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ul. Piotrowo 2

Poznań

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kontakt osobisty

Centrum Wykładowe, „blok informatyki”, pok. 7

Wyłączenie odpowiedzialności

Prezentowane materiały, będące dodatkiem pomocniczym do wykładów, z konieczności fragmentarycznym i niedopracowanym, należy wykorzystywać z pełną świadomością faktu, że mogą nie być pozbawione przypadkowych błędów, braków, wypaczeń i przeinaczeń :-)

Autor

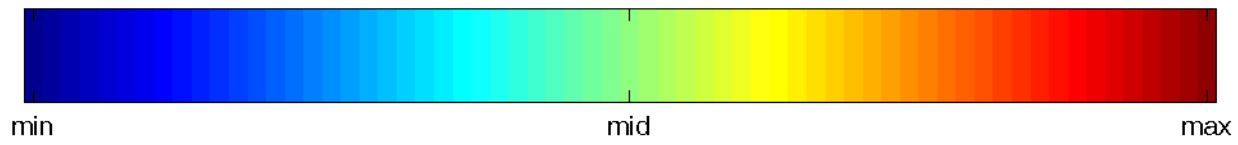
...

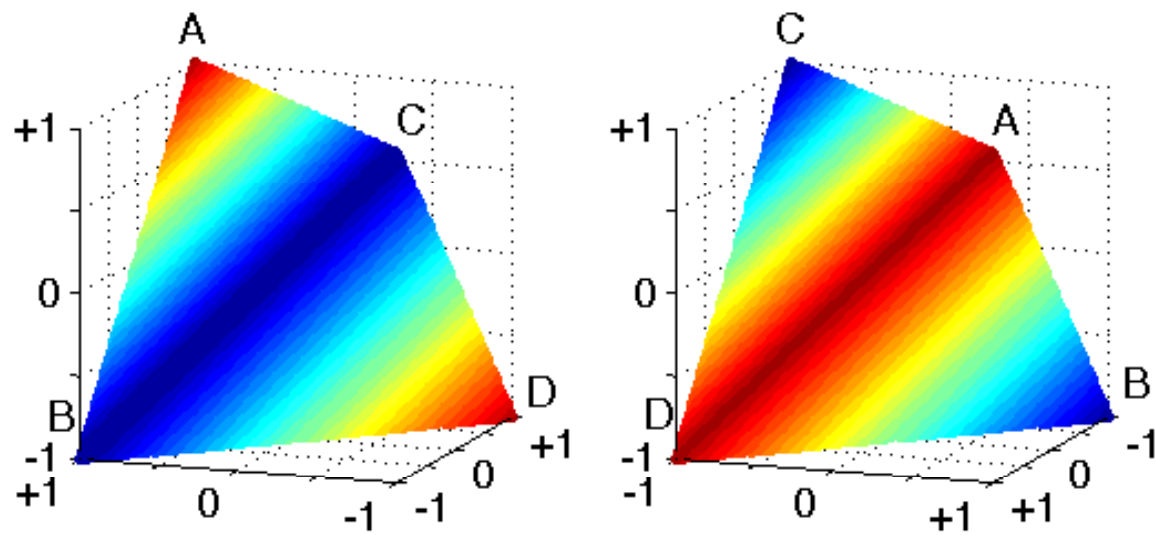
a	c
b	d

		predicted	
		1	0
original	1	TP	FN
	0	FP	TN

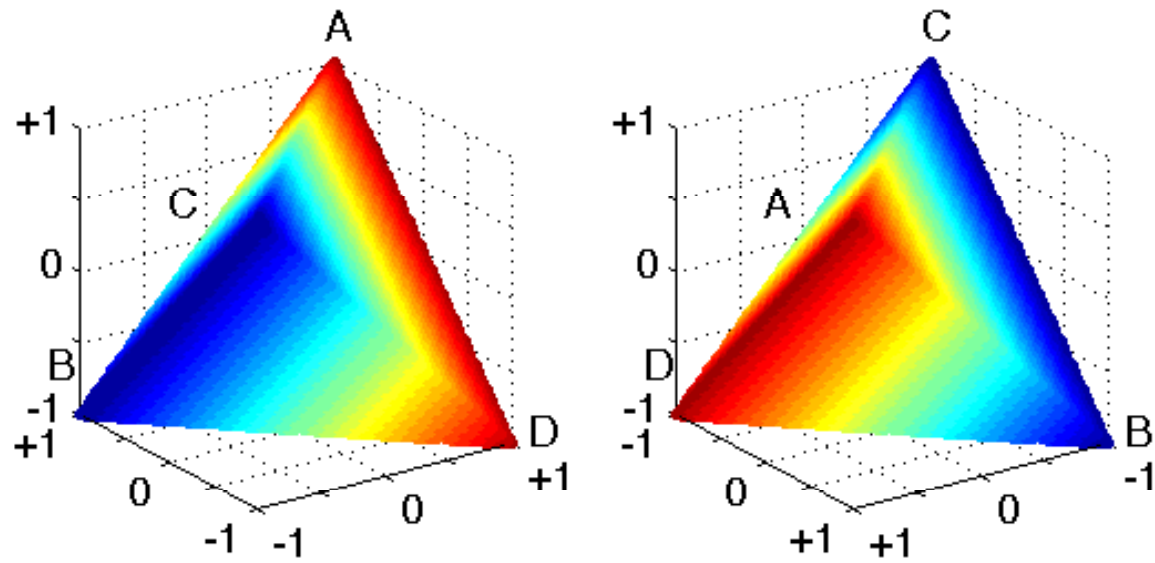
		predicted	
		1	0
original	1	TP A a	FN C c
	0	FP B b	TN D d

- Zakres wartości [min, max] = [0, 1]
 - (+ ewentualne NaN'y /fioletowe/)

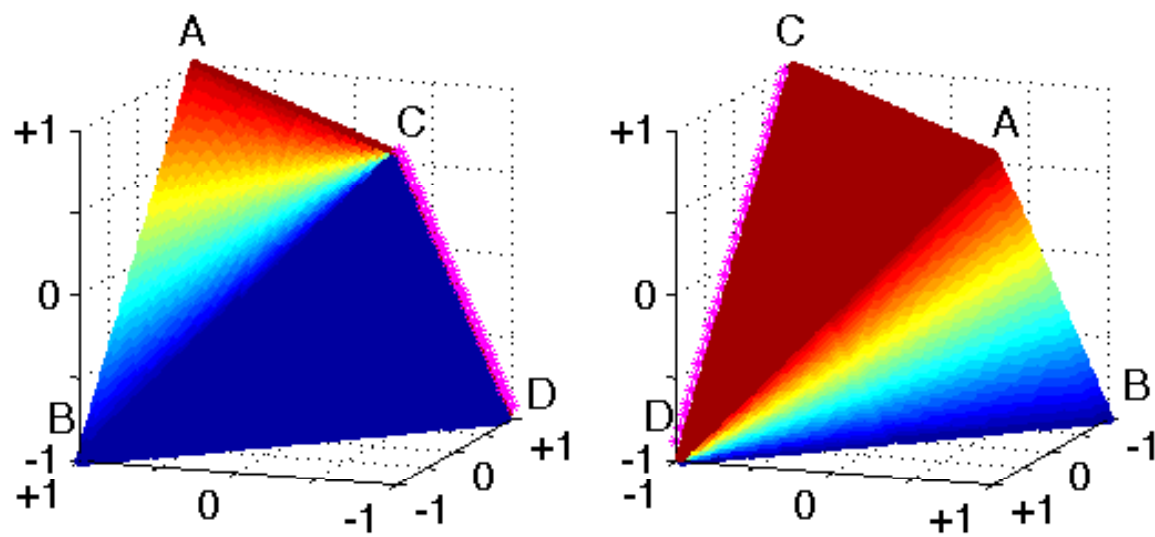




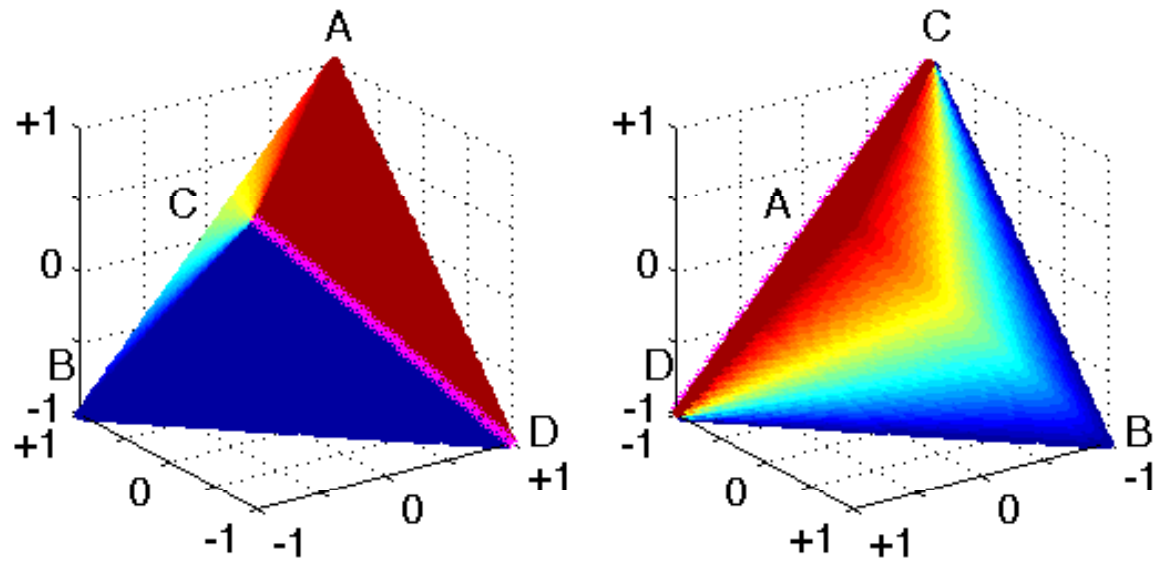
$$f(a,b,c,d) = (a+d)/(a+b+c+d) \text{ (accuracy)}$$



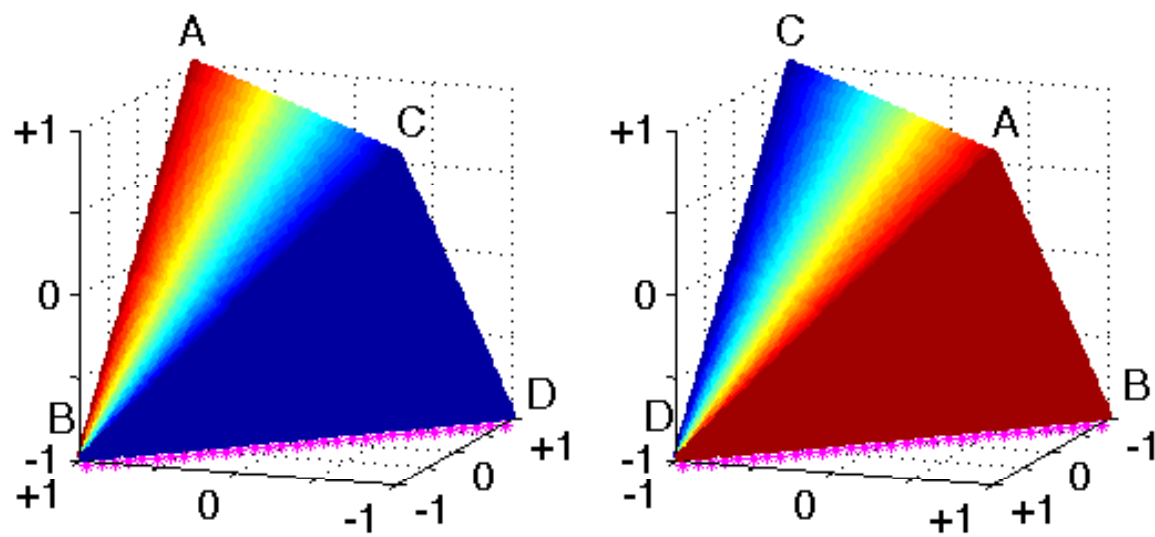
$$f(a,b,c,d) = \frac{a+d}{a+b+c+d} \text{ (accuracy)}$$



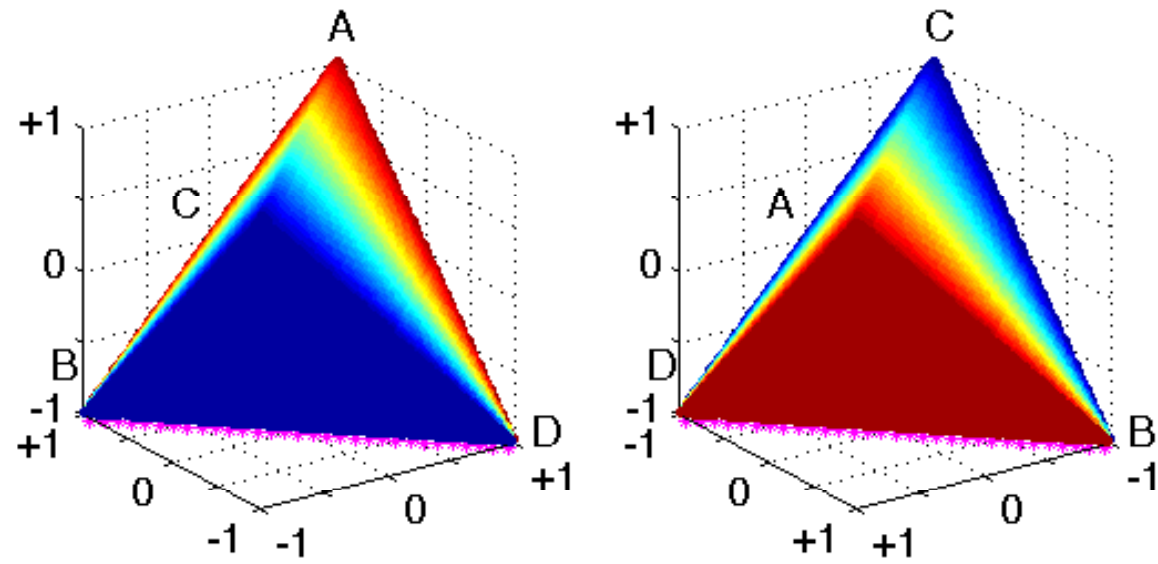
$$f(a,b,c,d) = a/(a+b) \text{ (precision)}$$



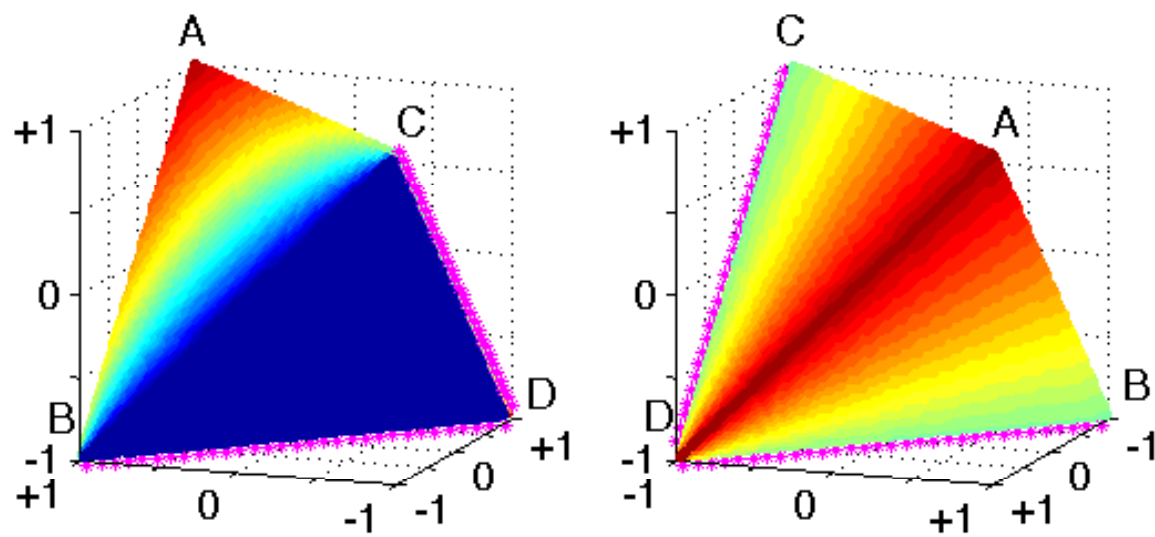
$$f(a,b,c,d) = a/(a+b) \text{ (precision)}$$



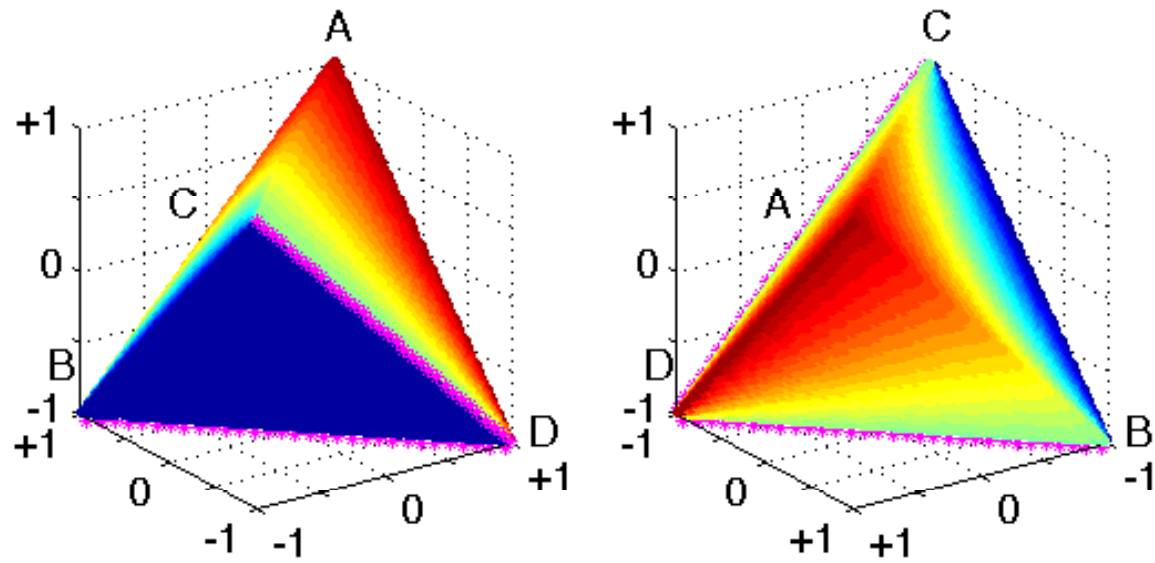
$$f(a,b,c,d) = a/(a+c) \text{ (recall)}$$



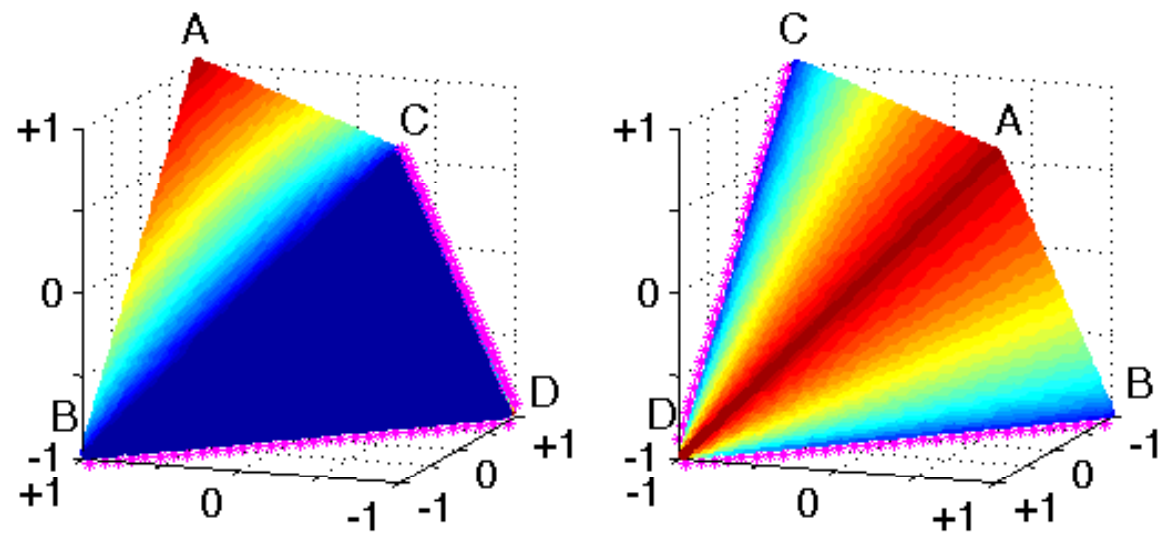
$$f(a,b,c,d) = a/(a+c) \text{ (recall)}$$



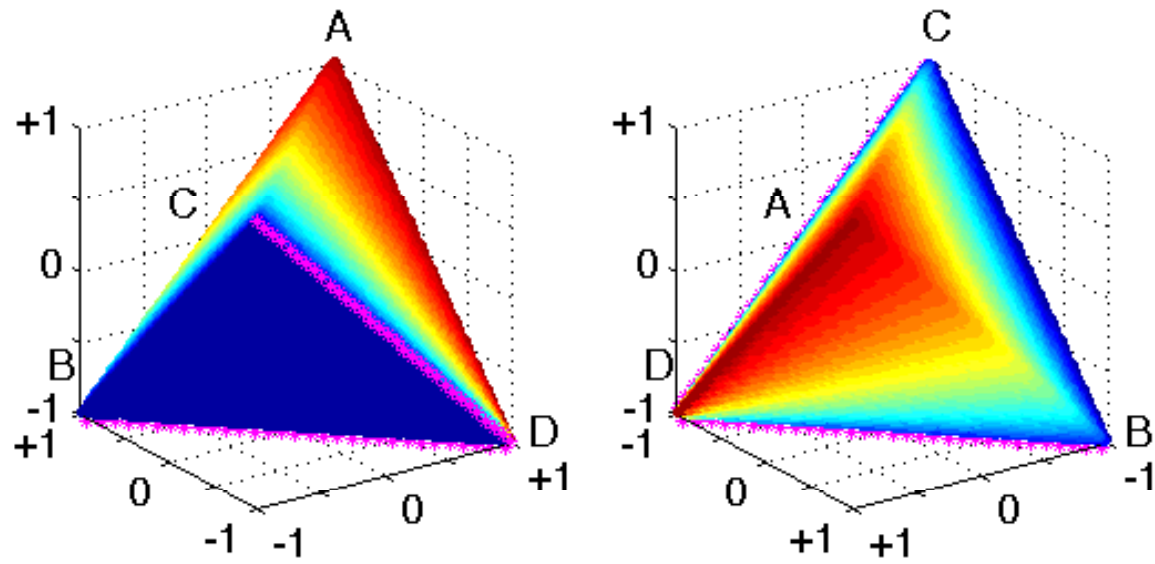
$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$



