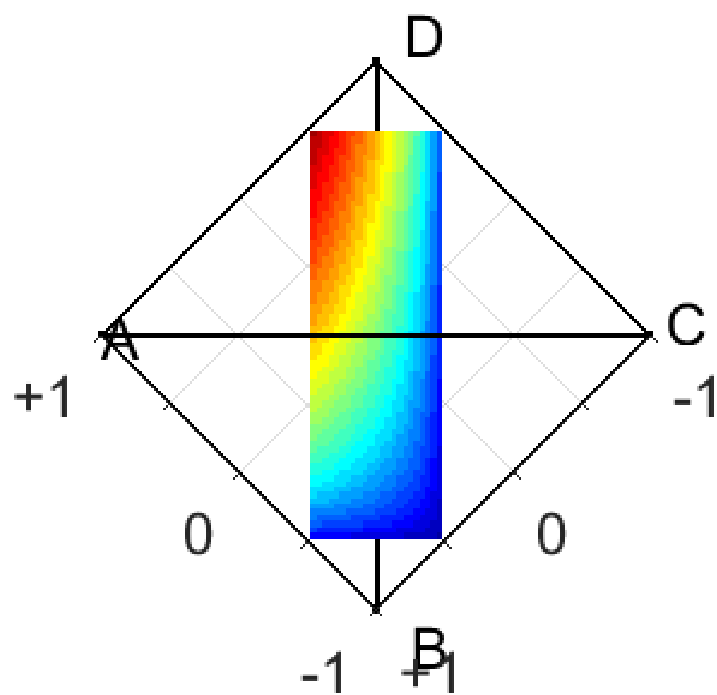
agregacja typu max

przewaga klasy 0



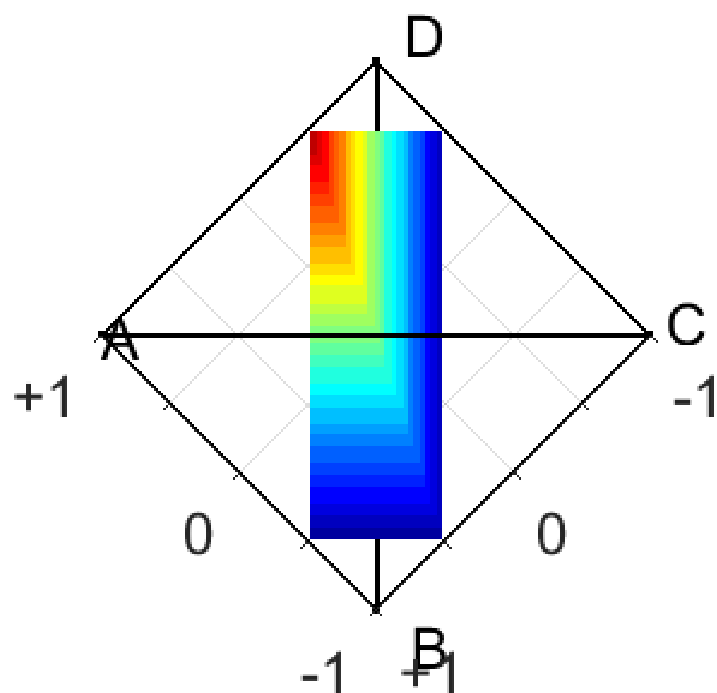
agregacja typu min  
(także gmean/hmean)

przewaga klasy 0



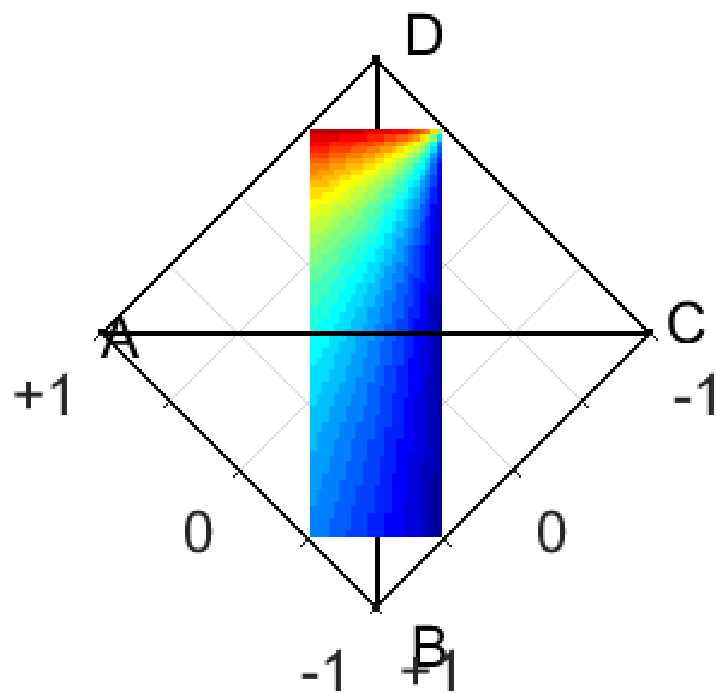
G-mean

przewaga klasy 0



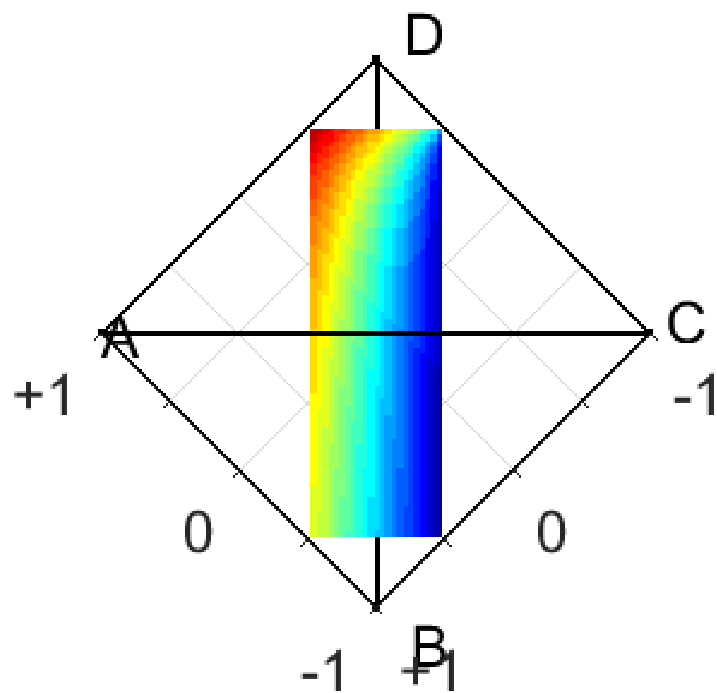
$\min(\text{specificity}, \text{recall})$

przewaga klasy 0



precision

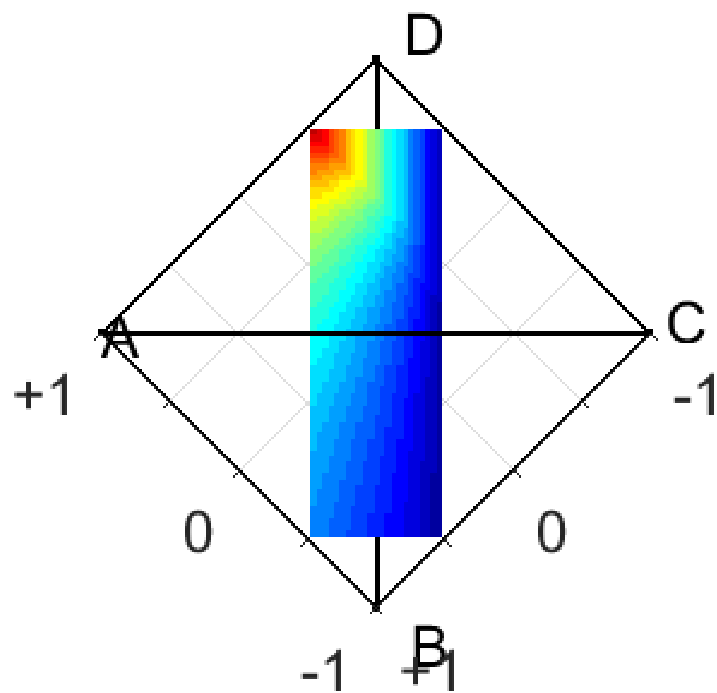
przewaga klasy 0



`amean(precision,recall)`

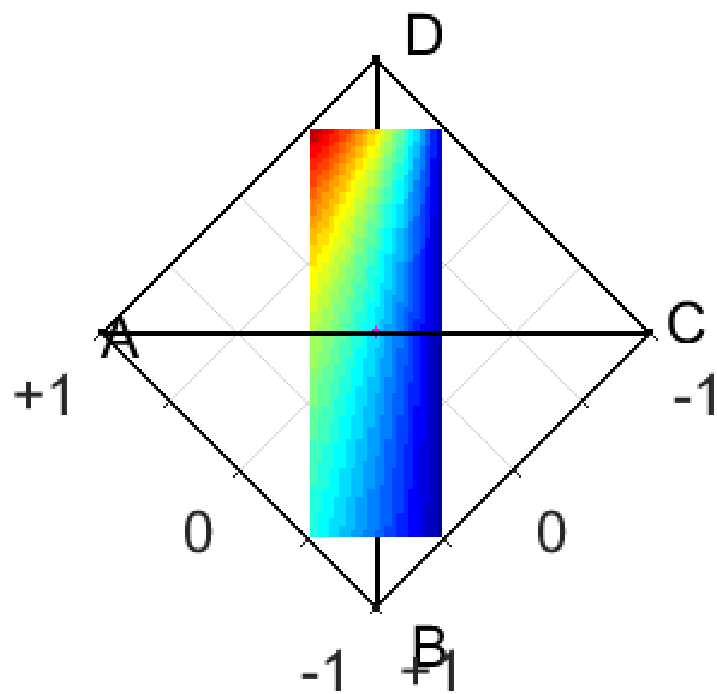


przewaga klasy 0



$\min(\text{precision}, \text{recall})$

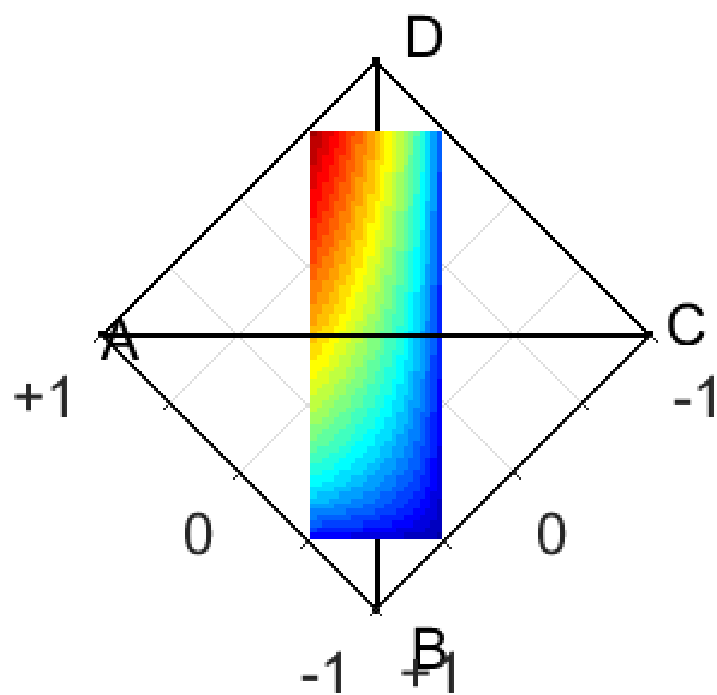
przewaga klasy 0



$F_1$

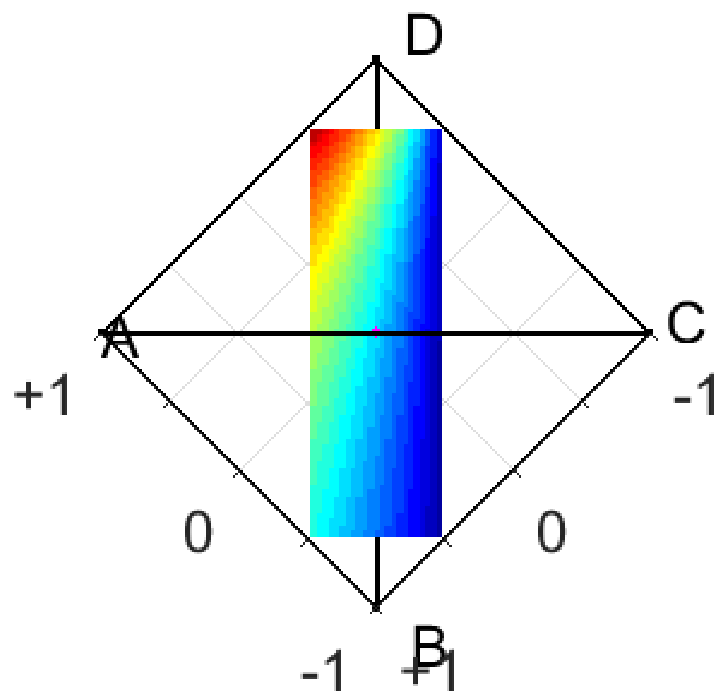
- $F_1$  vs G-mean dla rosnącej nierównowagi