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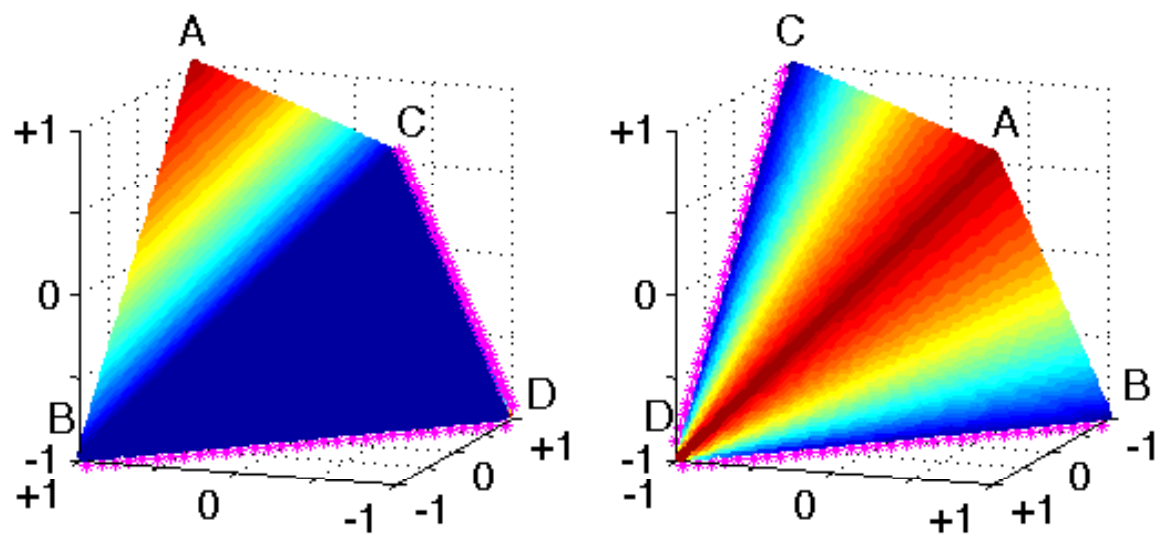
$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$



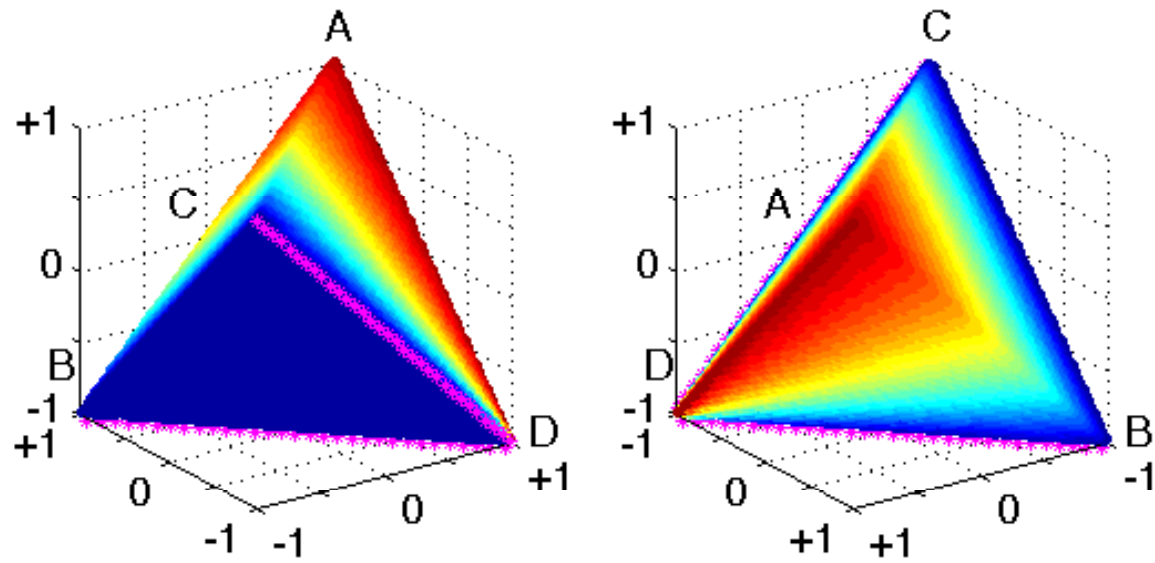
$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$

$\text{gmean}(\text{precision}, \text{recall}) = \text{G-measure}$

$$A \geq G \geq H$$



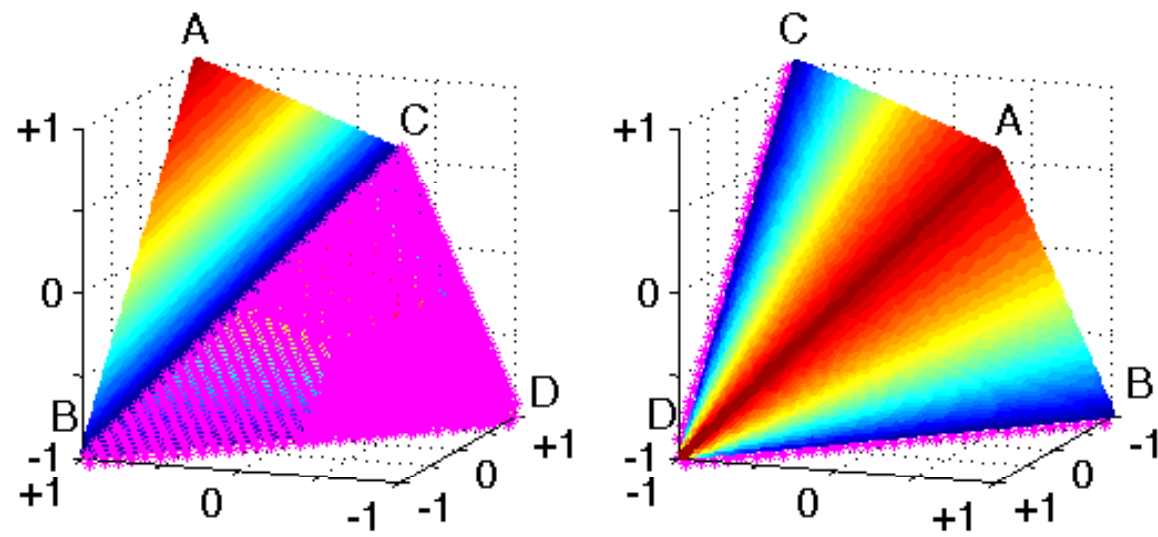
$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$



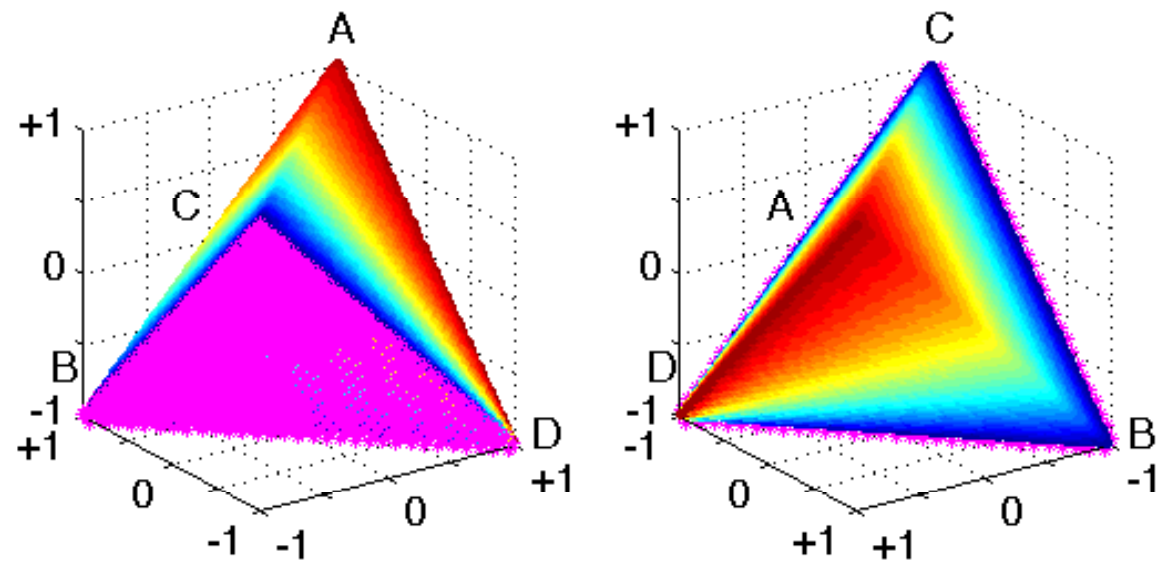
$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

$\text{hmean}(\text{precision}, \text{recall}) = F_1$

– wyjątki: wartości specjalne

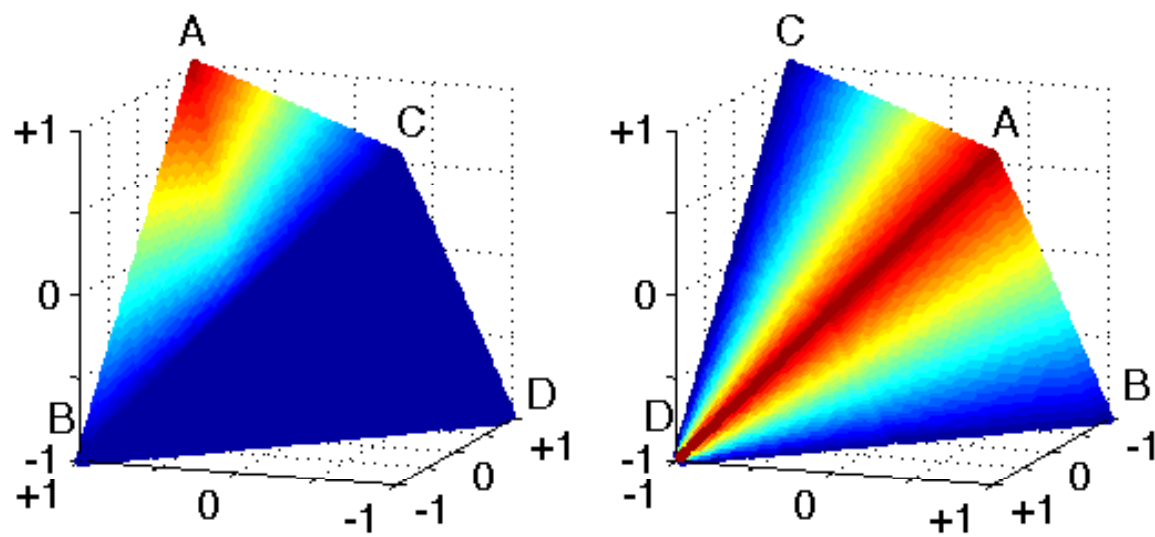


$$f(a,b,c,d) = F_1$$

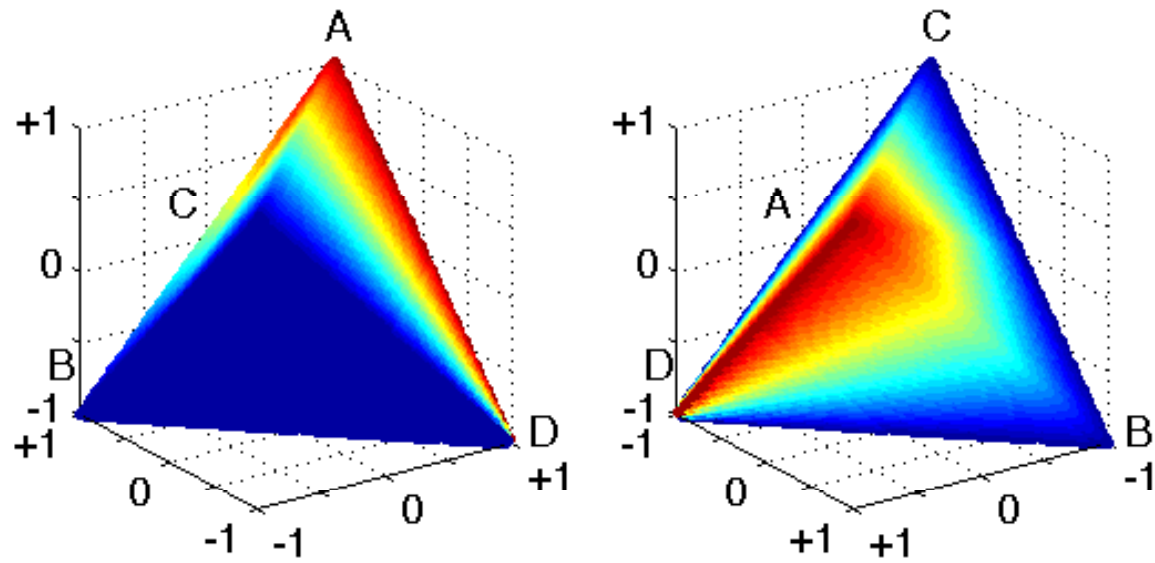


$$f(a,b,c,d) = F_1$$

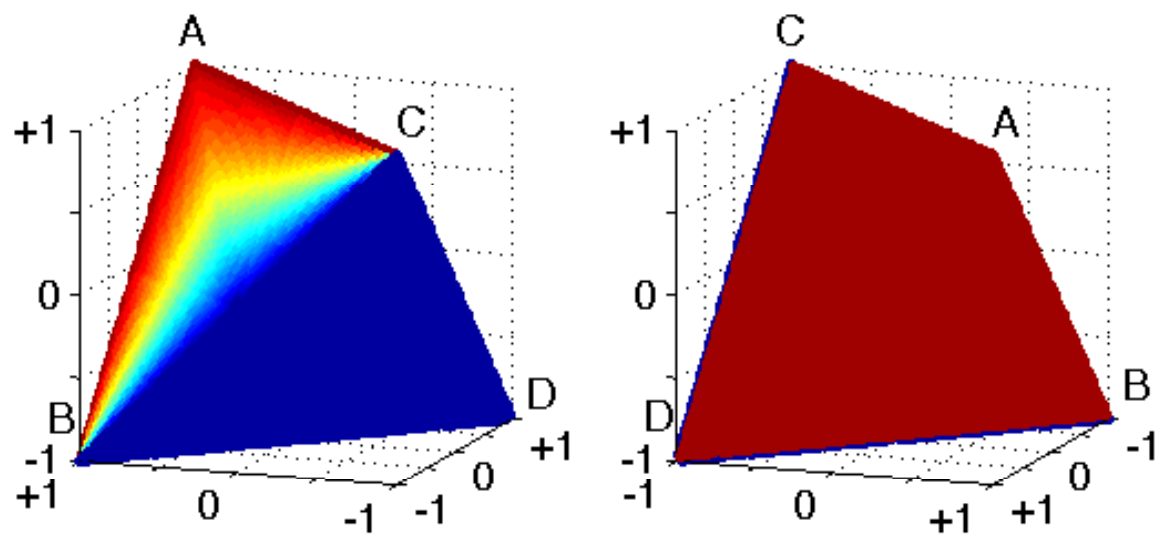
$\max \geq A \geq G \geq H \geq \min$



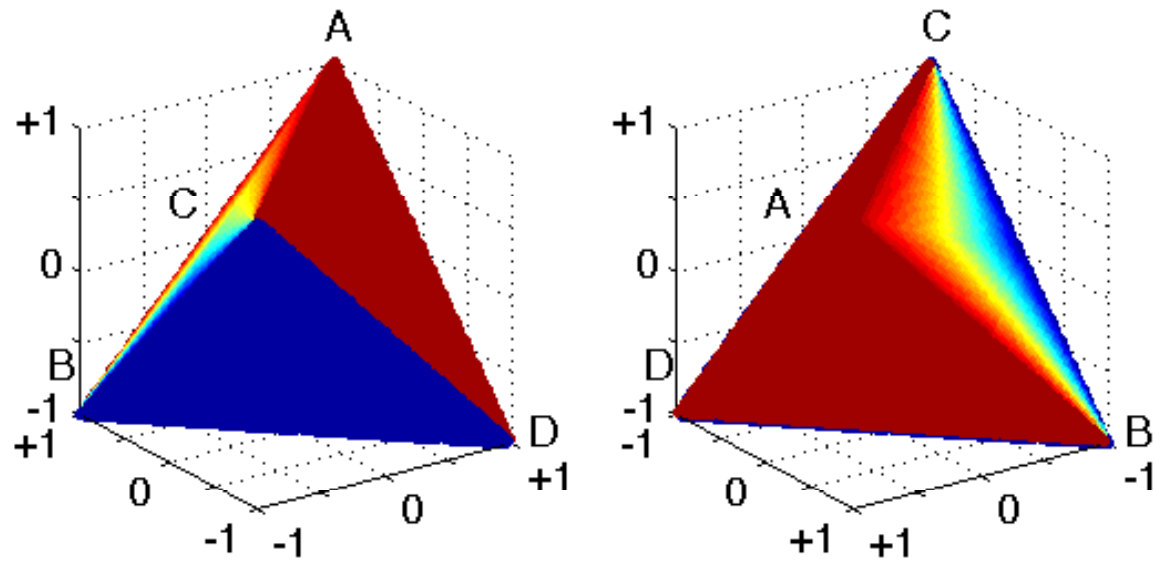
$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$



$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$

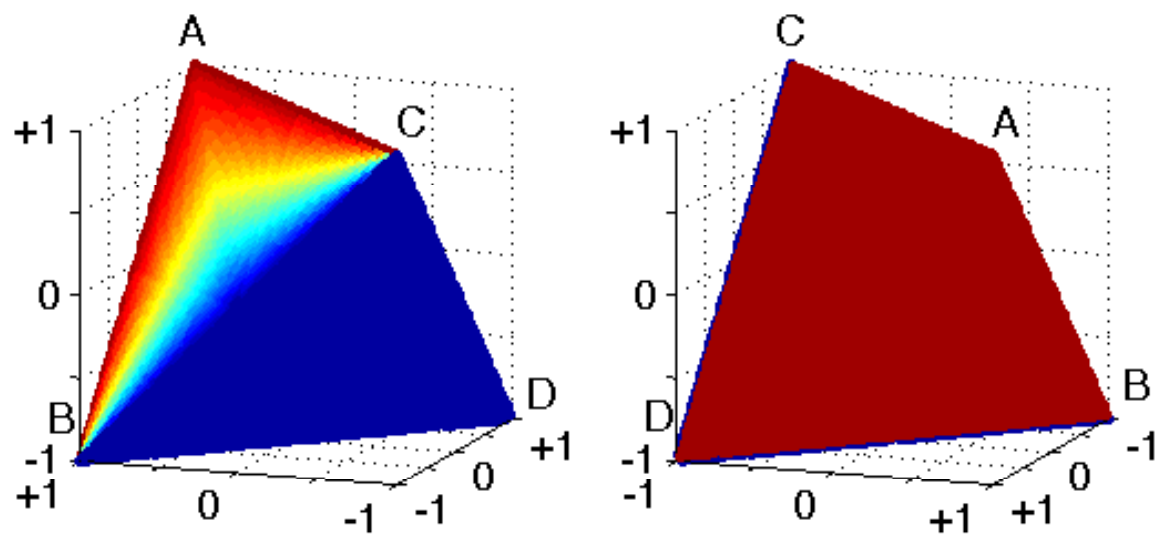


$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$

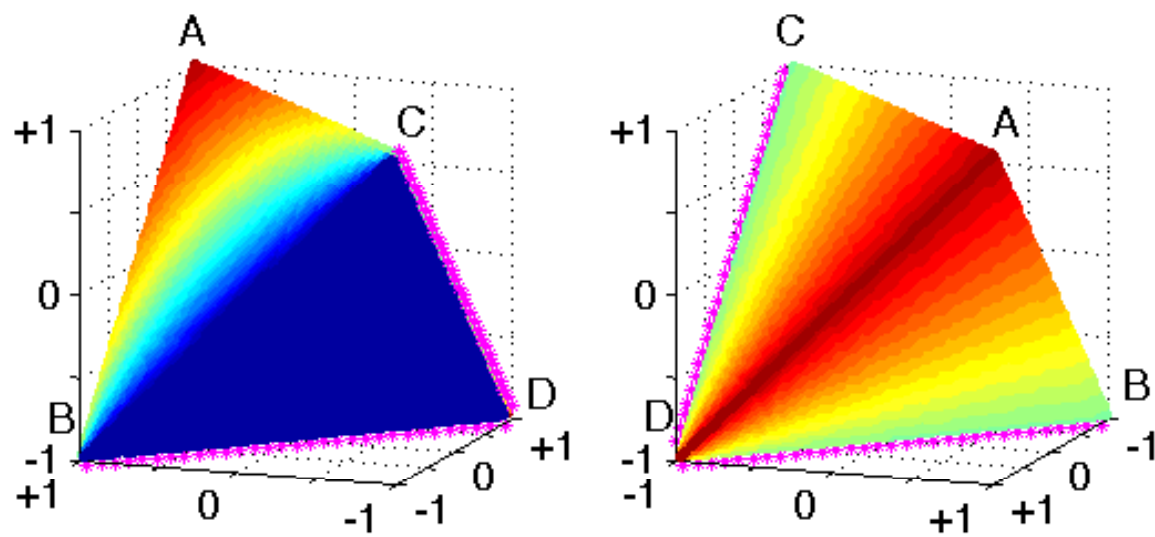


$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$

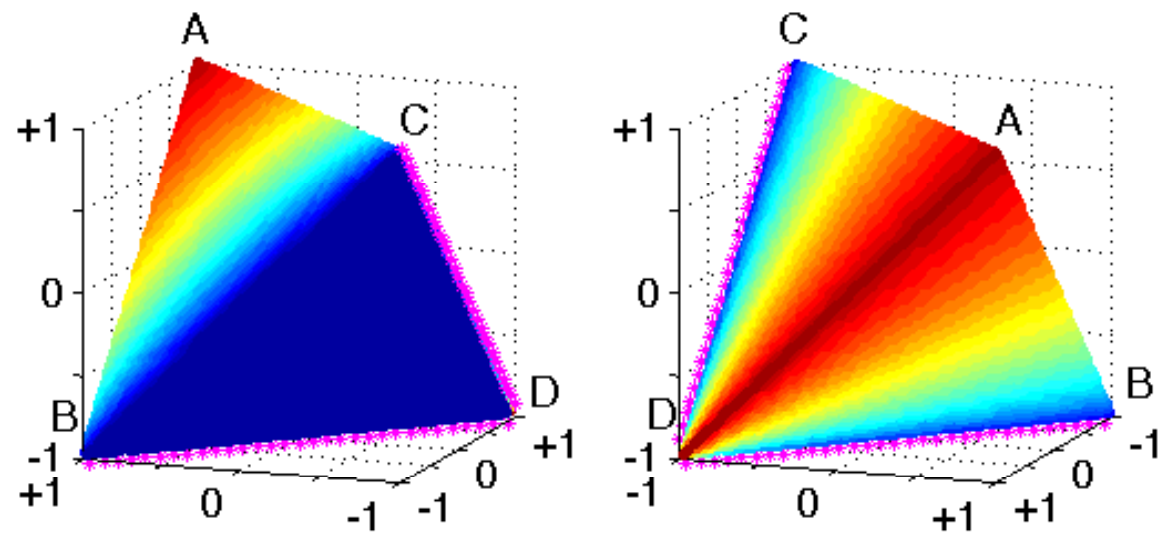
- sukcesja: $\max \rightarrow A \rightarrow G \rightarrow H \rightarrow \min$
 - wizualizacja a c / b d (+ rewers)