przewaga klasy 0



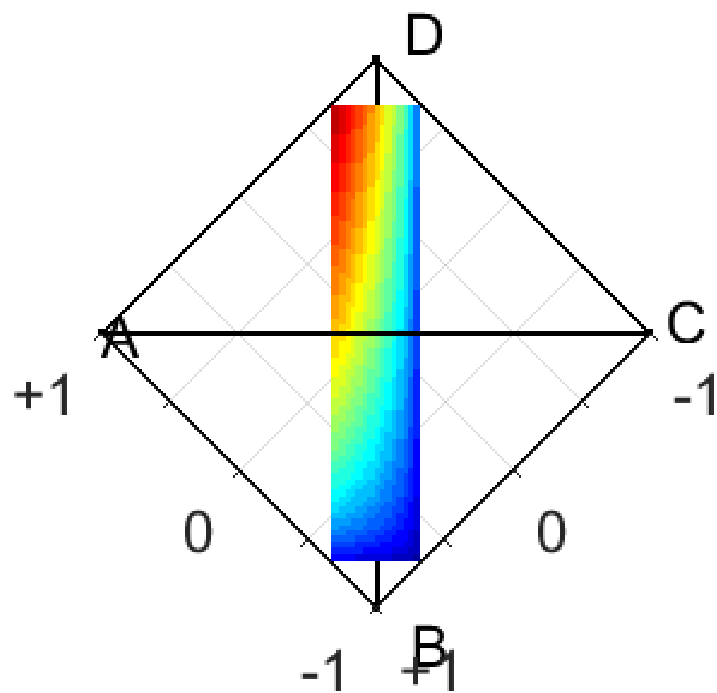
G-mean

# przewaga klasy 0



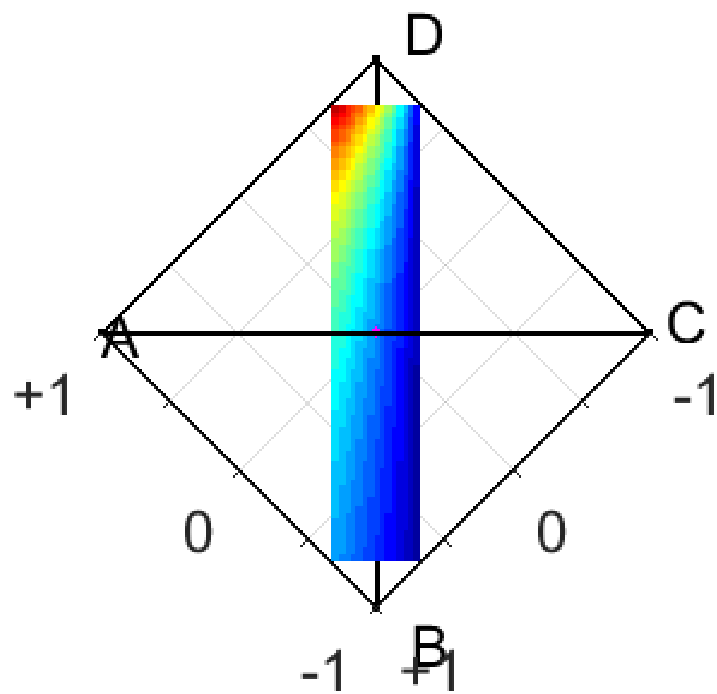
$F_1$

przewaga klasy 0



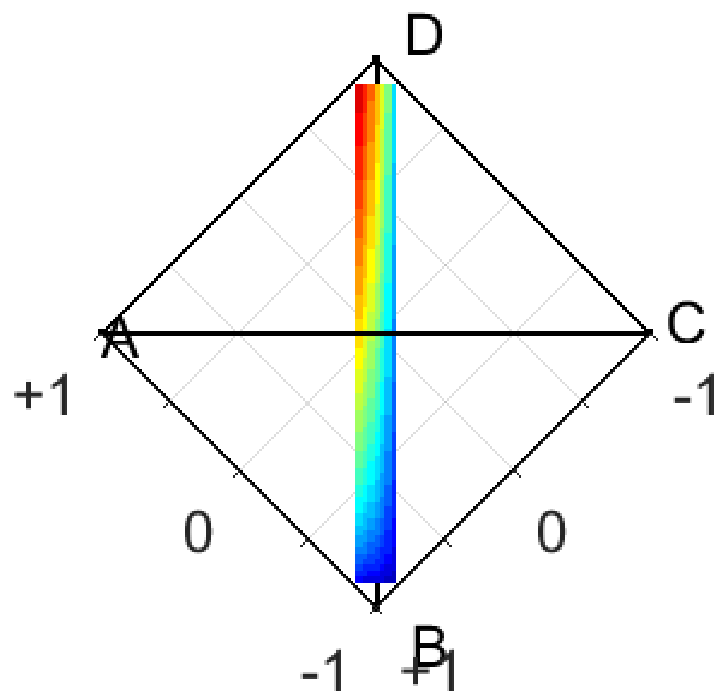
G-mean

przewaga klasy 0