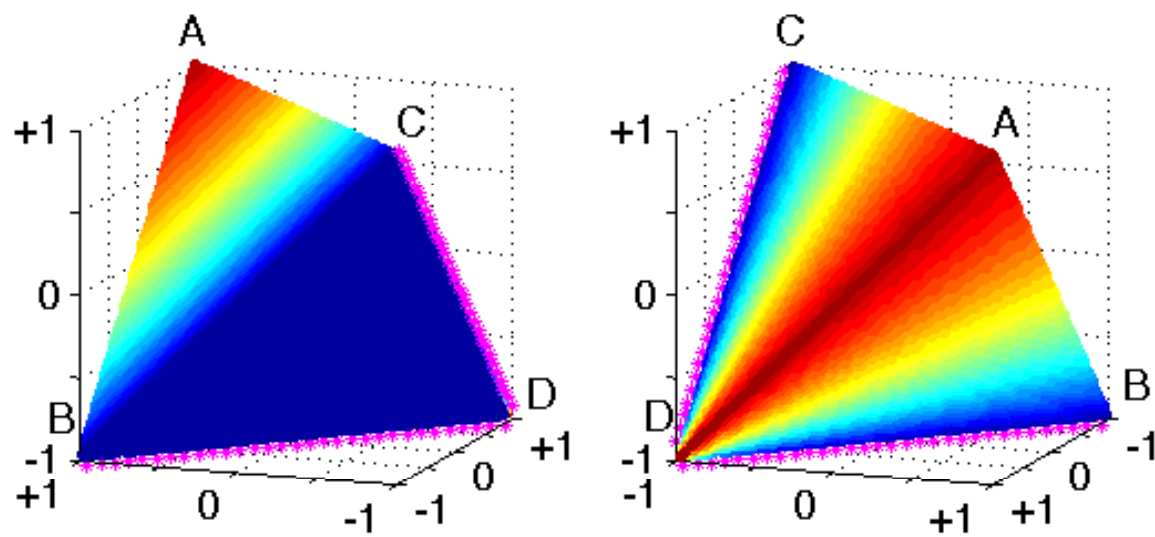
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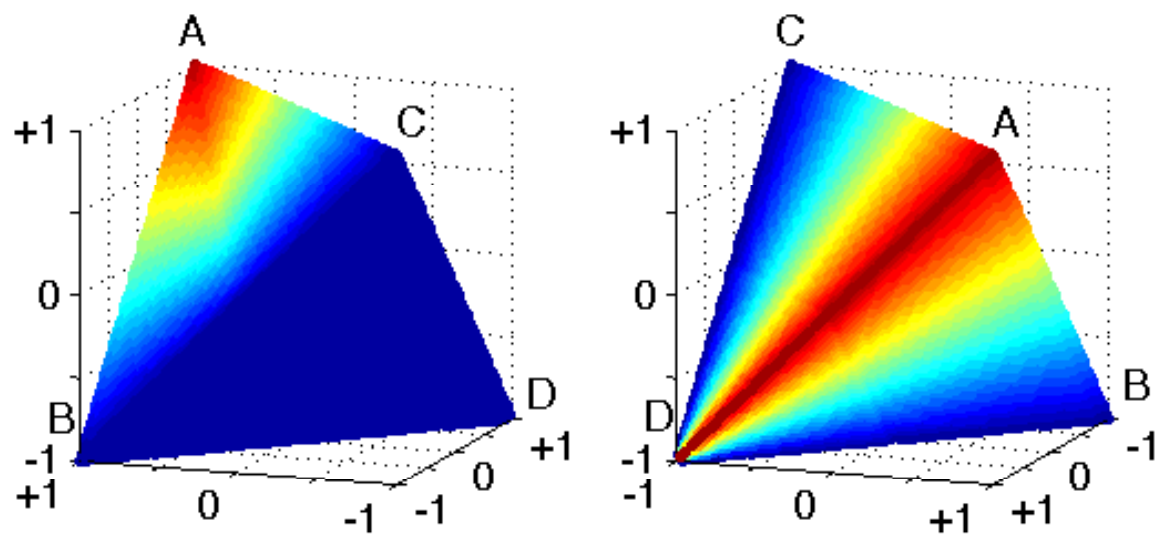
$$f(a,b,c,d) = \text{amean}(\text{precision}, \text{recall})$$



$$f(a,b,c,d) = \text{gmean}(\text{precision}, \text{recall})$$

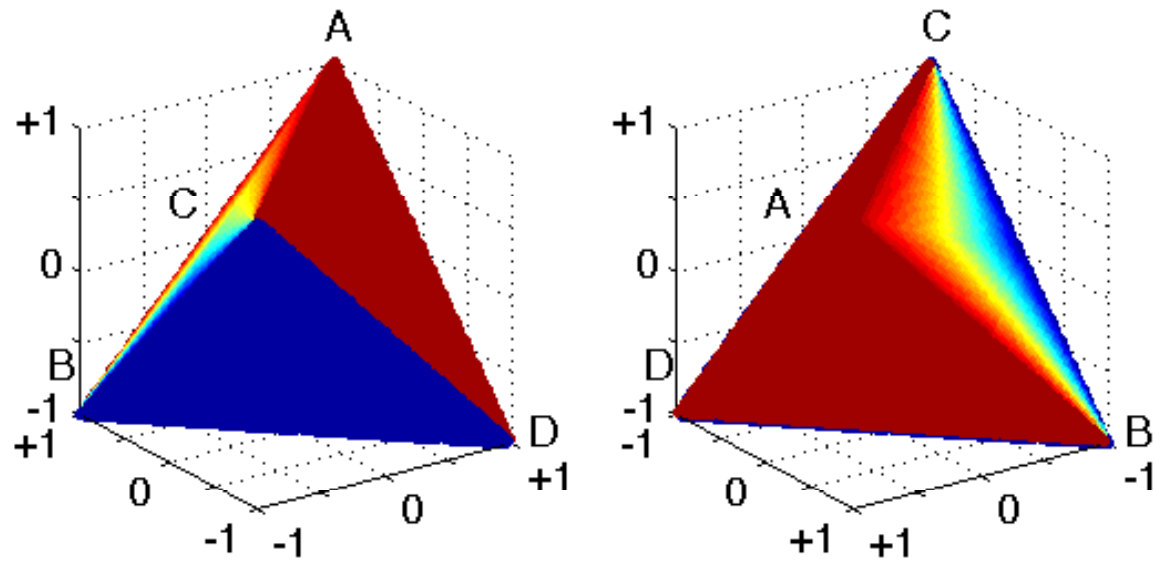


$$f(a,b,c,d) = \text{hmean}(\text{precision}, \text{recall})$$

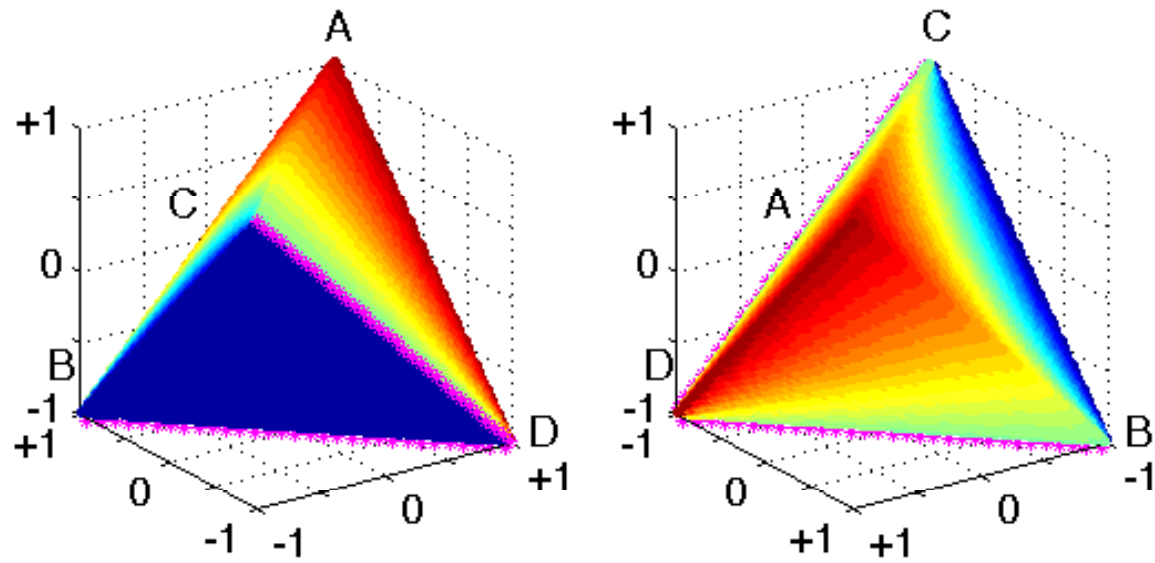


$$f(a,b,c,d) = \min(\text{precision}, \text{recall})$$

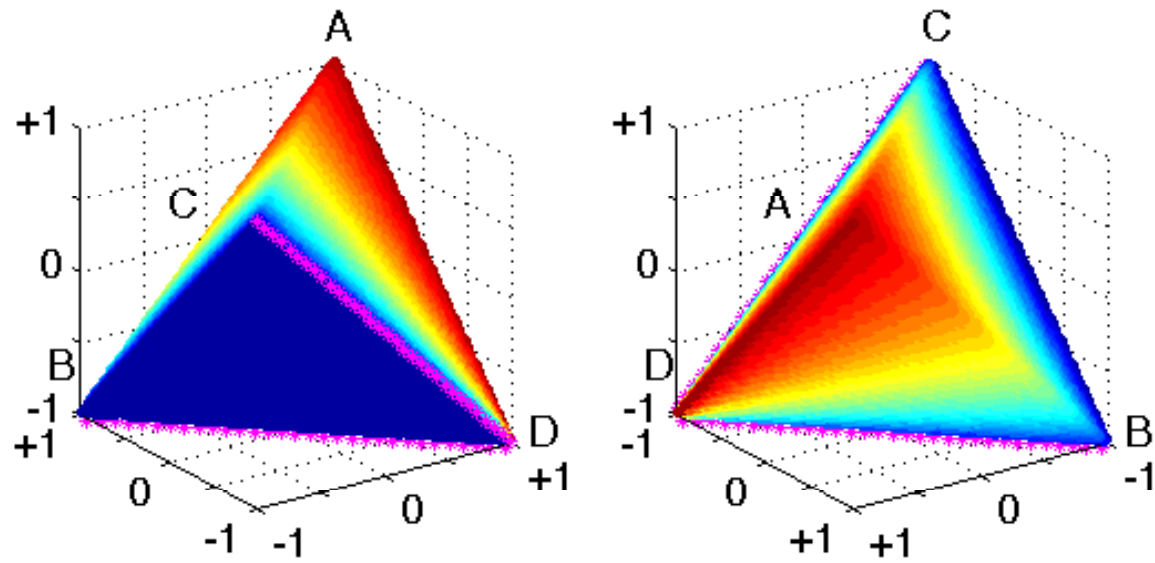
- sukcesja: $\max \rightarrow A \rightarrow G \rightarrow H \rightarrow \min$
 - wizualizacja c a / b d (+ rewers)



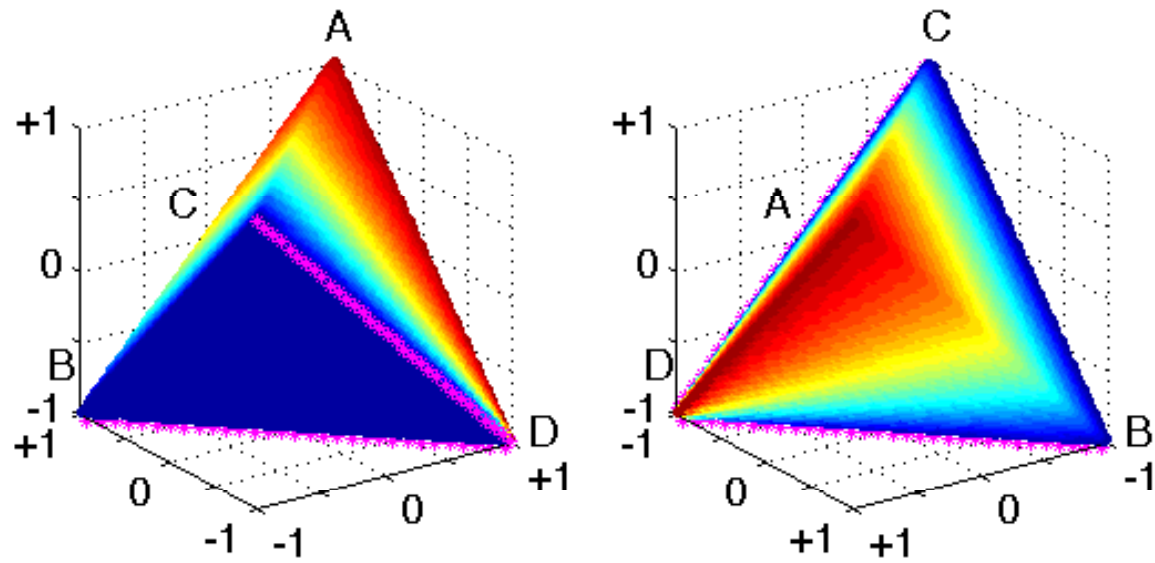
$$f(a,b,c,d) = \max(\text{precision}, \text{recall})$$



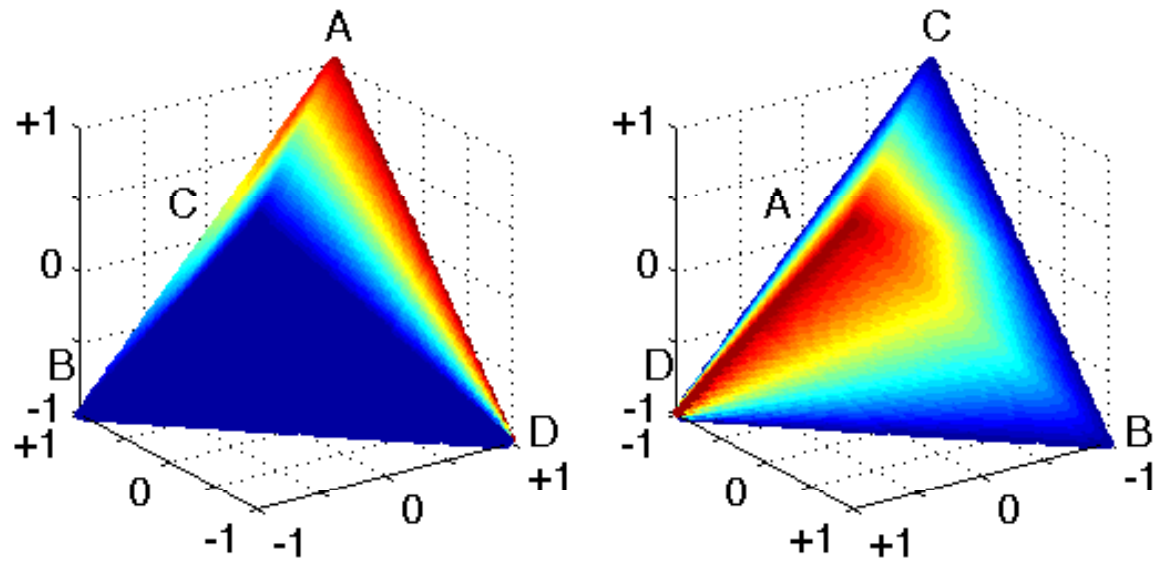
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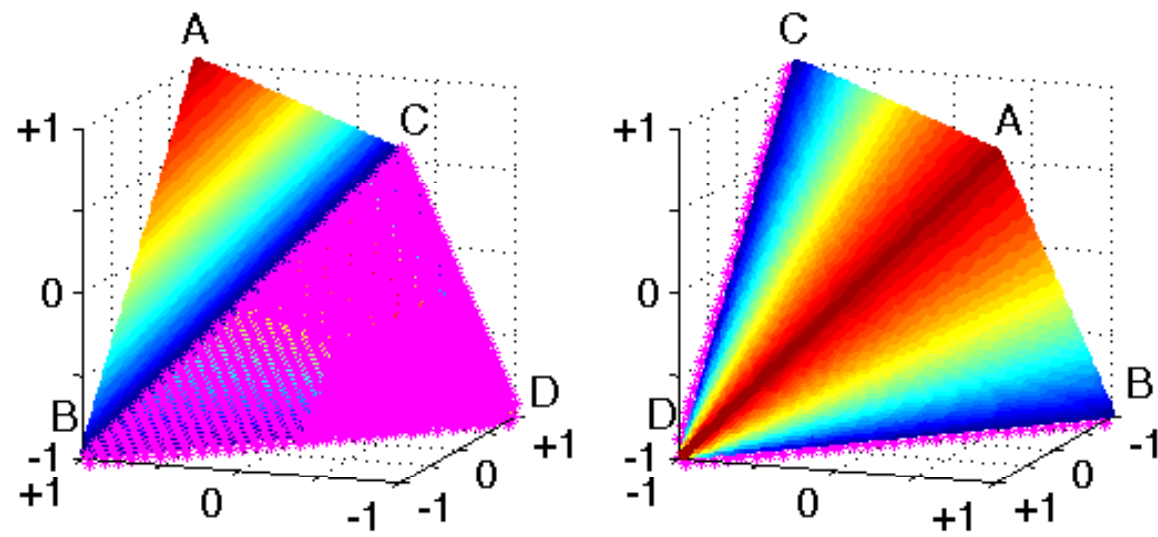


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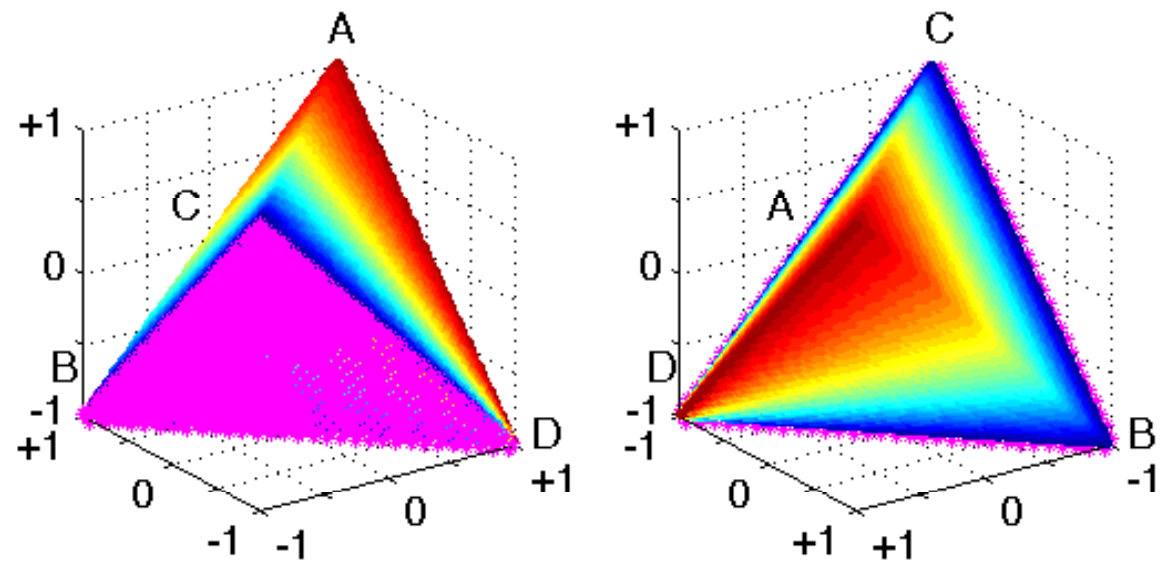