$F_1$

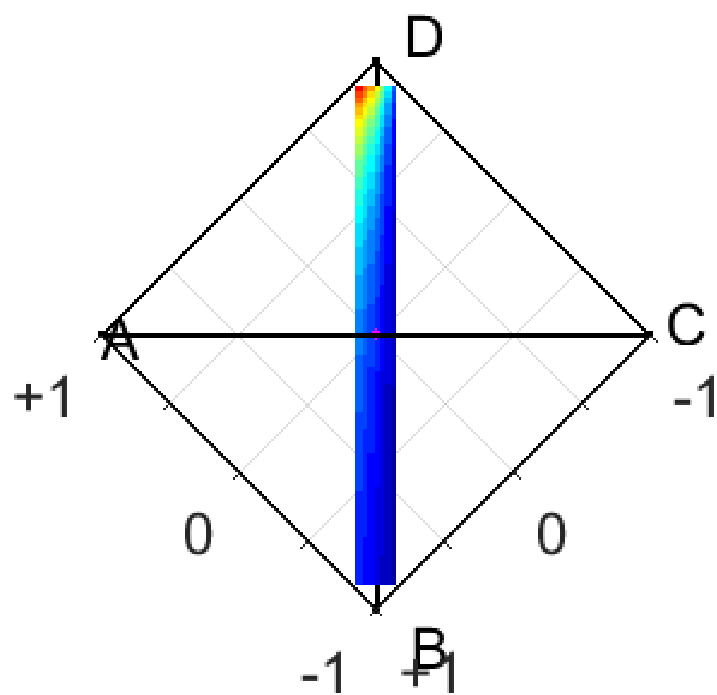
przewaga klasy 0



G-mean



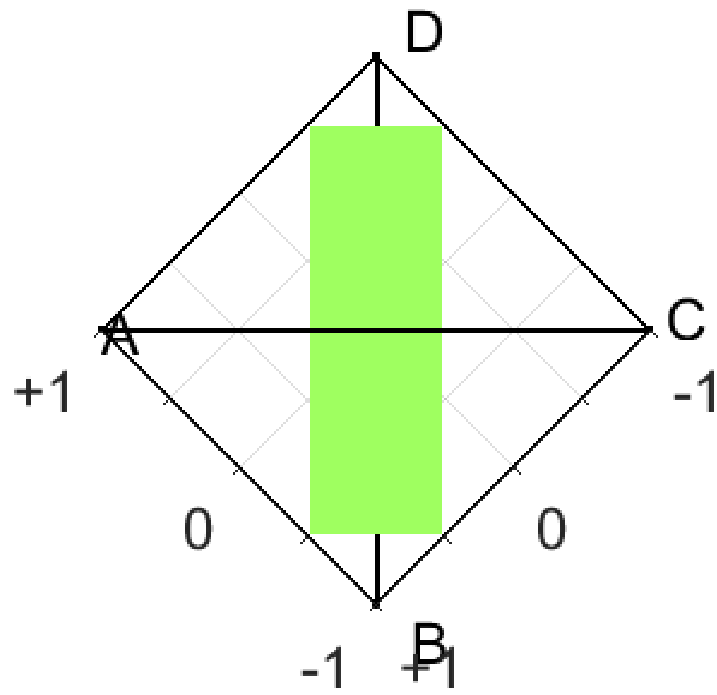
# przewaga klasy 0

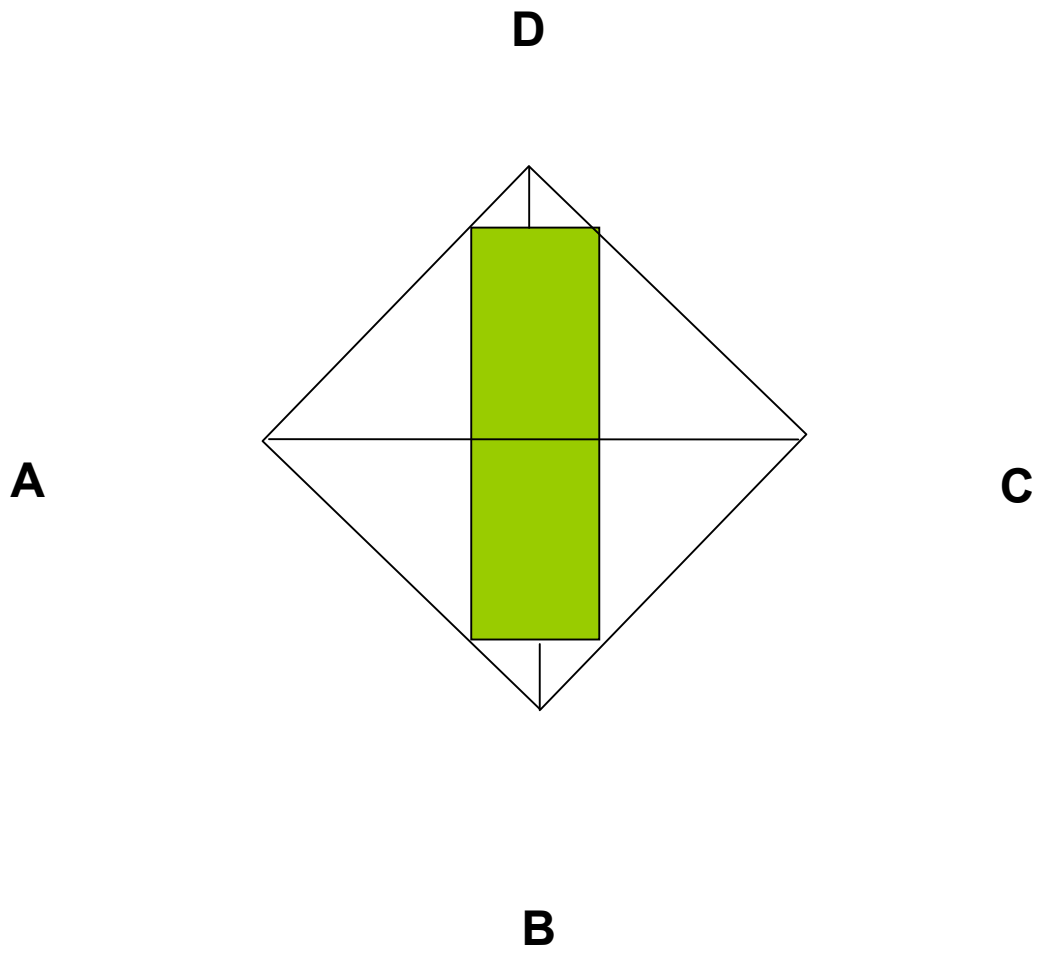


$F_1$

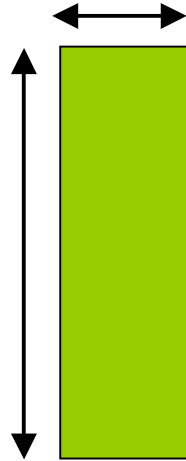
...