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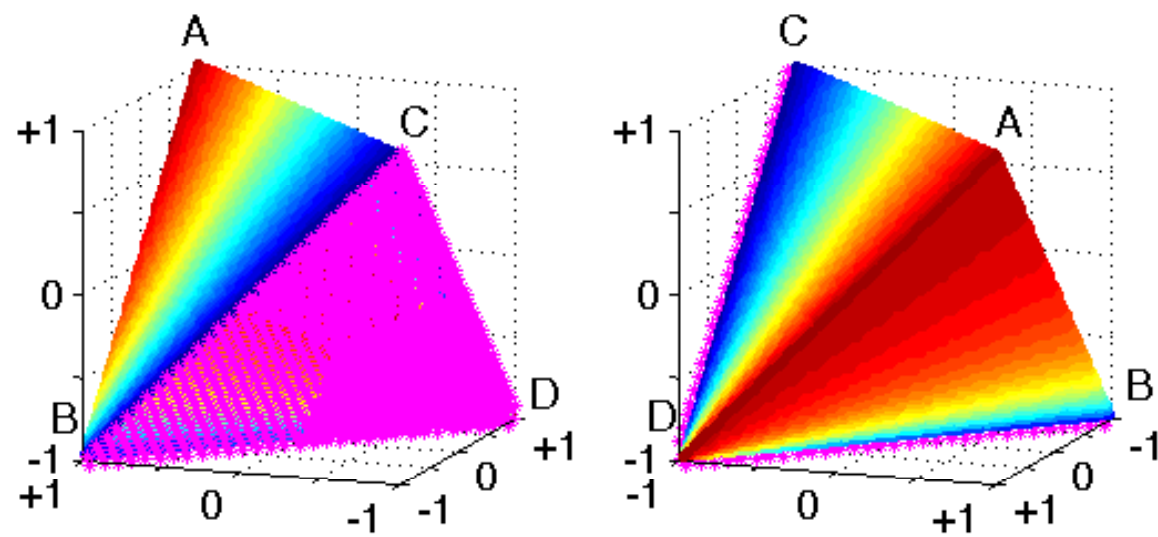
- Uogólnieniem F_1 (hmean) jest F_β (dla $\beta \neq 1$)



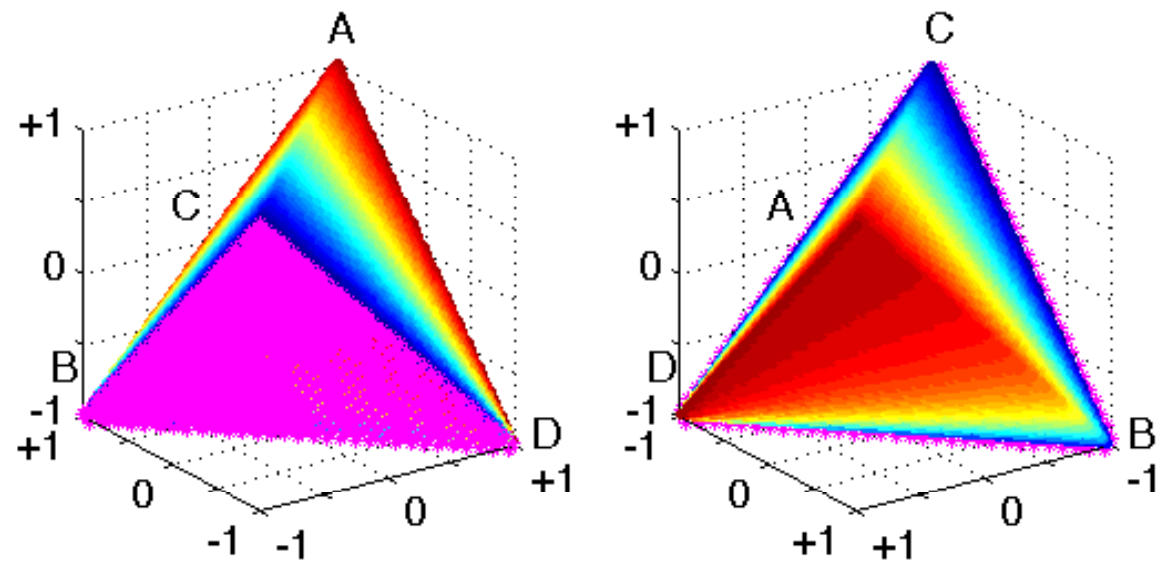
$$f(a,b,c,d) = F_1$$



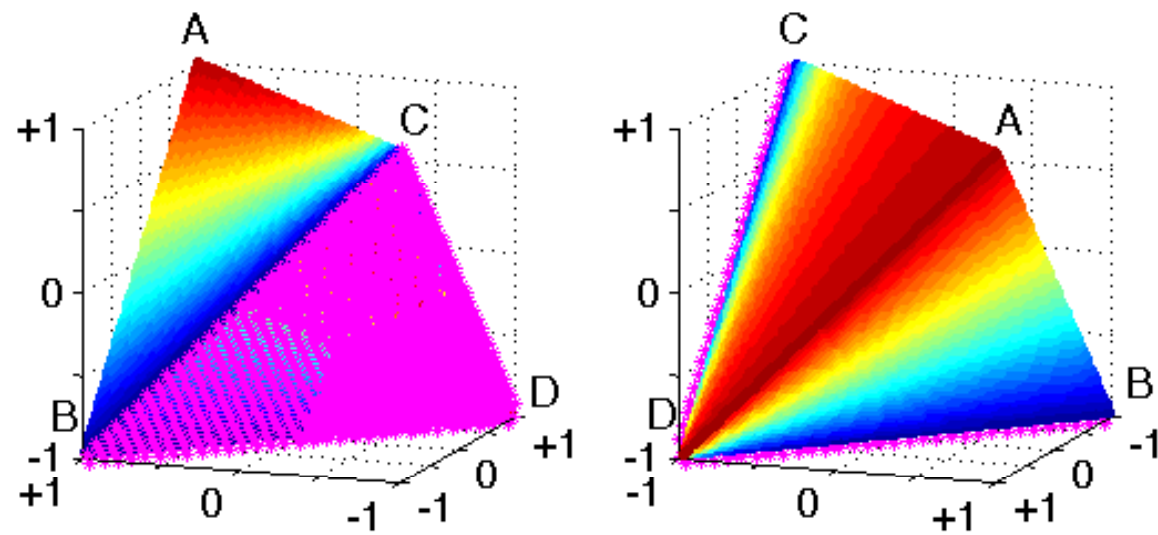
$$f(a,b,c,d) = F_1$$



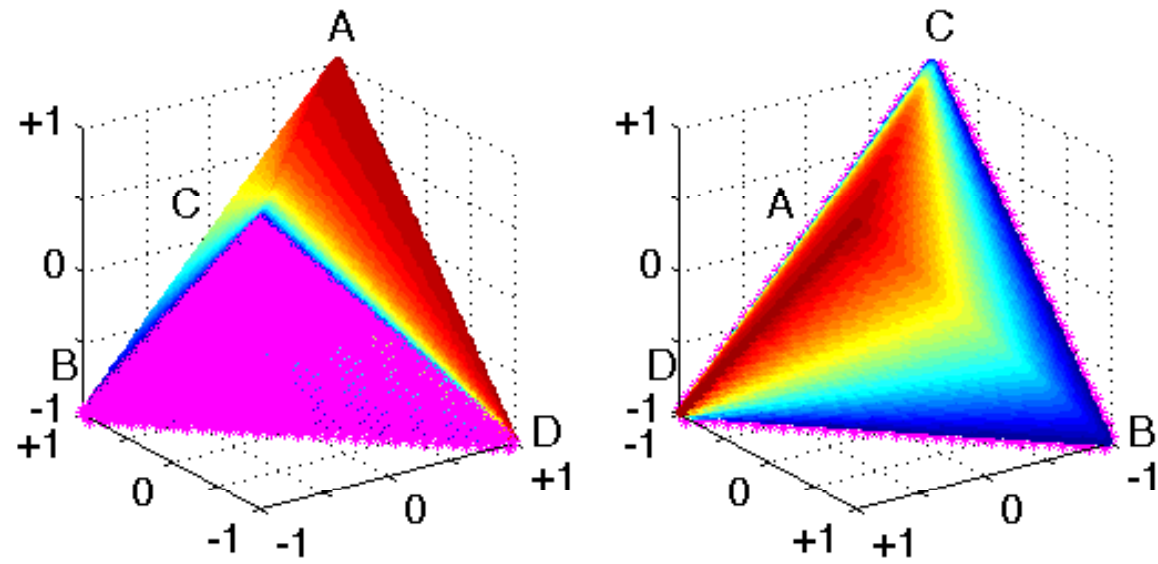
$$f(a,b,c,d) = F_3$$



$$f(a,b,c,d) = F_3$$



$$f(a,b,c,d) = F_{1/3}$$

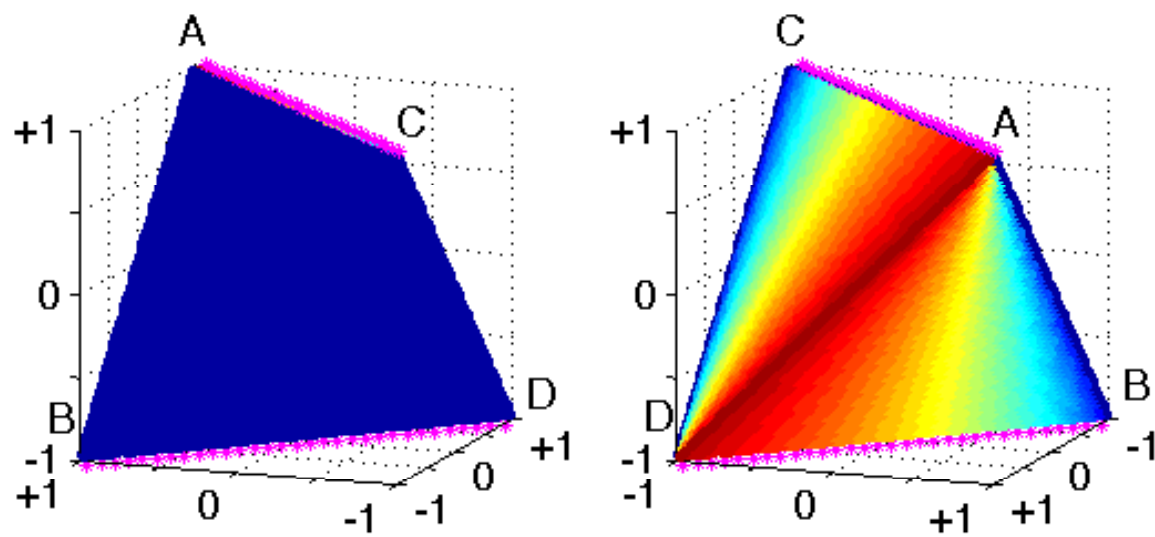


$$f(a,b,c,d) = F_{1/3}$$

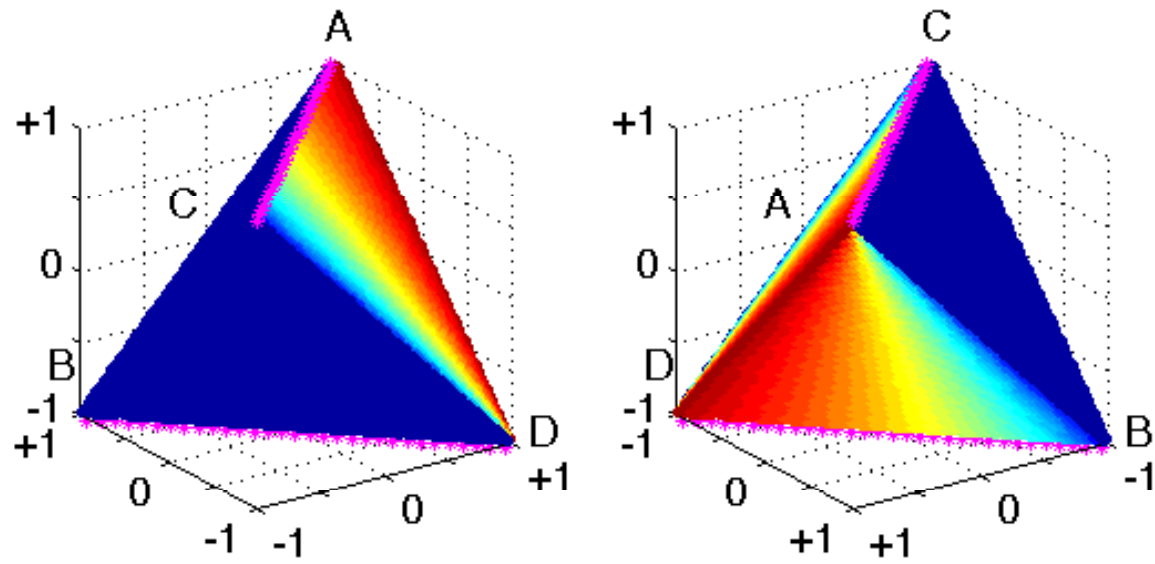
- G-mean:

- $\text{gmean}(\text{recall}, \text{specificity})$

- $\text{gmean}(\text{TP}/(\text{TP}+\text{FN}), \text{TN}/(\text{TN}+\text{FP})) = \text{gmean}(a/(a+c), d/(d+b))$

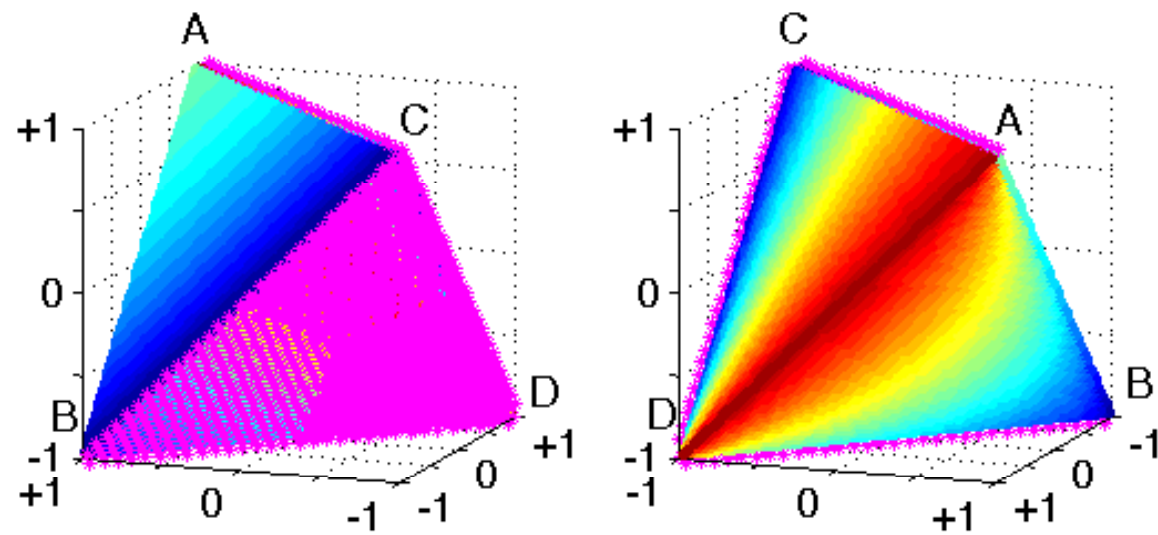


$$f(a,b,c,d) = \text{G-mean}$$

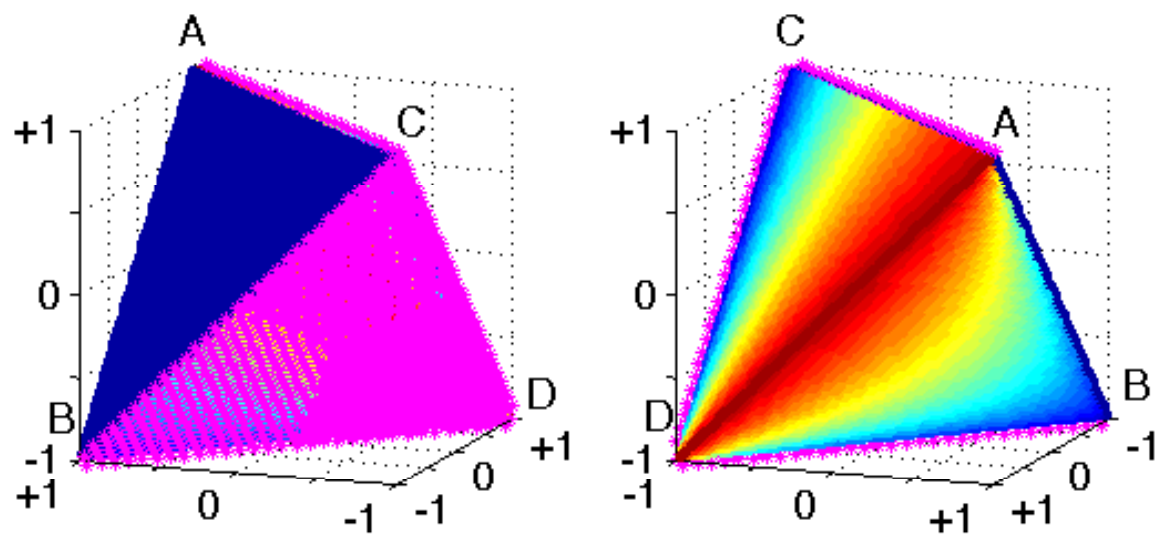


$$f(a,b,c,d) = \text{G-mean}$$

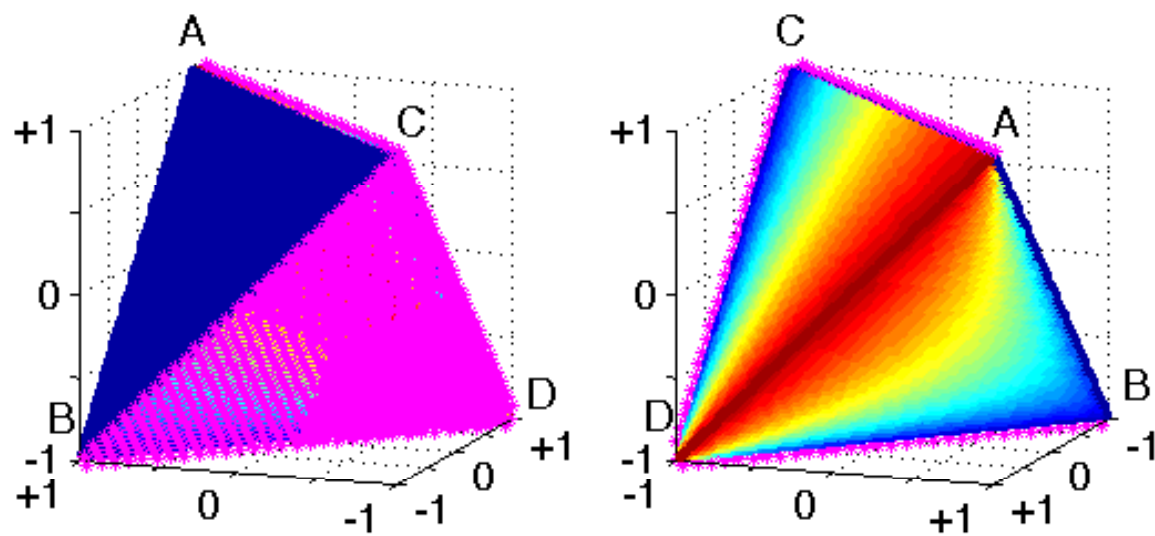
- kombinacije F_β i G-mean?