widok z góry





**A**



**D**

**B**

**C**

**A**



**D**

liczność klasy 1

liczność klasy 0

**C**

**B**

**A**



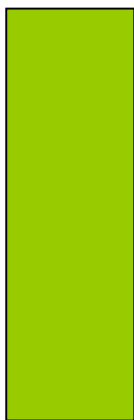
**D**

**B**

0 .. 1/2 .. 1  
d/(b+d)  
specificity

**C**

**D**



**A**

**C**

← ●  
1 .. 1/2 .. 0  
a/(a+c)  
recall

**B**



**D**



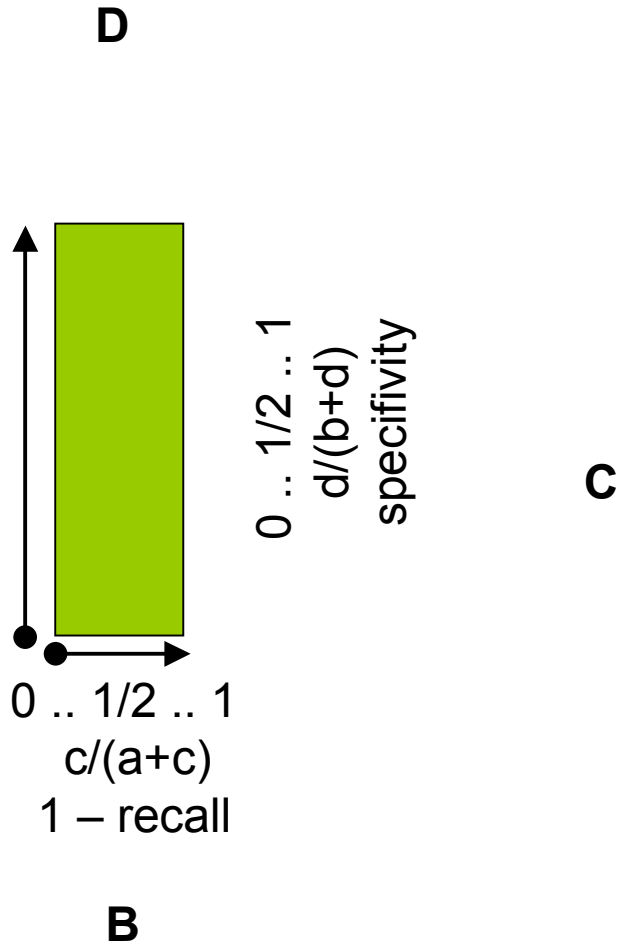
**A**

**C**

0 .. 1/2 .. 1  
c/(a+c)  
1 – recall

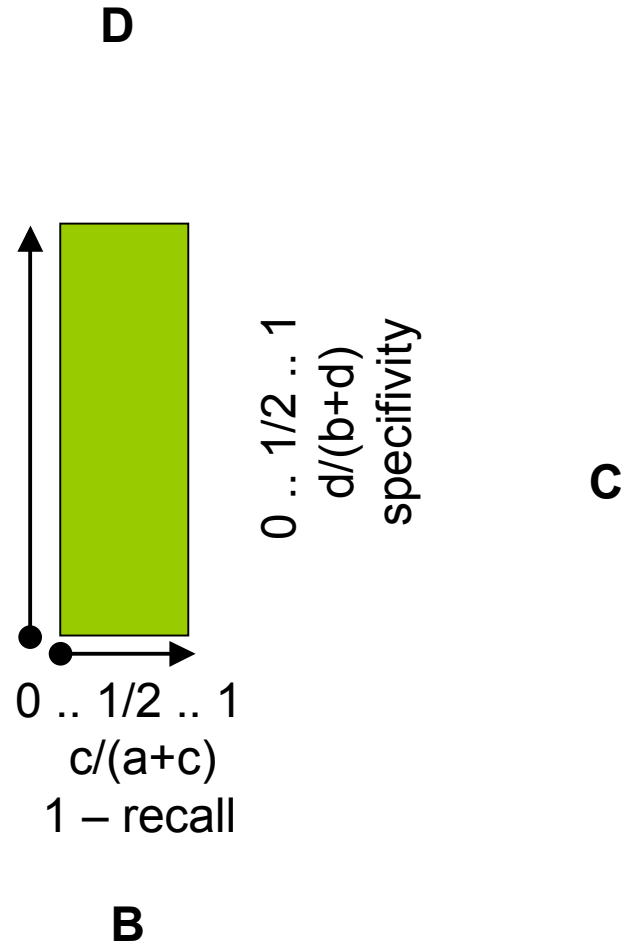
**B**

**A**



(1 – recall) vs specificity  
(układ: analog. jak dla AUC)

**A**



Uwaga...

**D**

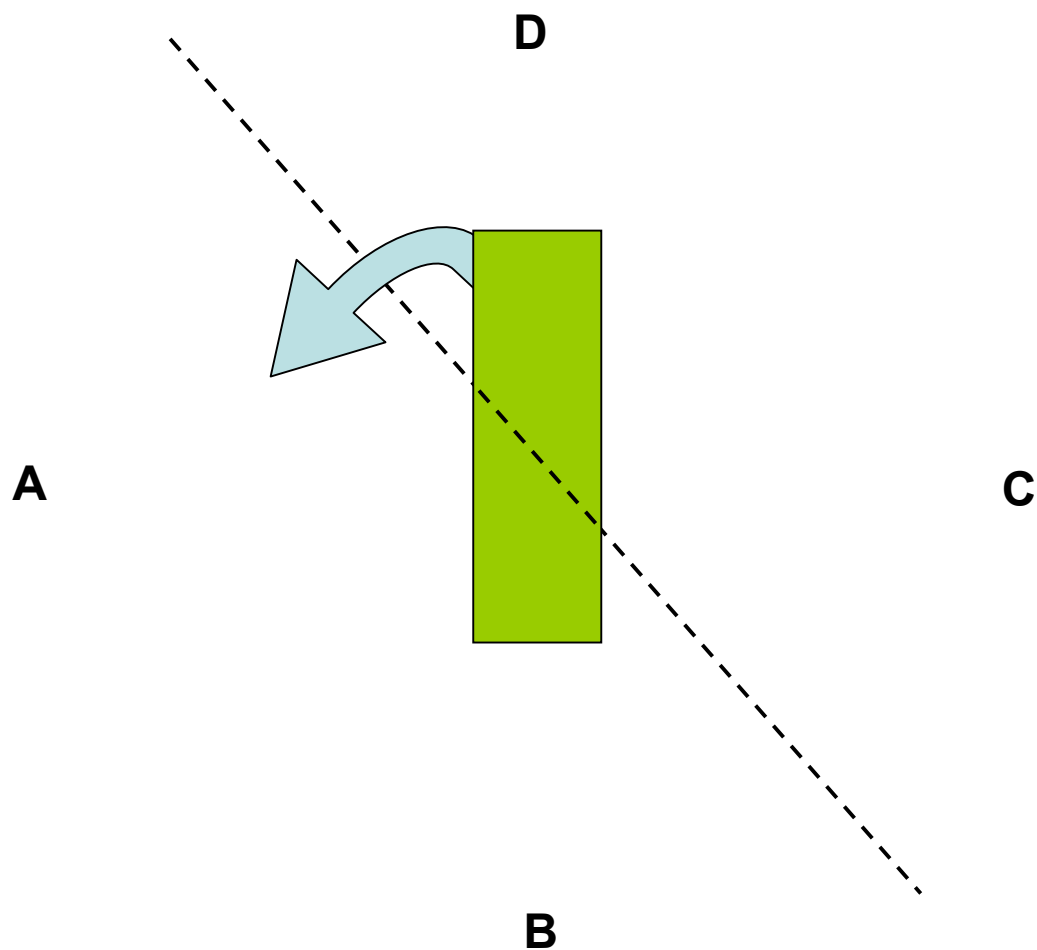


**A**

**C**

**B**

... dokonujemy obrotu w 3D...



... gotowe!