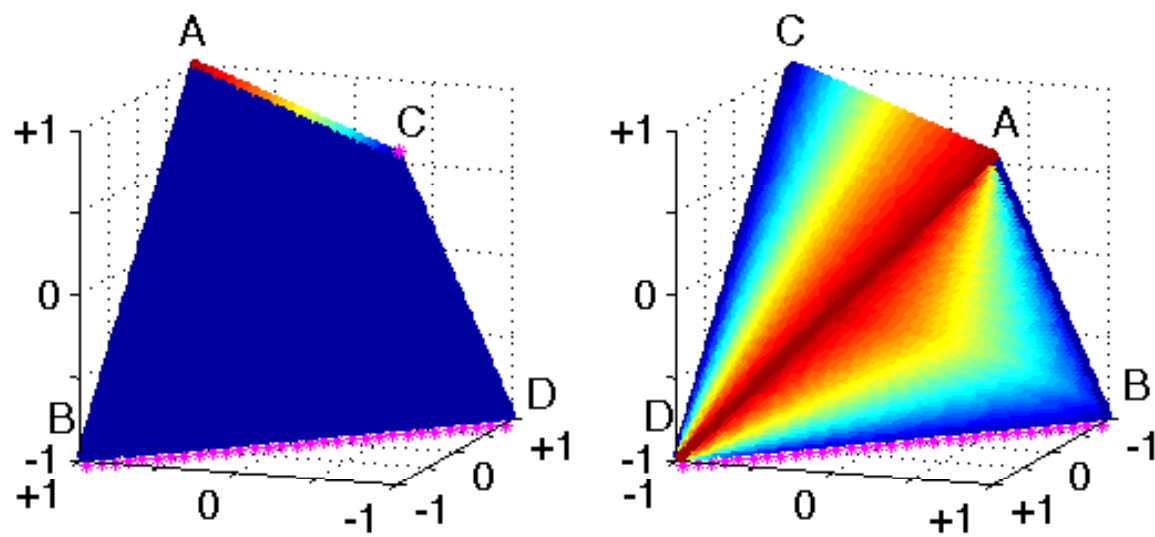
$$f(a,b,c,d) = \text{amean}(F_\beta, \text{G-mean})$$



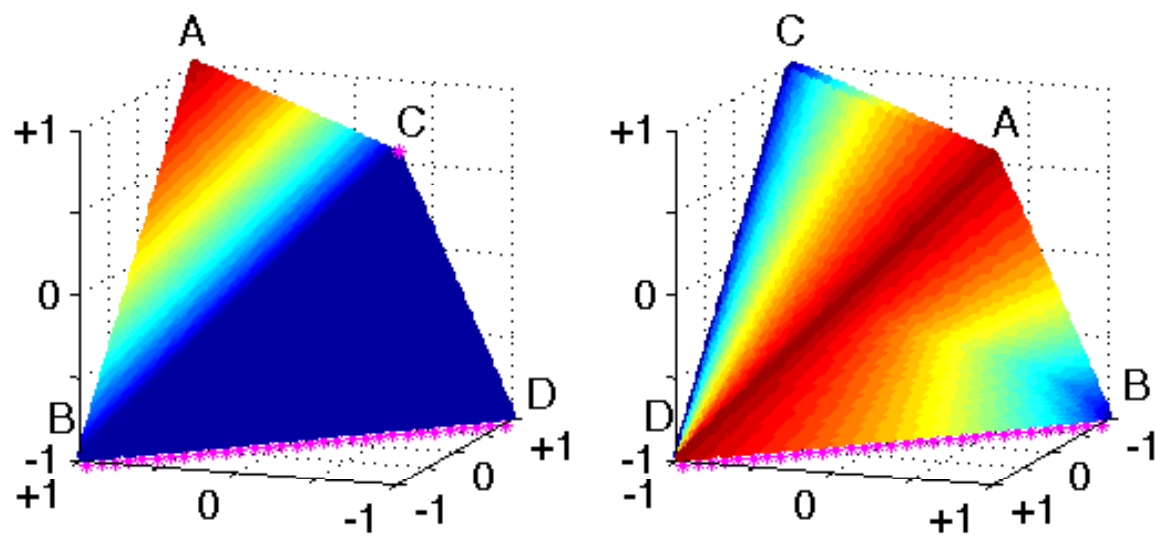
$$f(a,b,c,d) = \text{gmean}(F_{\beta}, \text{G-mean})$$



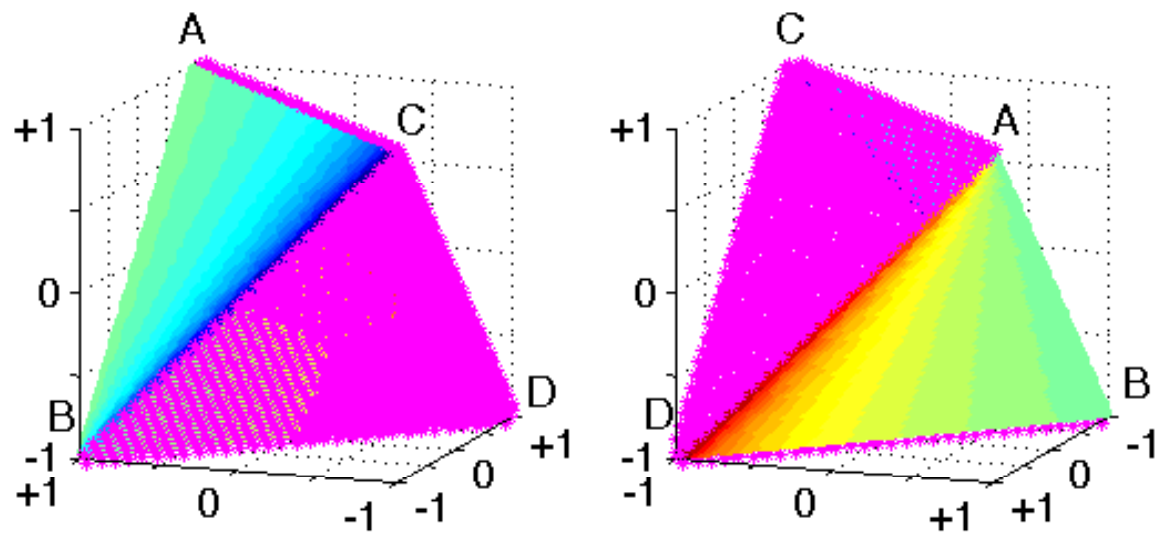
$$f(a,b,c,d) = \text{hmean}(F_{\beta}, \text{G-mean})$$



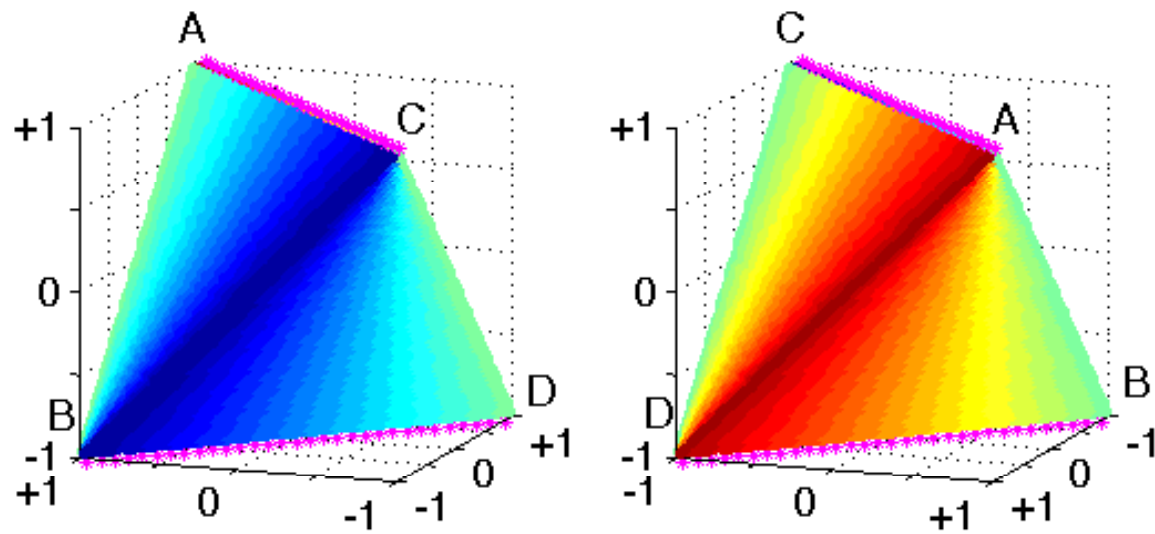
$$f(a,b,c,d) = \min(F_\beta, G\text{-mean})$$



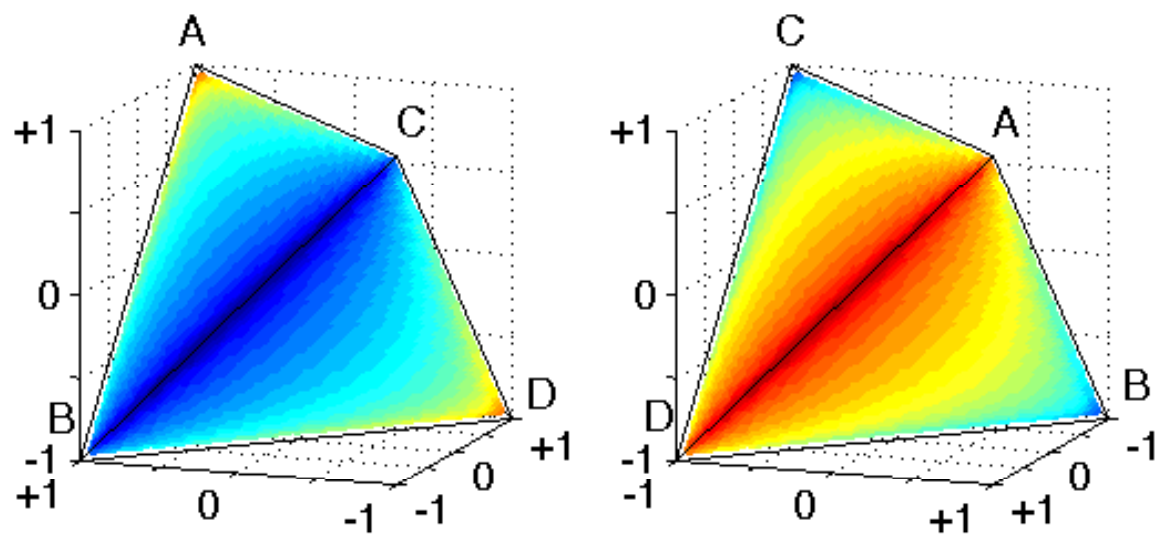
$$f(a,b,c,d) = \max(F_\beta, G\text{-mean})$$



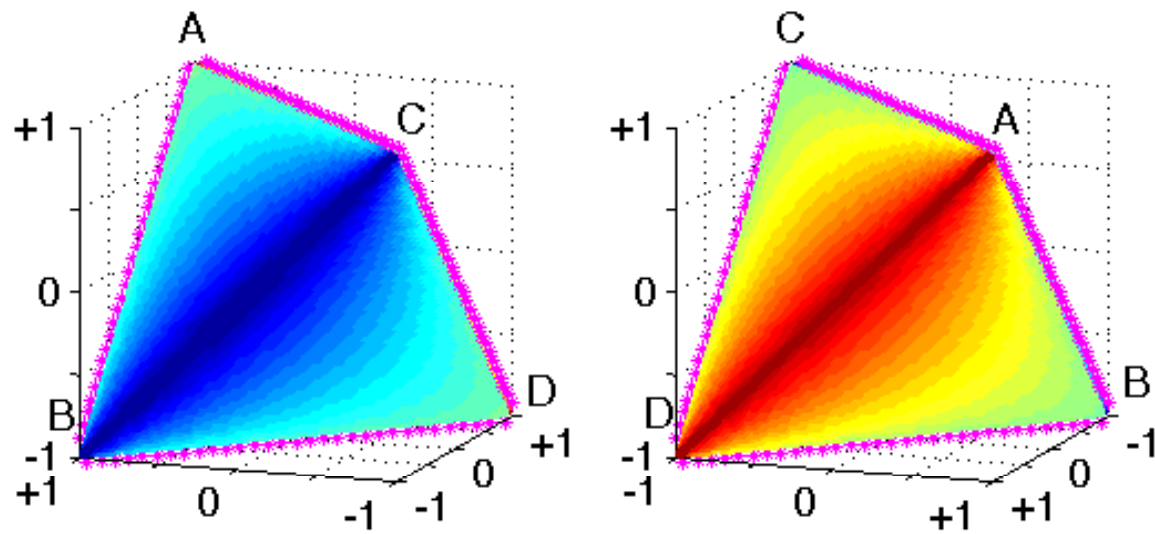
$f(a,b,c,d) = \text{Positive likelihood ratio}$



$f(a,b,c,d) = \text{Balanced accuracy}$



$f(a,b,c,d) = \text{Discriminant power}$



$$f(a,b,c,d) = \text{MCC}$$

...

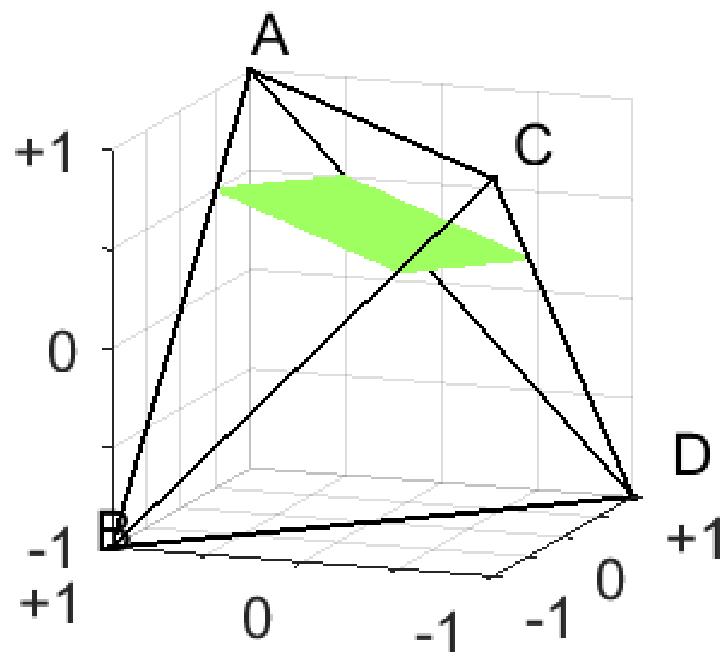
		predicted	
		1	0
original	1	TP A a	FN C c
	0	FP B b	TN D d

- Cel rozważań: relacje pomiędzy licznosciami klas

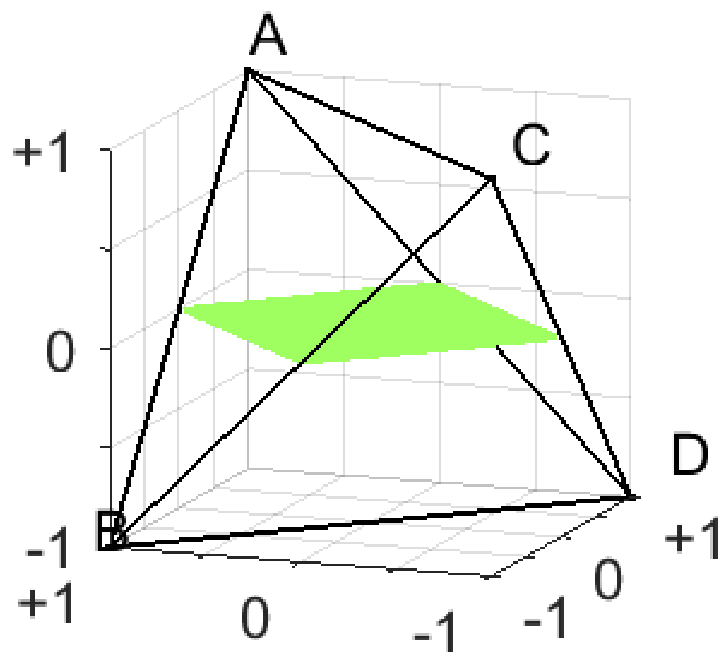
- liczność:
 - klasy 1: $a+c$
 - klasy 0: $b+d$

- wizualizacja ustalonej proporcji klas, np. $p : q$
(przy założeniu, że $a+b+c+d = 1$)
 - przecięcie czworościanu z pozioma płaszczyzną
(przebiegająca na poziomie wyznaczonym przez p i q)

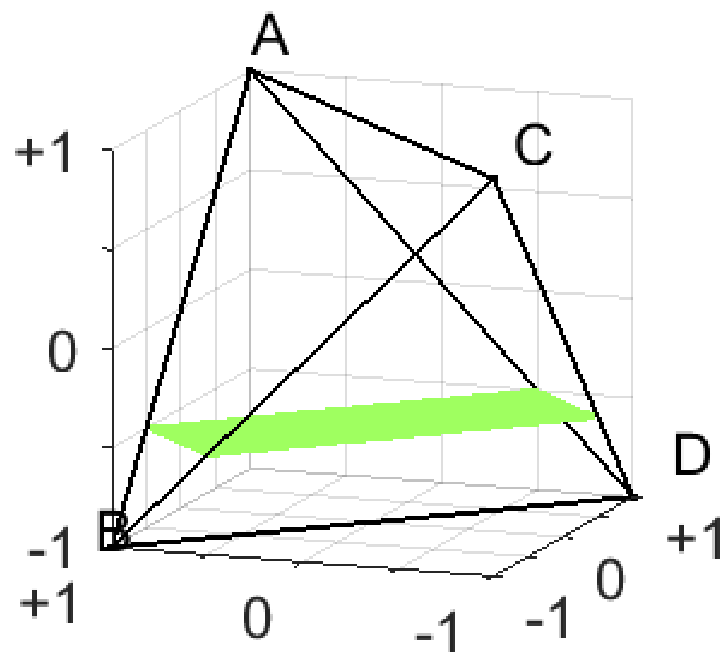
przewaga klasy 1



równowaga klas

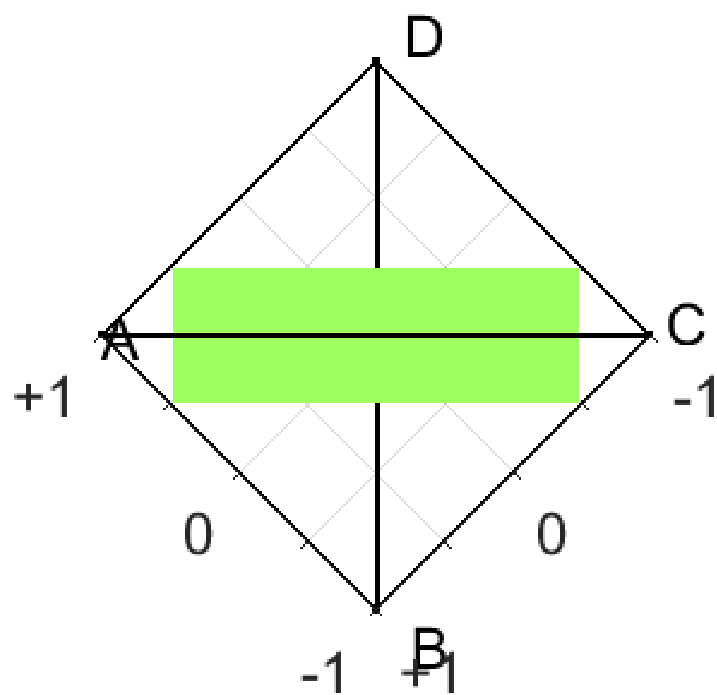


przewaga klasy 0

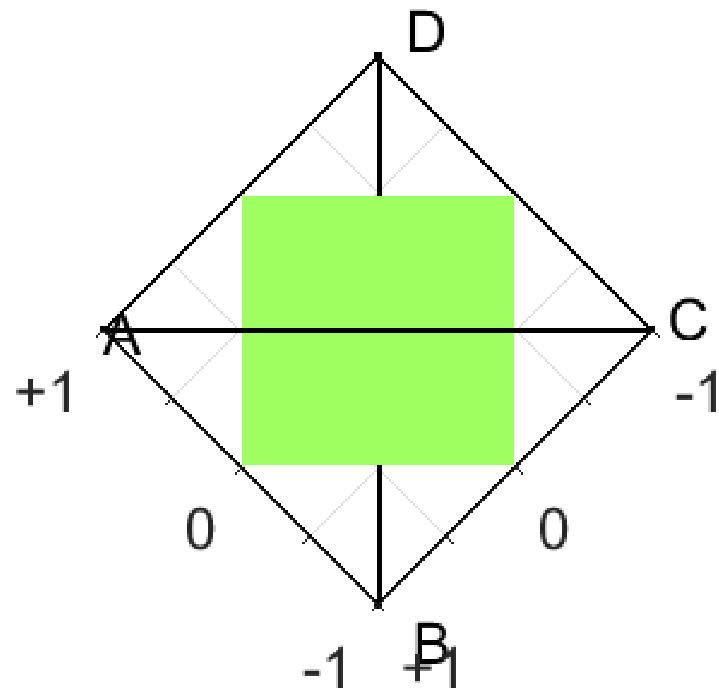


- dla ułatwienia: widok z góry
(figura dodatkowo lekko obrócona w lewo)

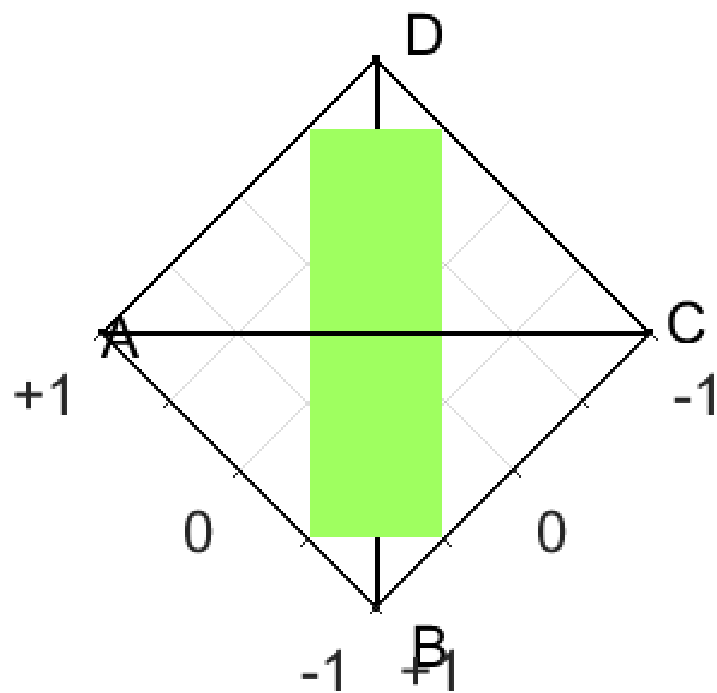
przewaga klasy 1



równowaga klas

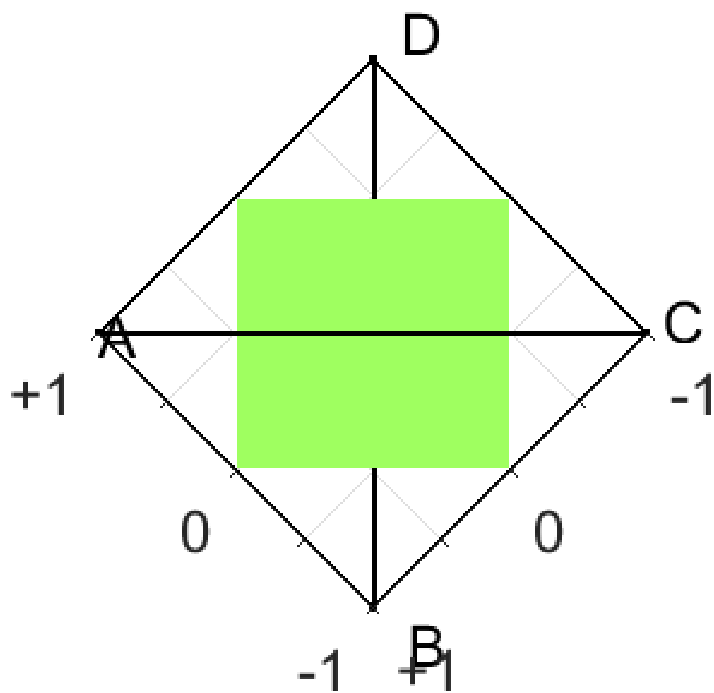


przewaga klasy 0



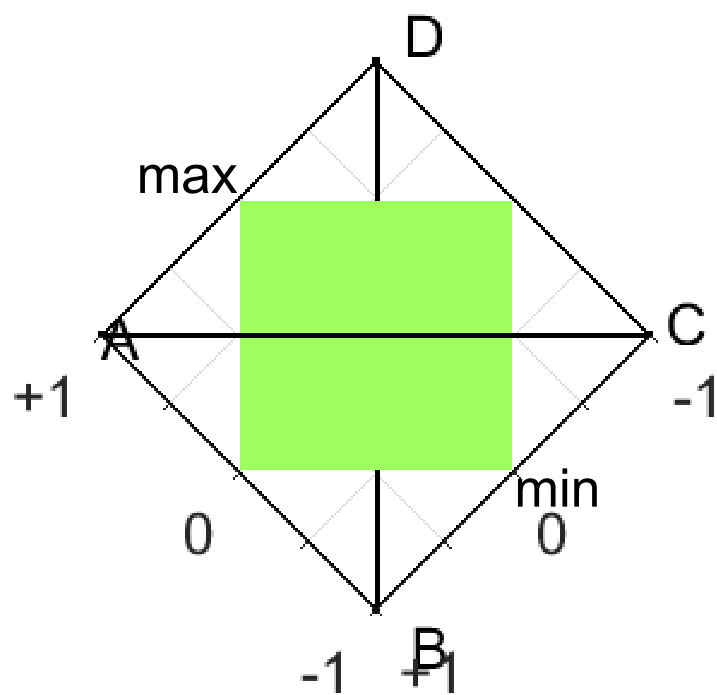
- na początek
 - równość klas: $a+c = b+d$

równowaga klas



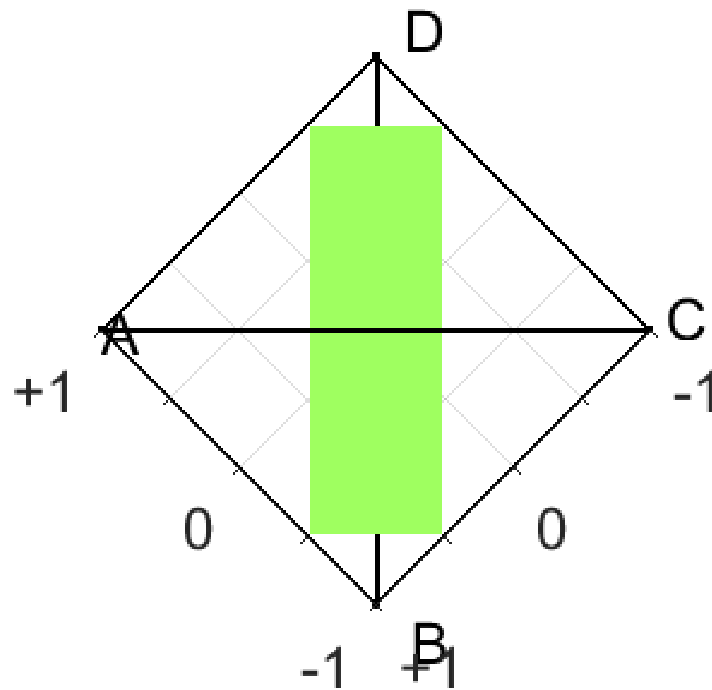
pożądany przebieg/wygląd miar?

równowaga klas

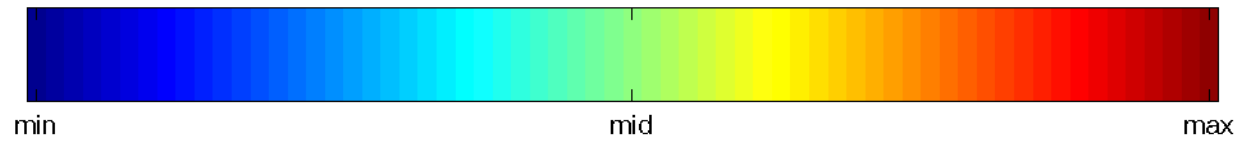


pożądany przebieg/wygląd miar

przewaga klasy 0

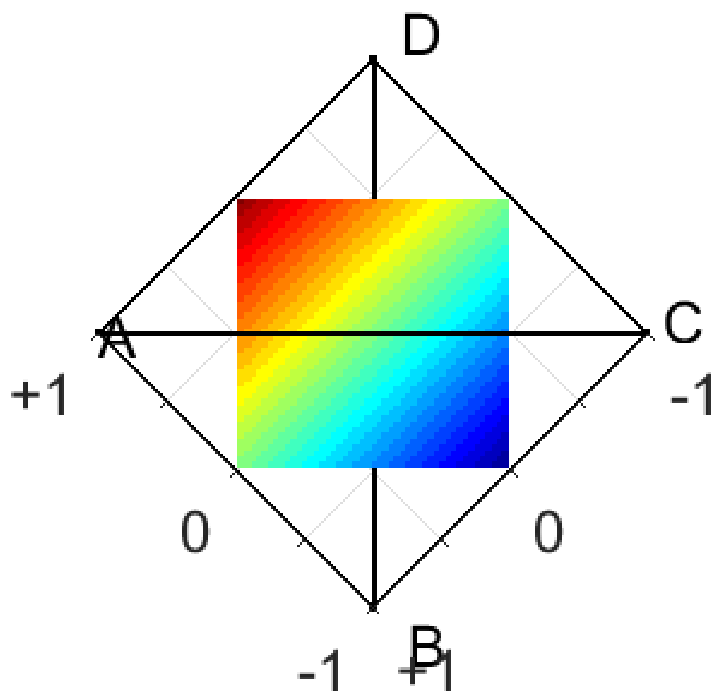


a teraz: pożądaný przebieg/wygląd miar???



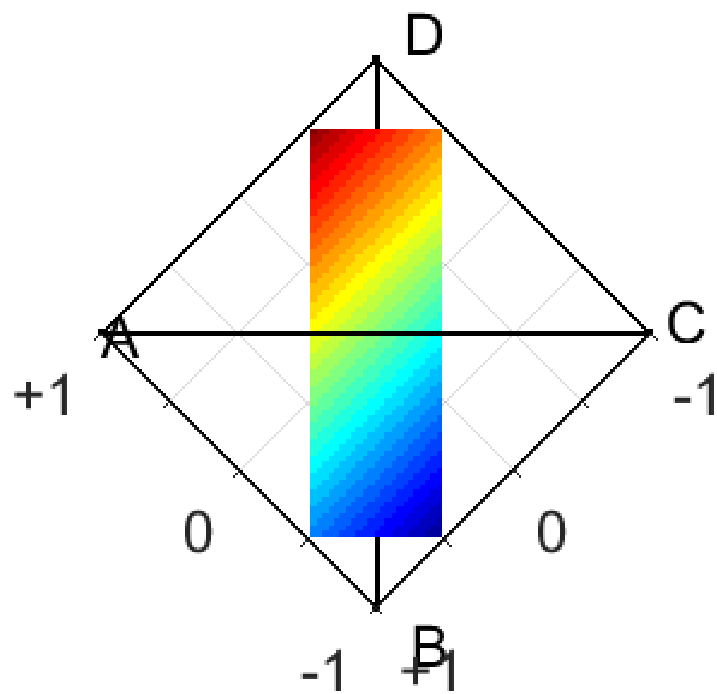
- miara na początek:
 - CA

równowaga klas



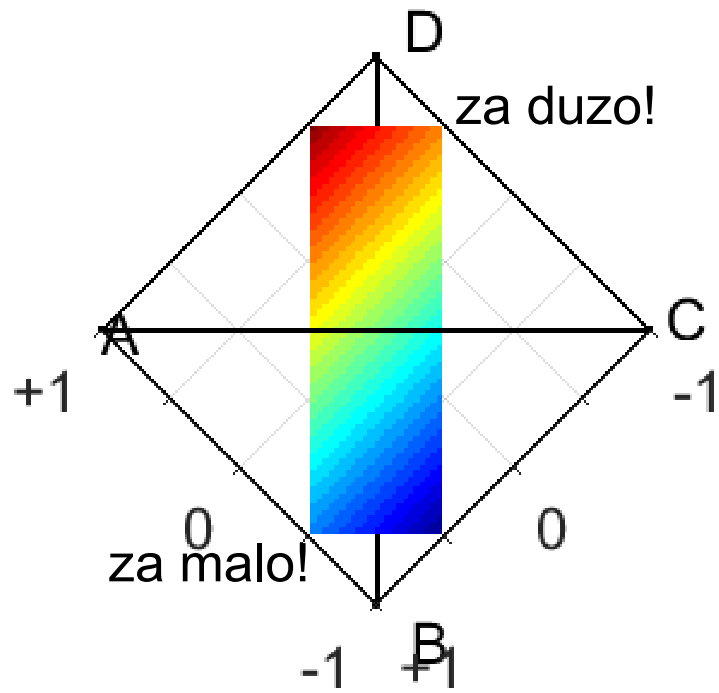
CA

przewaga klasy 0



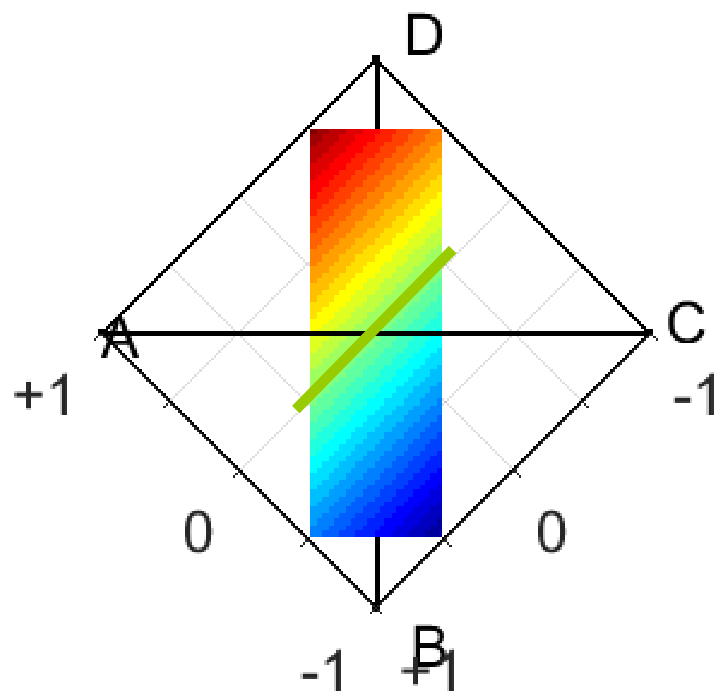
CA

przewaga klasy 0



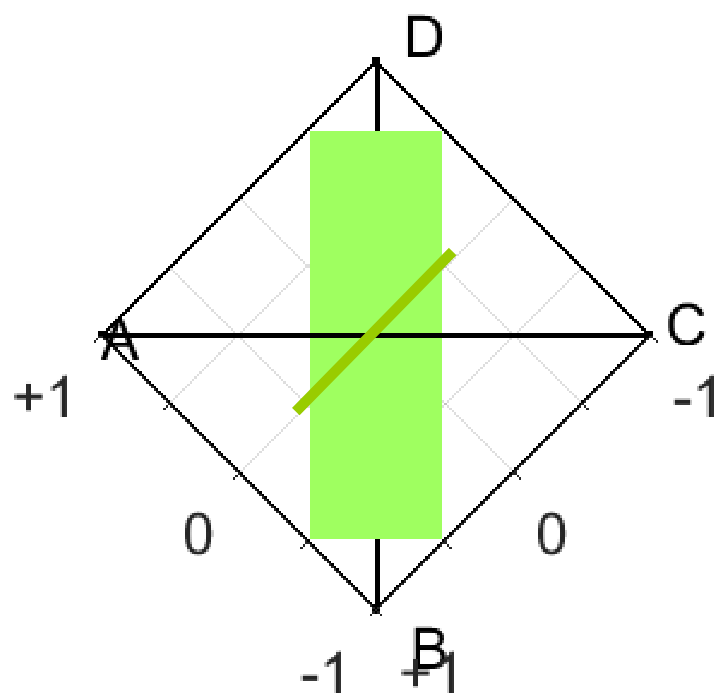
CA

przewaga klasy 0



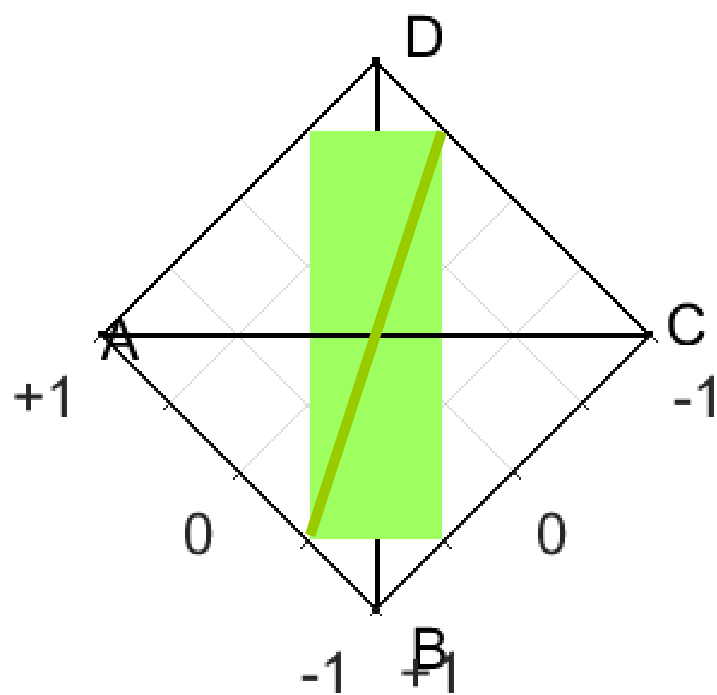
CA

przewaga klasy 0



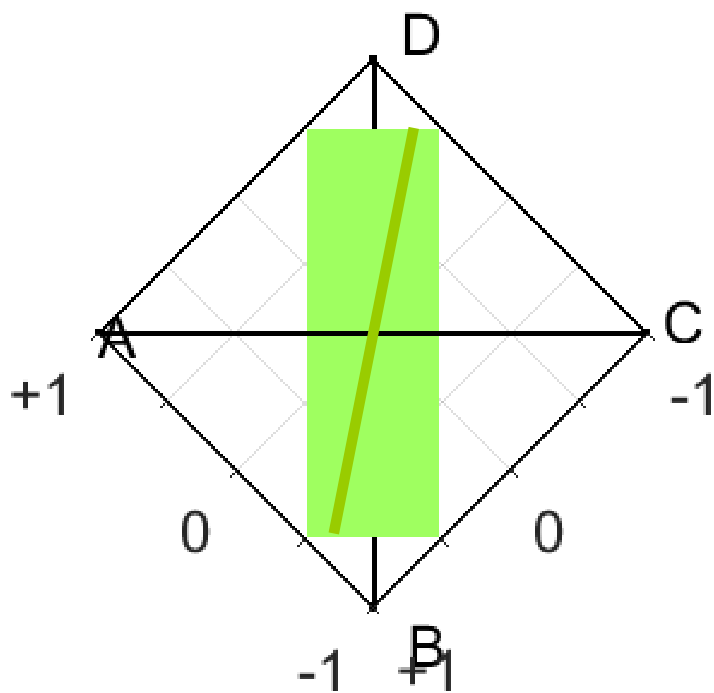
jak CA

przewaga klasy 0



lepiej? niż CA

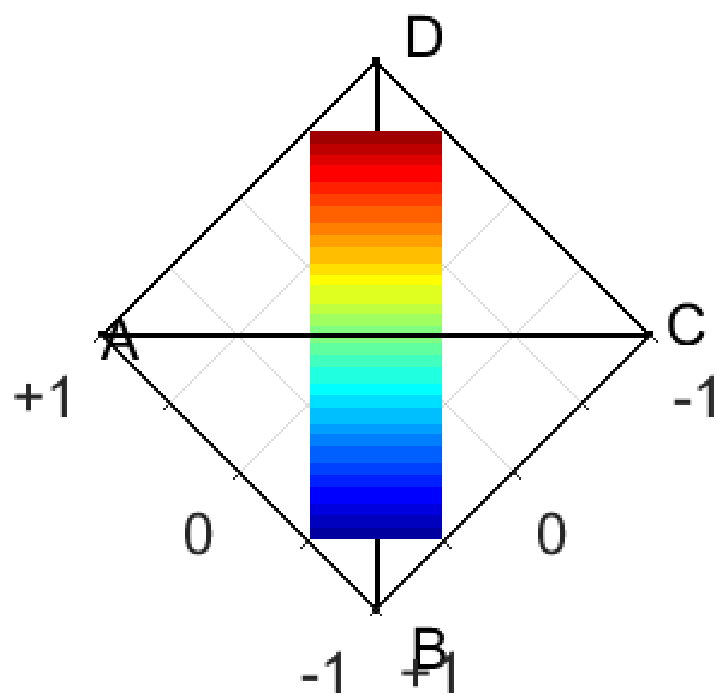
przewaga klasy 0



(przesadnie) lepiej? niż CA

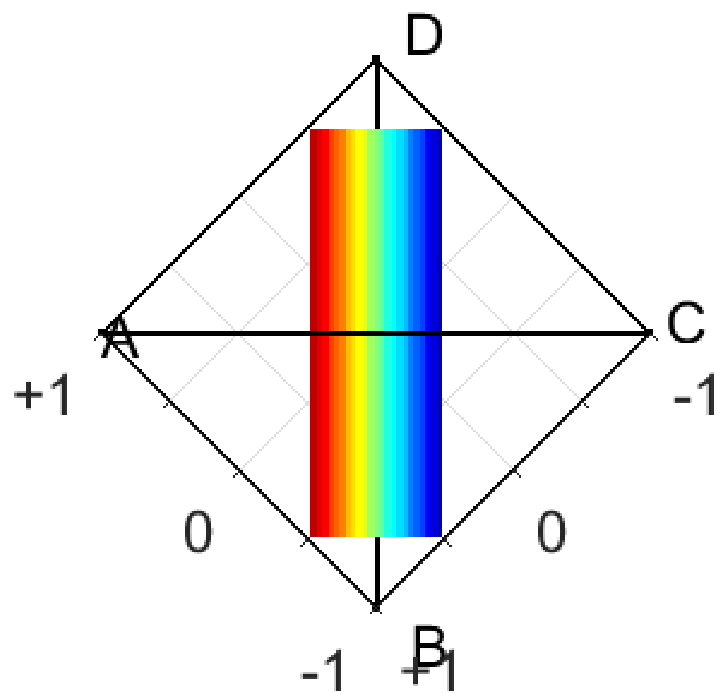
- inne miary:

przewaga klasy 0



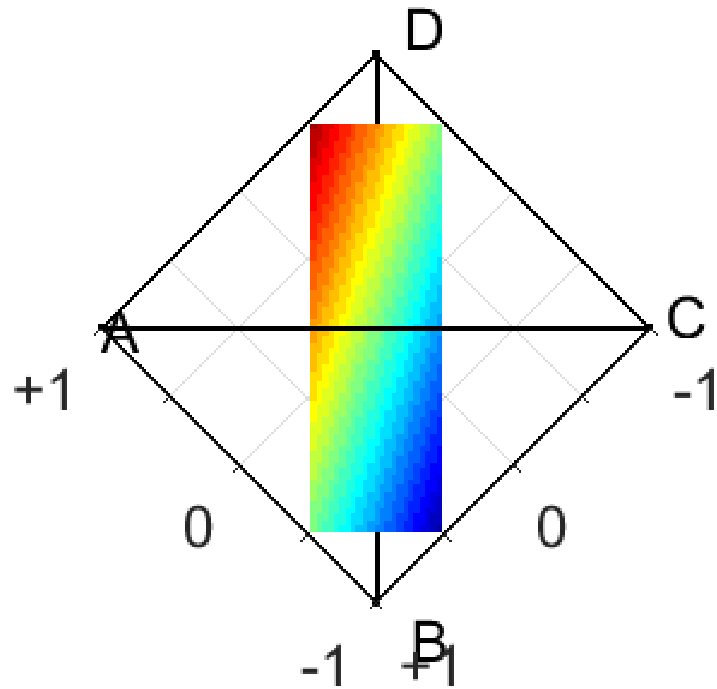
specificity

przewaga klasy 0



recall

przewaga klasy 0



`amean(specificity,recall)`

- kombinacje wypukłe

- $p, q \geq 0$

- $p+q = t$, gdzie $t > 0$

- wtedy

- $p/(p+q) \geq 0, q/(p+q) \geq 0$

- $p/(p+q) + q/(p+q) = 1$

- współczynniki kombinacji wypukłej

- ułamki $p/(p+q)$ i $q/(p+q)$ sterują proporcjami p i q

- $p/(p+q) = 0$ oznacza, że $p = 0, q = t$

- $p/(p+q) = 1/2$ oznacza, że $p = t/2, q = t/2$

- $p/(p+q) = 1$ oznacza, że $p = t, q = 0$

- (analogicznie dla $q/(p+q)$)