**A**

**D**



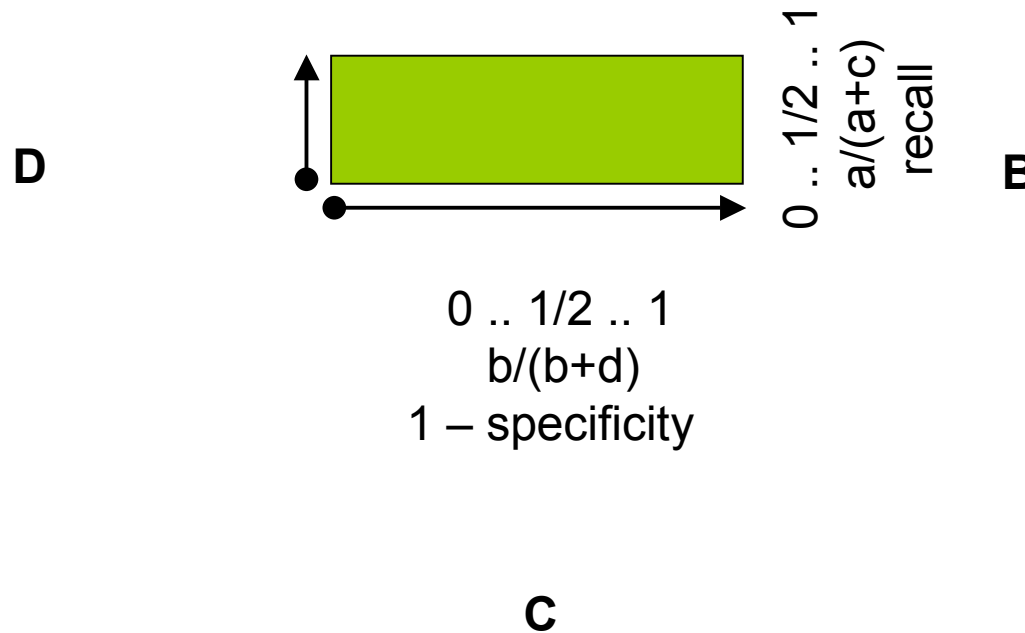
**B**

**C**

# To samo z drugiej strony

**A**

(1 – specificity) vs recall  
(układ: ident. jak dla AUC)



dodatkowo:

**A**

**D**

precision<sub>0</sub>

**B**

**C**



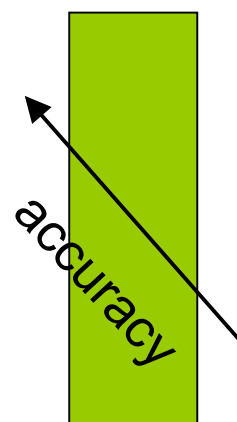
precision<sub>1</sub>



dodatkowo:

**A**

**D**



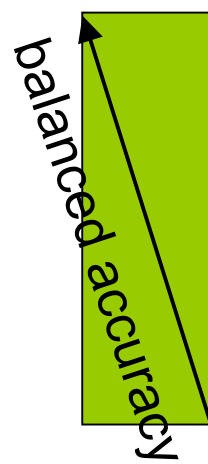
**C**

**B**

dodatkowo:

**A**

**D**



**C**

**B**

# Podsumowując

**D**



**A**

**C**

**B**

**A**



**B**

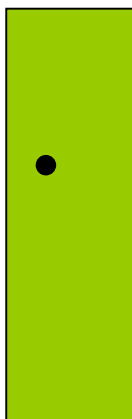
**C**

**D**

liczność klasy 1

liczność klasy 0

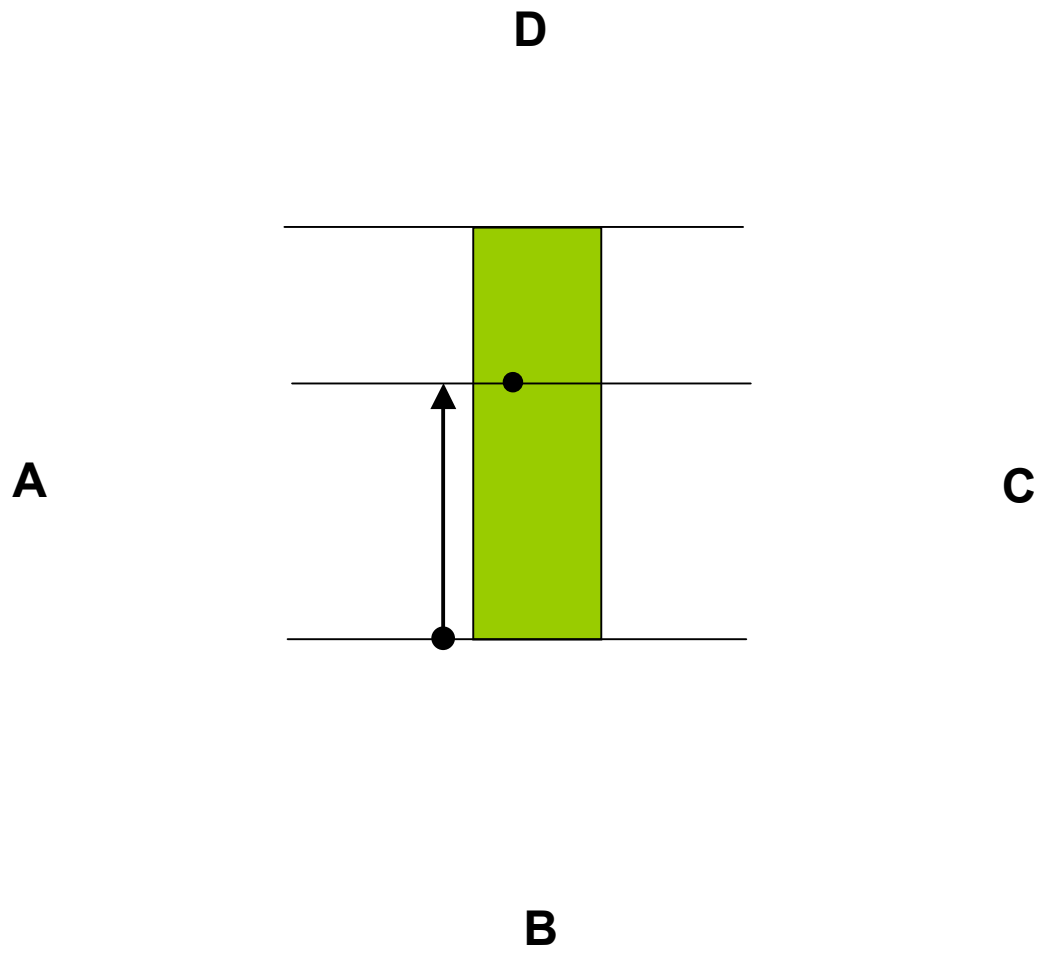
**D**



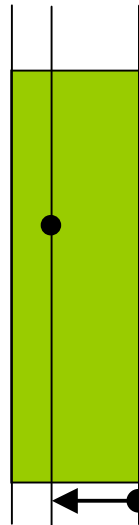