- warunki dot. a, b, c, d
 - $a, b, c, d \geq 0$
 - $a+b+c+d = 1$

- przypadki dotyczące proporcji klas

- $a+c = p$

- $b+d = q$

gdzie

- $p, q \geq 0$

- $p+q = 1 > 0$

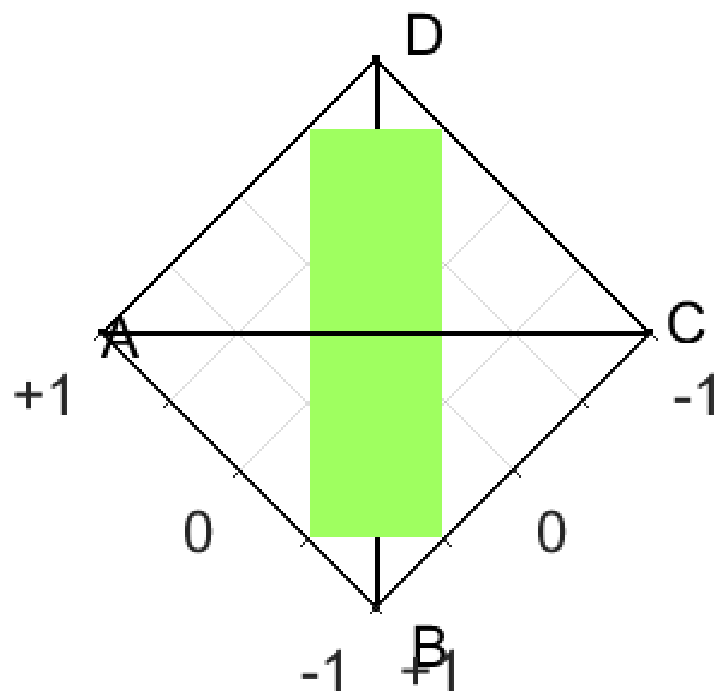
- wtedy

- ułamki $(a+c)/(a+b+c+d)$ i $(b+d)/(a+b+c+d)$ sterują proporcjami klas

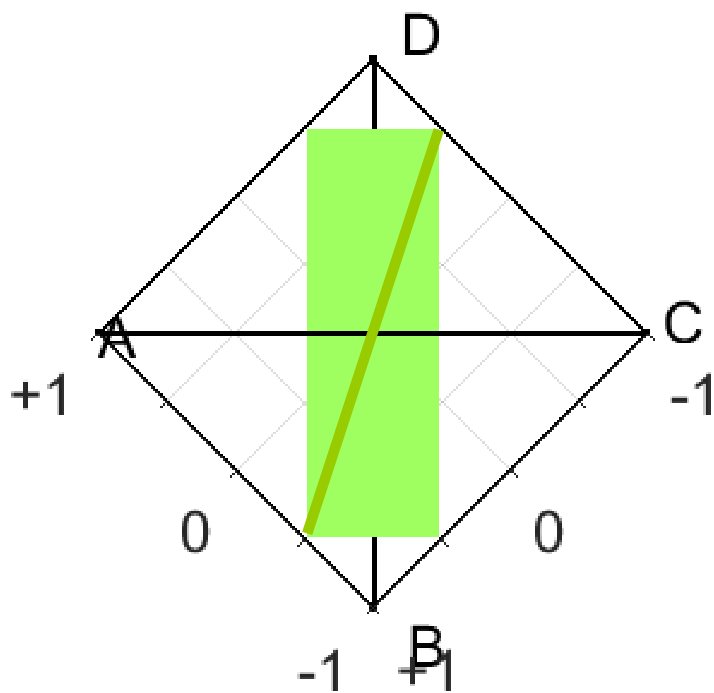
- dla $(a+c)/(a+b+c+d) = p$ mamy $(b+d)/(a+b+c+d) = 1 - p = q$

- dla ustalonych $a+c = p$ i $b+d = q$ zmianom mogą podlegać
 - a i c (w ramach sumy $a+c = p$)
 - współczynniki kombinacji wypukłej
 - ułamki $a/(a+c)$ i $c/(a+c)$ sterują proporcjami a i c
 - $a/(a+c) = \text{recall}$
 - $c/(a+c) = 1 - \text{recall}$
 - b i d (w ramach sumy $b+d = q$)
 - współczynniki kombinacji wypukłej
 - ułamki $b/(b+d)$ i $d/(b+d)$ sterują proporcjami b i d
 - $b/(b+d) = 1 - \text{specificity}$
 - $d/(b+d) = \text{specificity}$

przewaga klasy 0

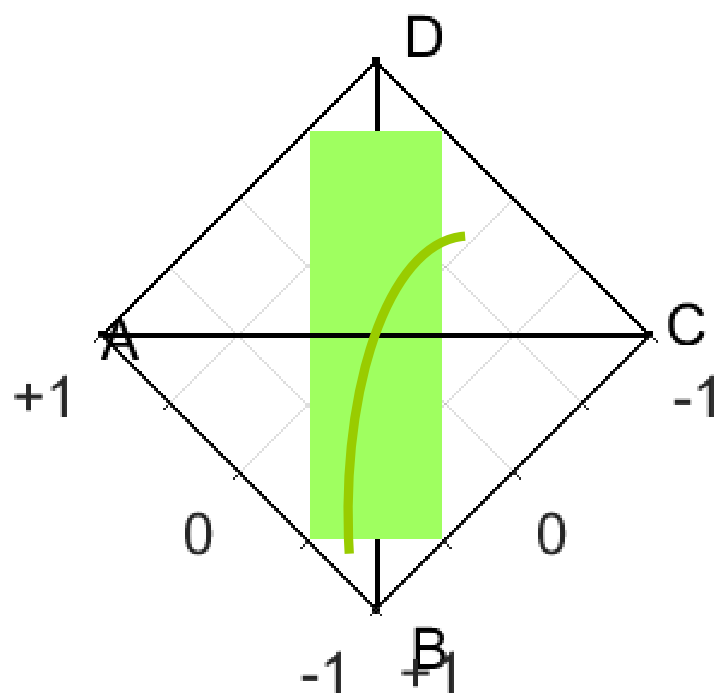


przewaga klasy 0



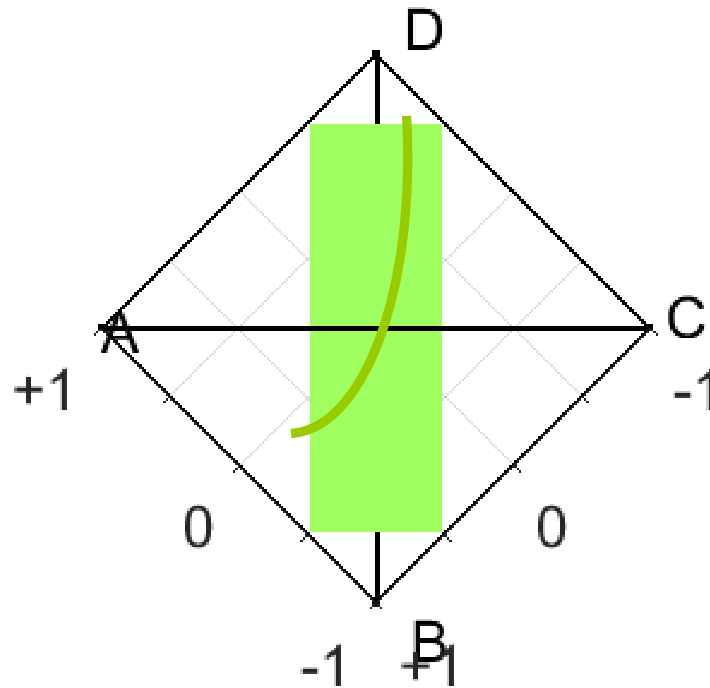
agregacja typu amean

przewaga klasy 0



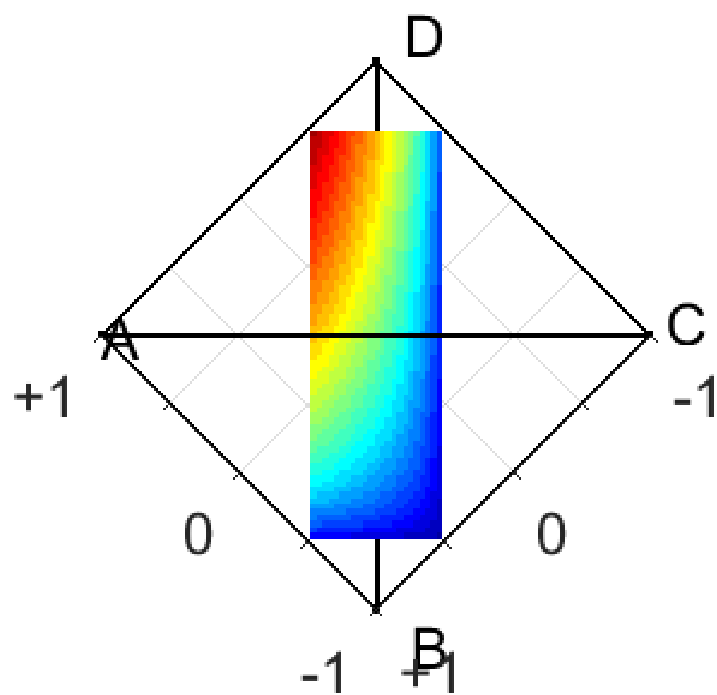
agregacja typu max

przewaga klasy 0



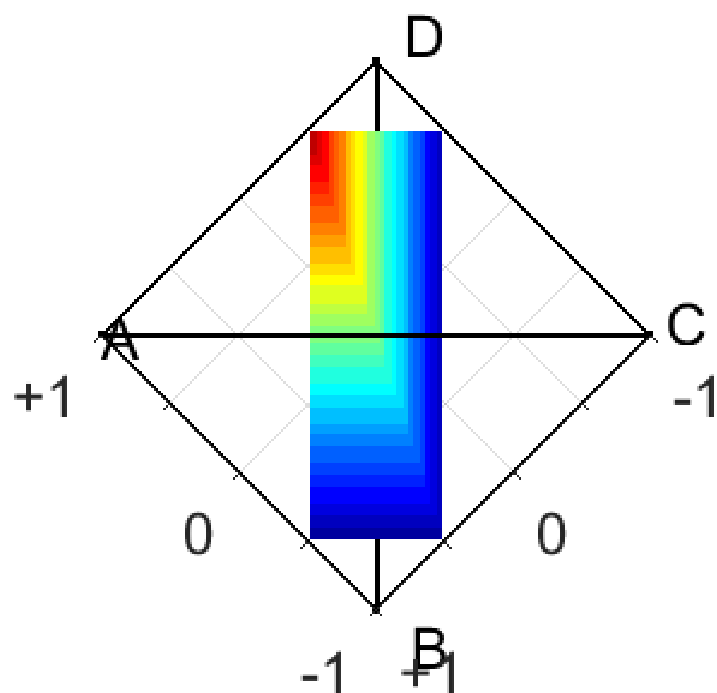
agregacja typu min
(także gmean/hmean)

przewaga klasy 0



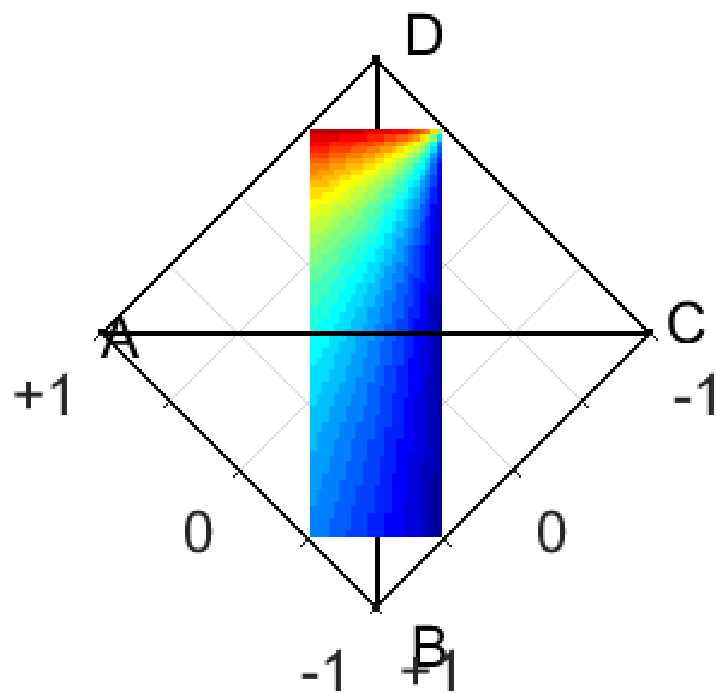
G-mean

przewaga klasy 0



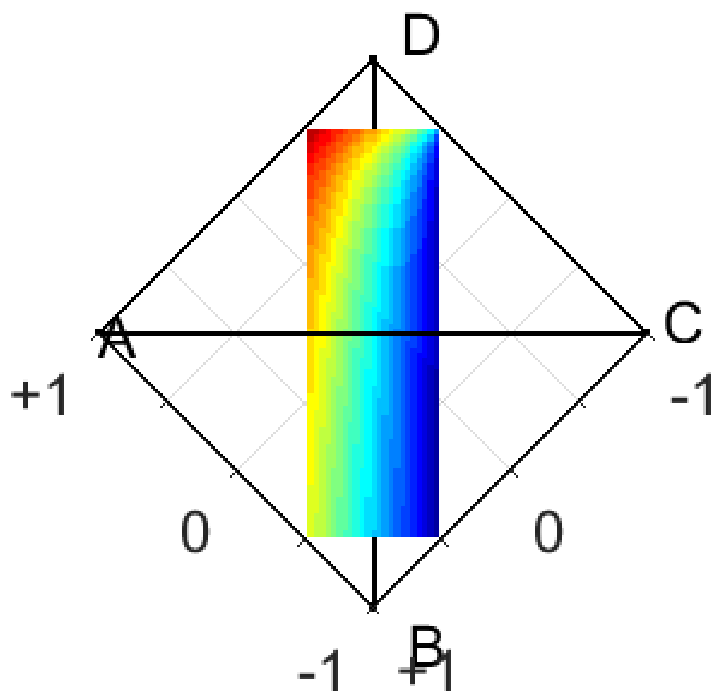
$\min(\text{specificity}, \text{recall})$

przewaga klasy 0



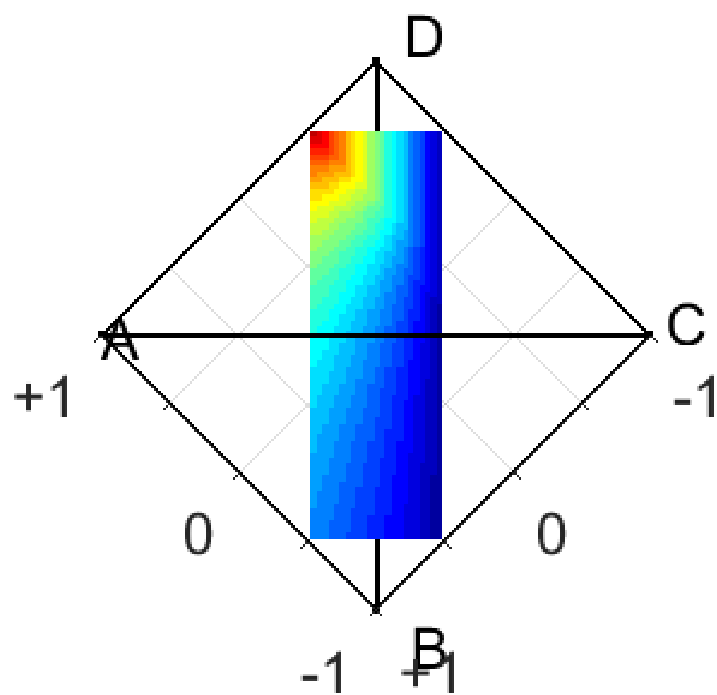
precision

przewaga klasy 0



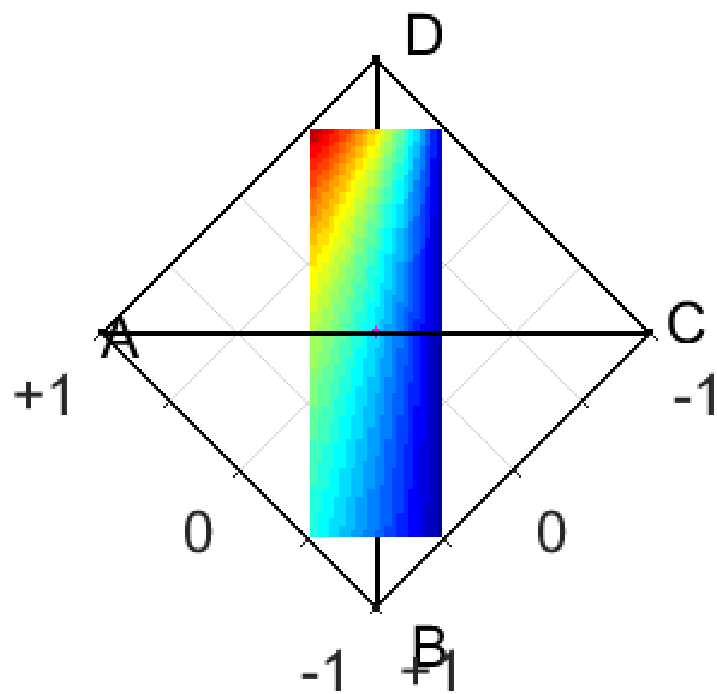
`amean(precision,recall)`

przewaga klasy 0



$\min(\text{precision}, \text{recall})$

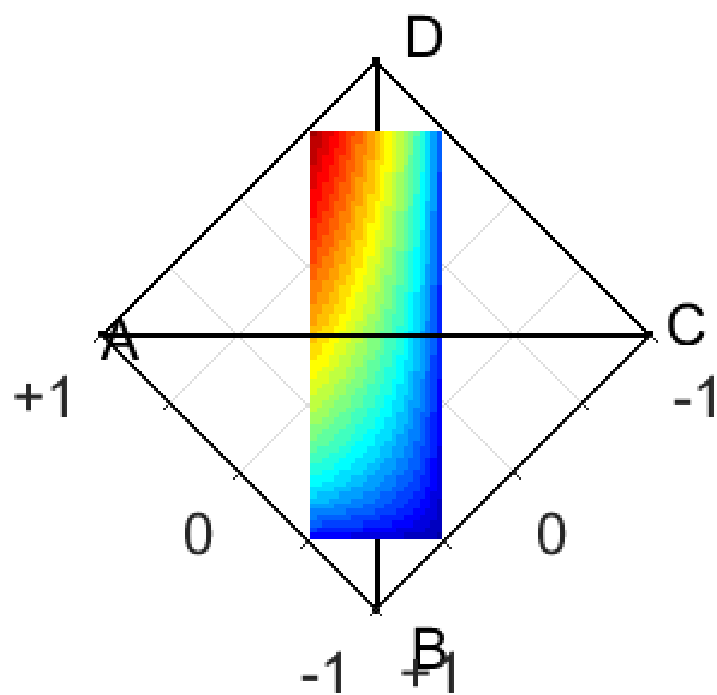
przewaga klasy 0



F₁

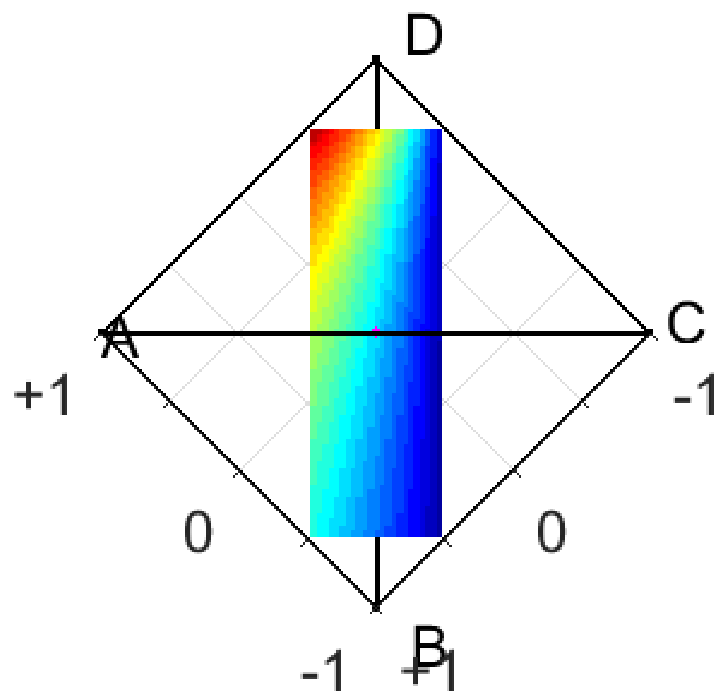
- F_1 vs G-mean dla rosnącej nierównowagi