**A**

**C**

**B**



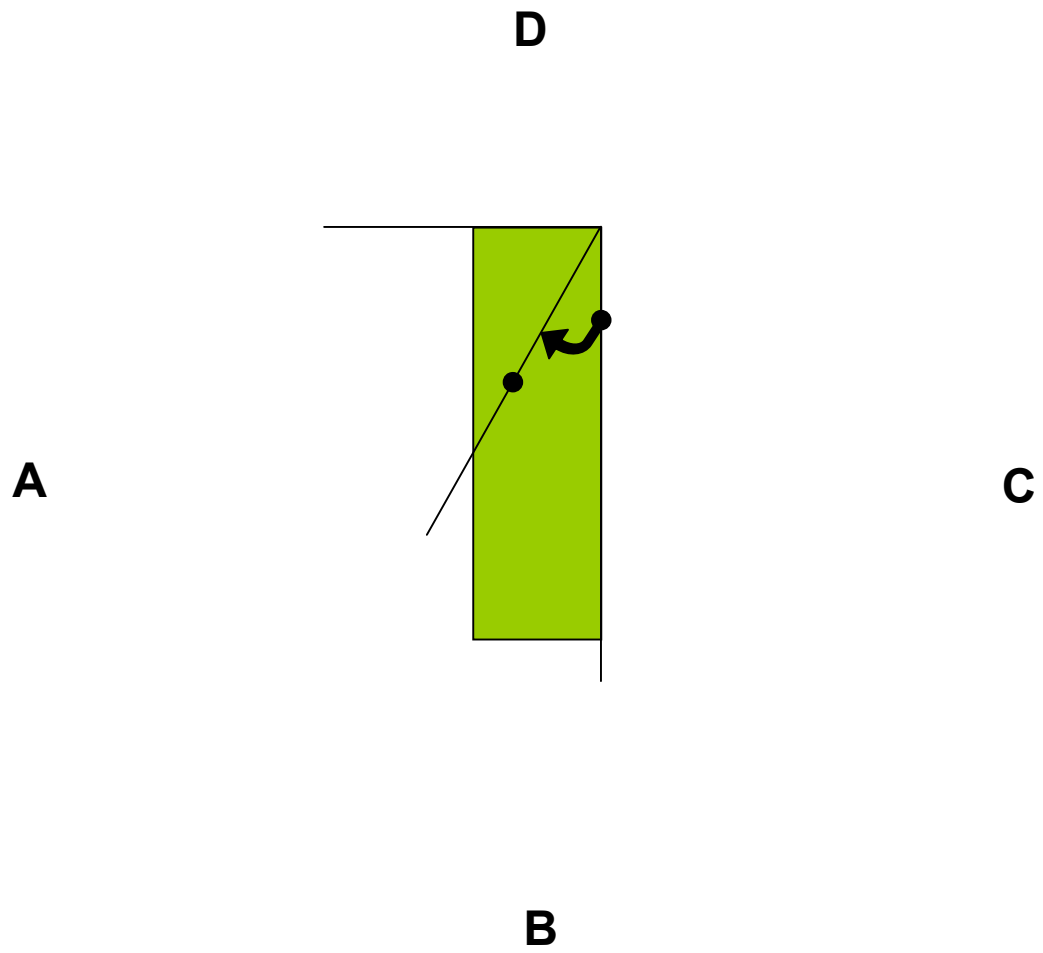
**A**



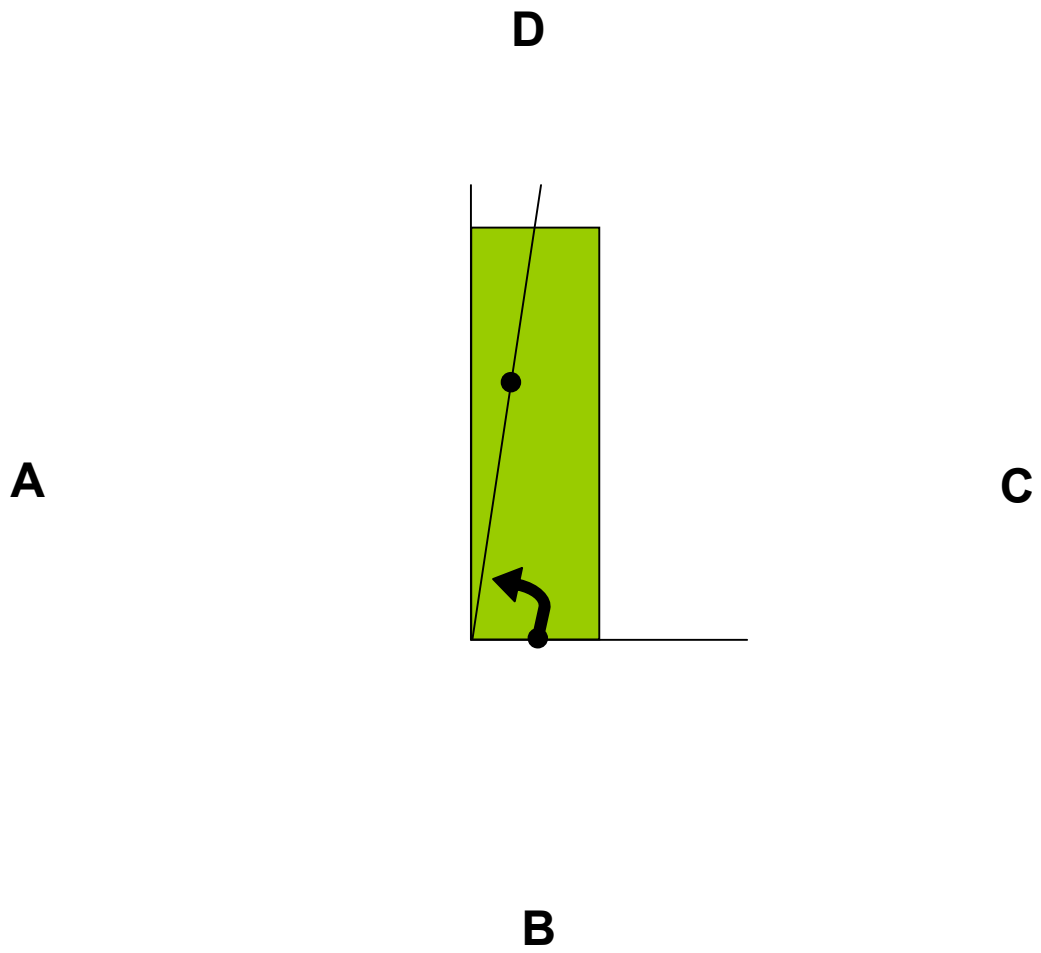
**D**

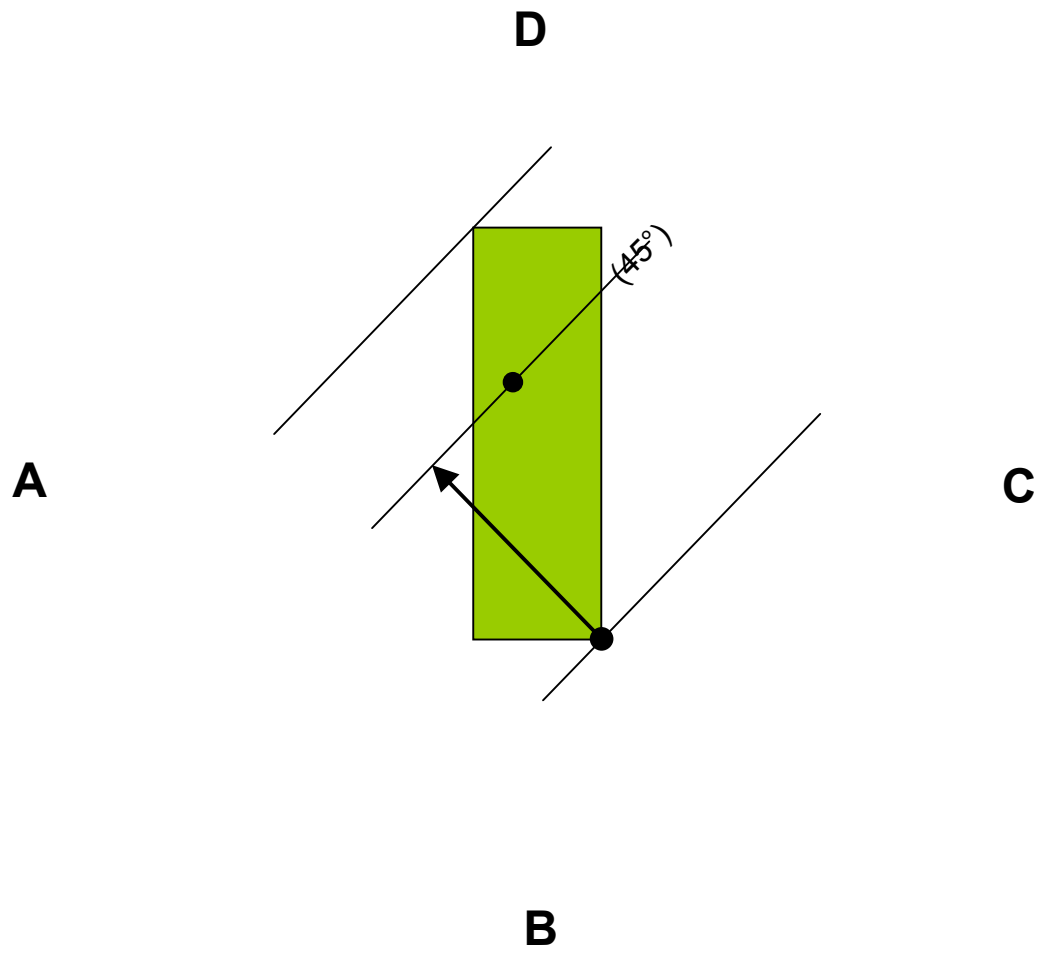
**C**

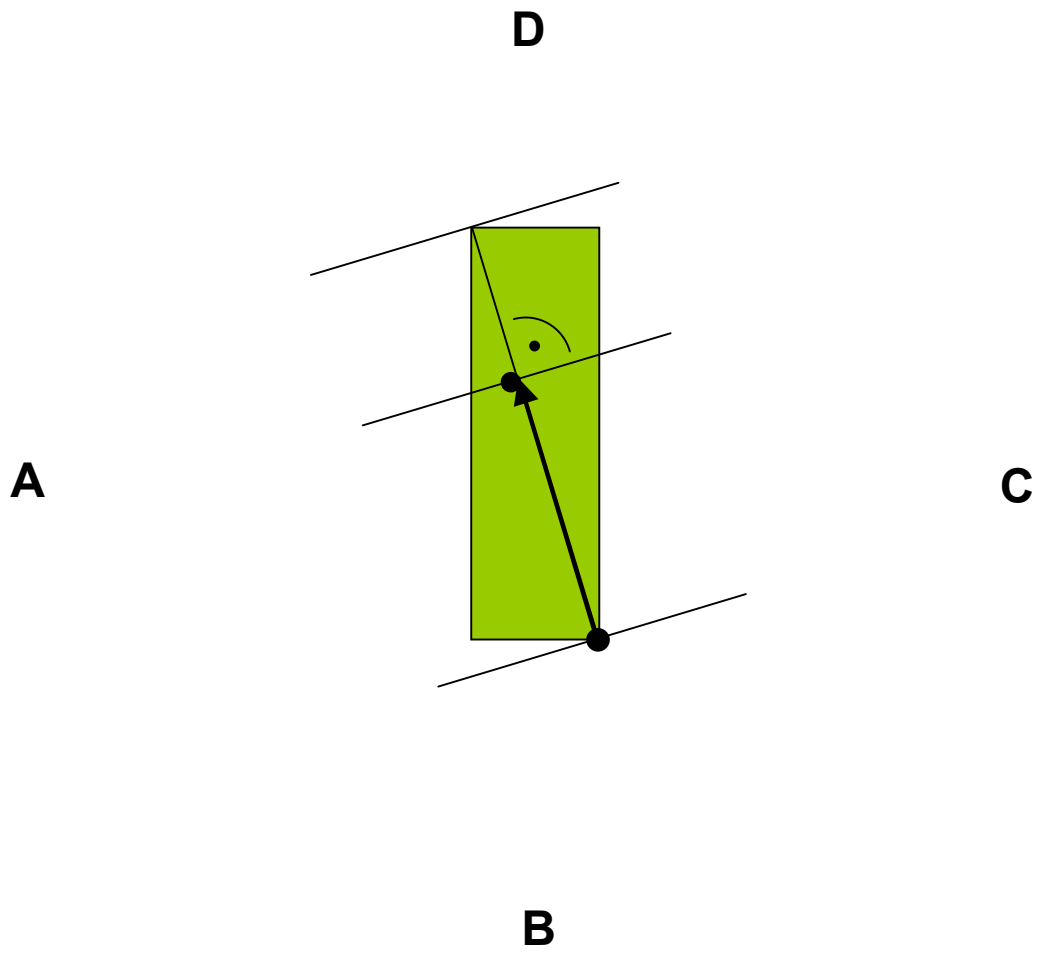
**B**











...