przewaga klasy 0



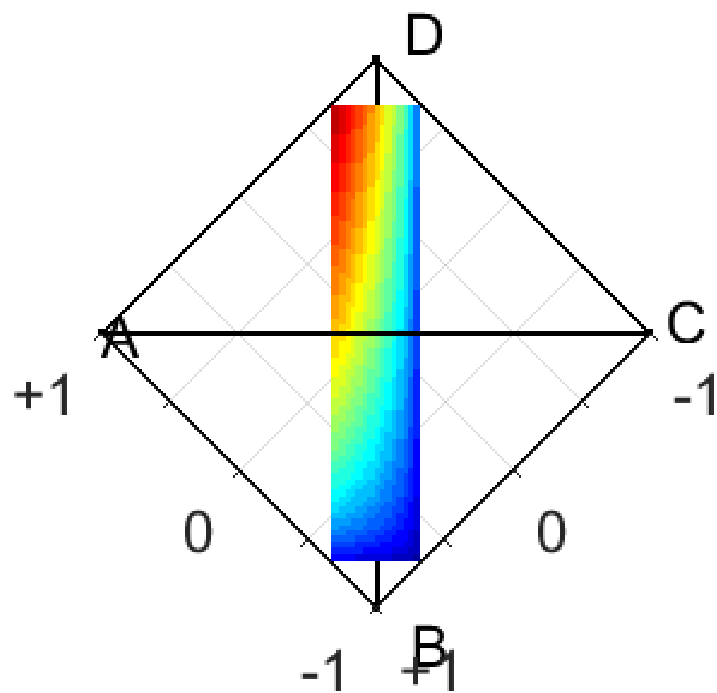
G-mean

przewaga klasy 0



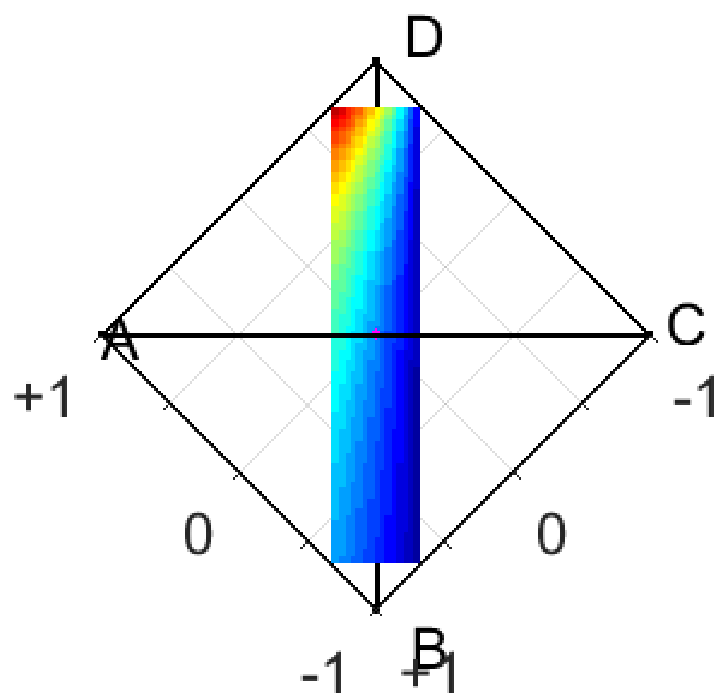
F_1

przewaga klasy 0



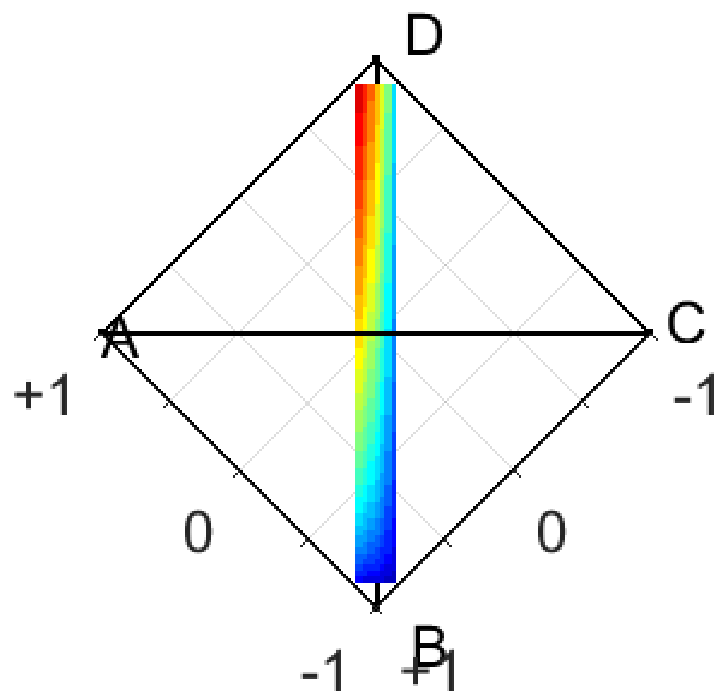
G-mean

przewaga klasy 0



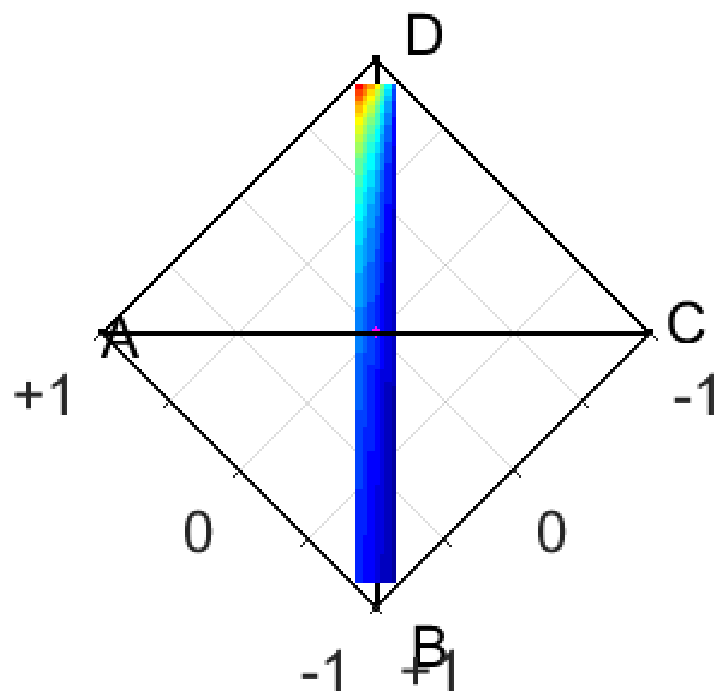
F_1

przewaga klasy 0



G-mean

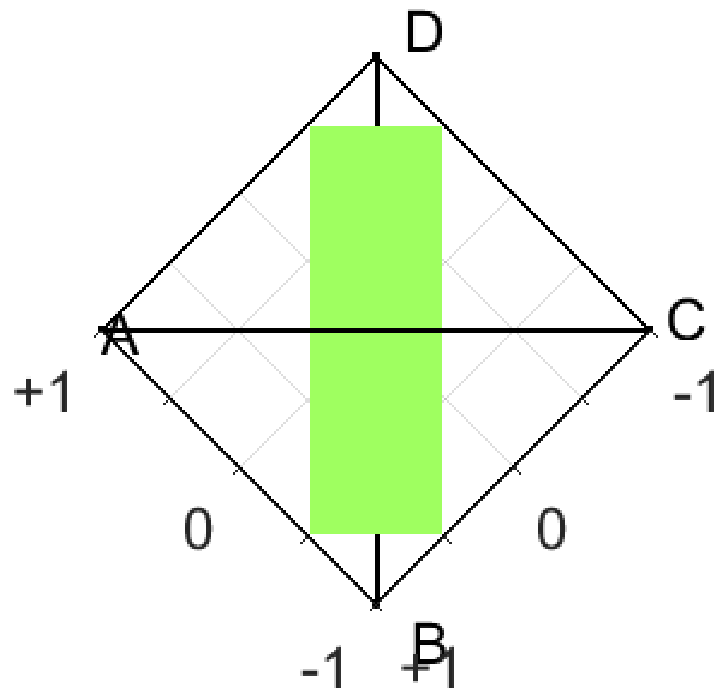
przewaga klasy 0

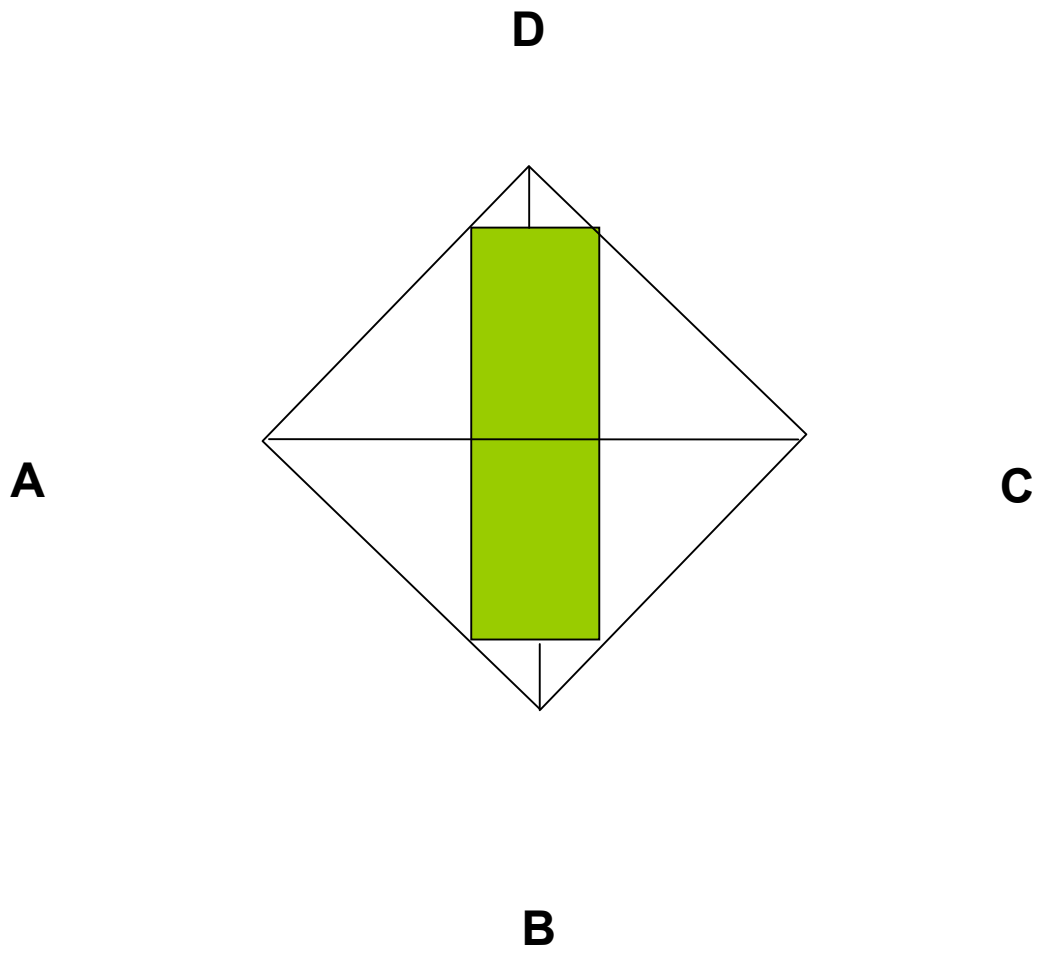


F_1

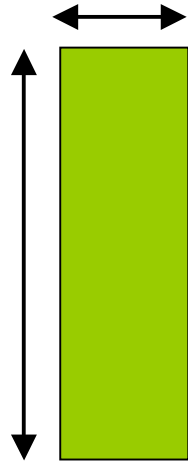
...

widok z góry





A



D

B

C

A



B

C

D

liczność klasy 1

liczność klasy 0

A



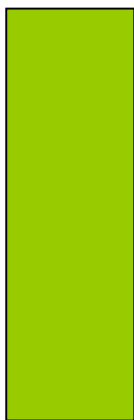
D

B

0 .. 1/2 .. 1
d/(b+d)
specificity

C

D



A

C

← ●
1 .. 1/2 .. 0
a/(a+c)
recall

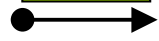
B

D



A

C



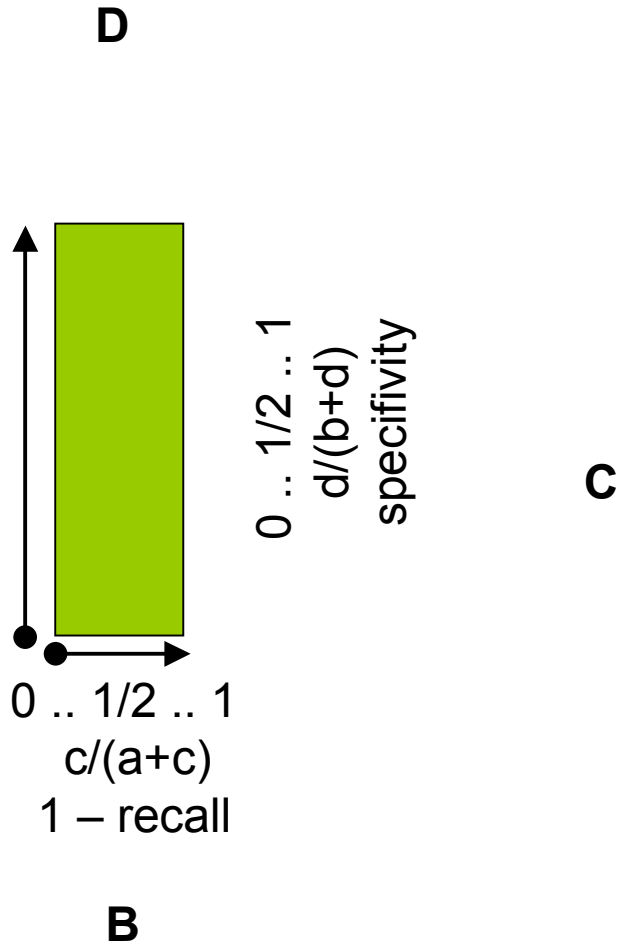
0 .. 1/2 .. 1

$c/(a+c)$

1 – recall

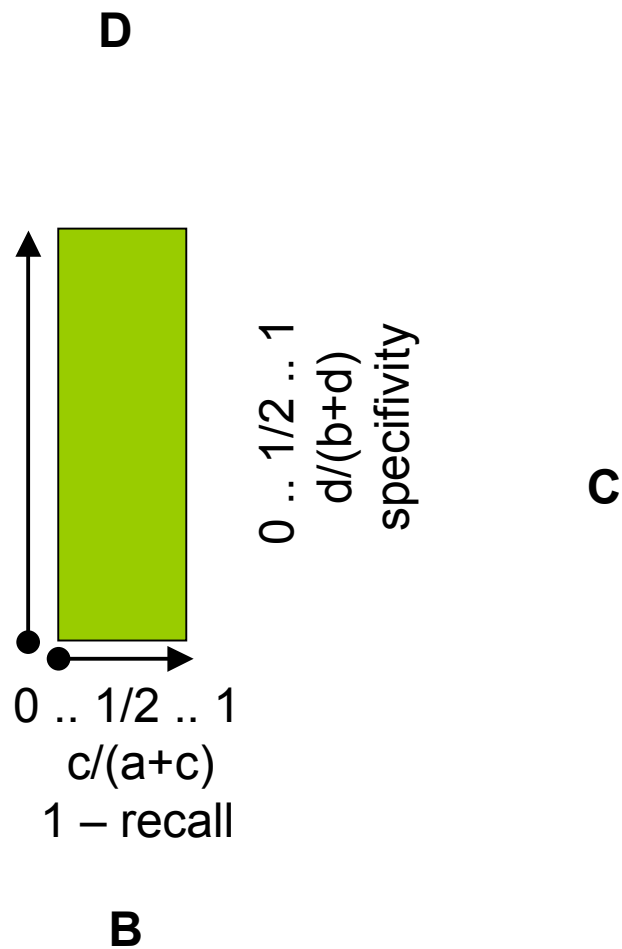
B

A



(1 – recall) vs specificity
(układ współ. jak dla AUC)

A



dodatkowo:

A

D

precision₀

B



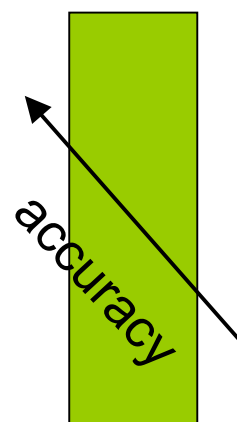
precision_d

C

dodatkowo:

A

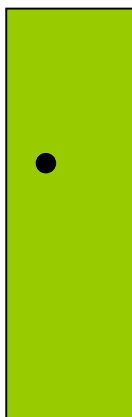
D



C

B

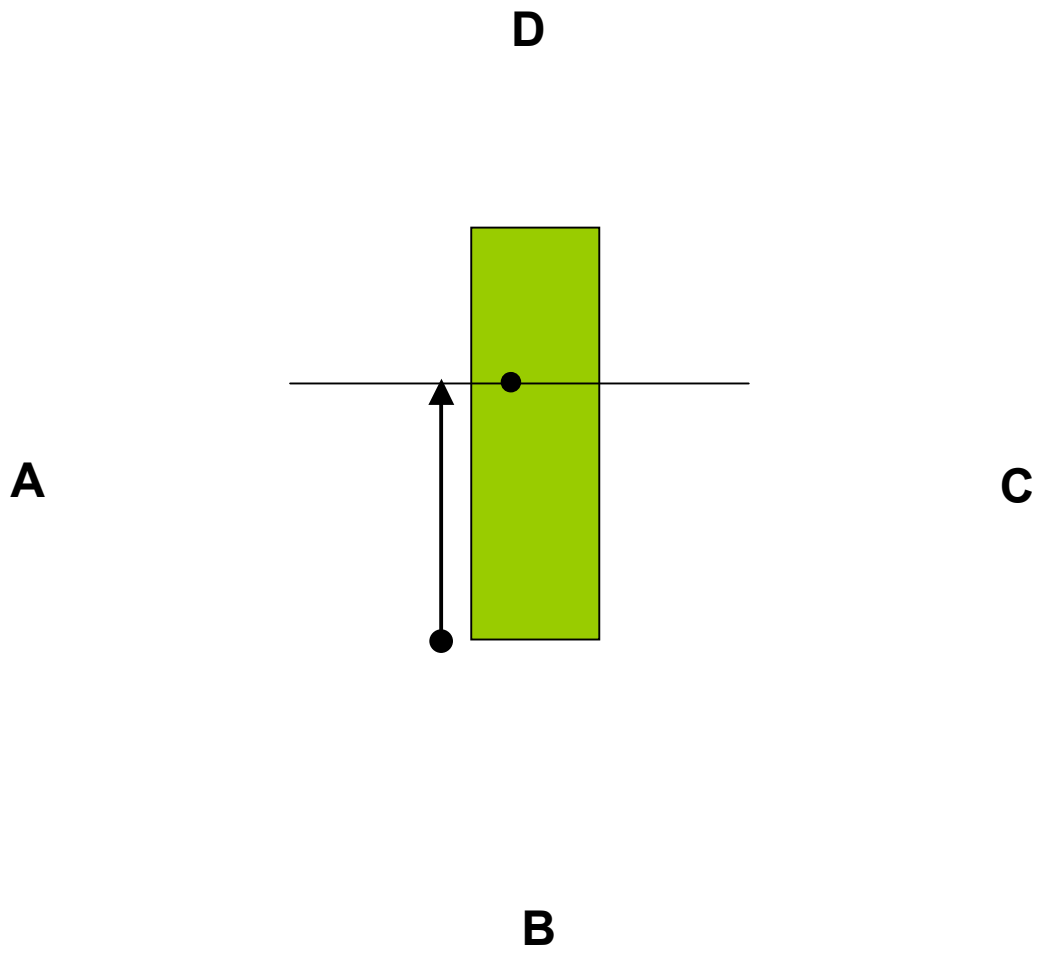
D



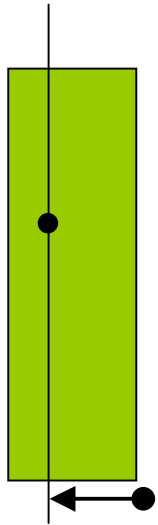
A

C

B



A

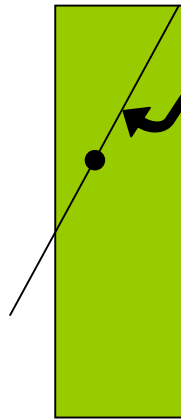


D

B

C

A



D

B

C

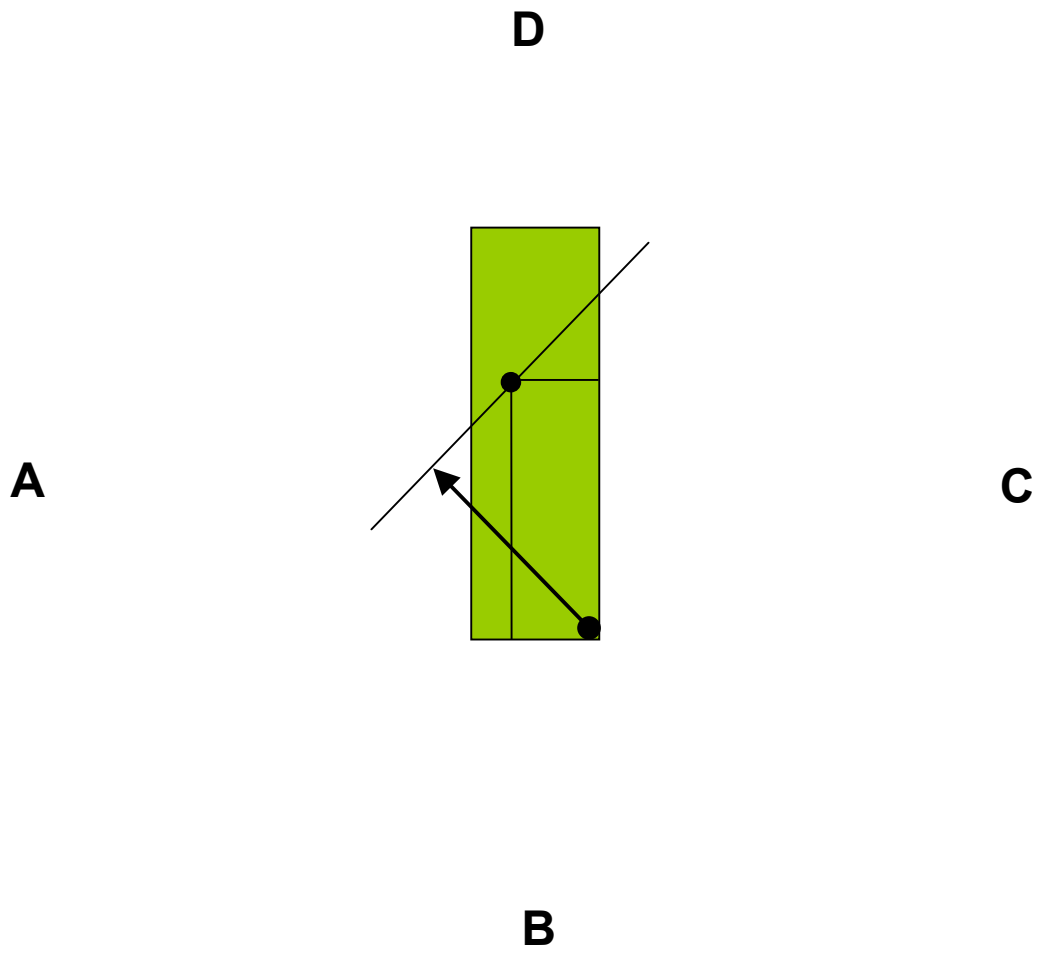
A



D

B

C



...