Rough set approach to classification of incomplete data

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Abstract. In this report, we consider different ways of dealing with missing attribute values in ordinal and non-ordinal classification data when analyzed using Indiscernibility-based Rough Set Approach (IRSA) or Dominance-based Rough Set Approach (DRSA). Moreover, we propose some desirable properties that a rough set approach capable of dealing with missing attribute values should possess. Then, we analyze which of these properties are satisfied by the considered approaches.

Keywords: Rough set, Indiscernibility-based Rough Set Approach, Dominance-based Rough Set Approach, Ordinal classification with monotonicity constraints, Missing values

1 Introduction

In data mining concerning classification problems, it is quite common to have missing values for attributes describing objects [10]. Thus, different ways of handling missing values, or more generally, incomplete data, have been proposed. The usual approach is to assume that some value(s) can represent correctly the missing one. Then, the missing values are replaced in some way by so-called representative values. In this case, the question is how to avoid data distortion [10].

Rough set approach to handling missing values avoids making changes in the data. The problem is addressed by a proper definition of the relation employed to form granules of knowledge. Extensions of the rough set model [12], that introduce relations forming granules of indiscernible or similar objects, include [4, 5, 8, 9, 11, 14].

In this work, we consider both Indiscernibility-based Rough Set Approach (IRSA), in which attributes describing objects are not considered to be ordered, and Dominance-based Rough Set Approach (DRSA), which takes into account the order of attributes. We focus on extensions the following indiscernibility-based rough set approaches:

- the classical rough set approach proposed by Pawlak (CRSA) [12],
- Variable Precision Rough Set (VPRS) model proposed by Ziarko [15],

– Variable Consistency Indiscernibility-based Rough Set Approach (VC-IRSA) proposed by Błaszczyński, Greco, Słowiński, and Szeląg [2],

and on extensions of the following dominance-based rough set approaches:

– classical Dominance-based Rough Set Approach (CDRSA) proposed by Greco, Matarazzo, and Słowiński [6],
– Variable Consistency Dominance-based Rough Set Approach (VC-DRSA) proposed by Błaszczyński, Greco, Słowiński, and Szeląg [2].

In VC-IRSA and VC-DRSA we employ object consistency measure ϵ proposed in [1, 2].

Some propositions of handling missing values in dominance-based rough set approaches were given in [3–5]. We review these approaches and consider some new ones.

The rest of this paper is structured as follows. Section 2 covers basics of IRSA and DRSA. In Section 3, we present ways of handling missing values in IRSA and DRSA. Section 4 concludes the paper.

2 Basics of IRSA and DRSA

Data analyzed by IRSA and DRSA concern a finite universe U of objects described by attributes from a finite set A. Moreover, A is divided into disjoint sets of condition attributes C and decision attributes Dec. The value set of $q \in C \cup Dec$ is denoted by V_q, and $V_P = \prod_{q=1}^{|P|} V_q$ is called P-evaluation space, where $P \subseteq C$. For simplicity, we assume that $Dec = \{d\}$. Values of d are class labels.

We consider a given set $P \subseteq C$ of attributes. To simplify notation, where possible, we will skip P in all expressions valid for any $P \subseteq C$. Moreover, for any $q_i \in P$, we denote by $q_i(x)$ the evaluation of object $x \in U$ on attribute q_i.

Decision attribute d makes a partition of set U into n disjoint sets of objects, called *decision classes*. We denote this partition by $\mathcal{X} = \{X_1, \ldots, X_n\}$.

2.1 Basics of IRSA

In IRSA, the attributes are not considered to be ordered, and thus *indiscernibility relation* is employed. Indiscernibility relation makes a partition of universe U into disjoint blocks of objects that have the same description and are considered indiscernible. Such blocks are called *granules*. Given an object $x \in U$, $I(x)$ denotes a set of objects indiscernible with x.

Given a non-ordinal classification problem, two objects $x, y \in U$ are said to be *inconsistent* with respect to (w.r.t.) indiscernibility relation if they have the same evaluation on each condition attribute (i.e., they belong to the same block) but they are assigned to different decision classes. In order to handle inconsistency observed in non-ordinal classification data, one calculates lower approximations (or positive regions) of considered decision classes.

CRSA In CRSA [12], lower approximation of class X_i, $i \in \{1, \ldots, n\}$ is defined as

$$\underline{X_i} = \{x \in U : I(x) \subseteq X_i\}. \tag{1}$$

VPRS In VPRS [15], lower approximation (positive region) of class X_i, $i \in \{1, \ldots, n\}$ is defined as

$$\underline{X_i} = \{x \in U : \mu_{X_i}(x) \geq l\}, \tag{2}$$

where consistency threshold $l \in [0, 1]$ and μ_{X_i} is a gain-type object consistency measure [2] called *rough membership*, defined as

$$\mu_{X_i}(x) = \frac{|I(x) \cap X_i|}{|I(x)|}, \tag{3}$$

where $| \cdot |$ denotes cardinality of a set of objects.

VC-IRSA We employ VC-IRSA [2] in which probabilistic lower approximation of class X_i, $i \in \{1, \ldots, n\}$ is defined as

$$\underline{X_i} = \{x \in X_i : \epsilon_{X_i}(x) \leq \theta_{X_i}\}, \tag{4}$$

where consistency threshold $\theta_{X_i} \in [0, 1]$ and ϵ_{X_i} is a cost-type object consistency measure, defined as

$$\epsilon_{X_i}(x) = \frac{|I(x) \cap \neg X_i|}{|\neg X_i|}, \tag{5}$$

where $\neg X_i$ denotes the complement of class X_i. Value $\epsilon_{X_i}(x)$ reflects the consistency of object x w.r.t. class X_i (or, the evidence for the membership of x to X_i). ϵ_{X_i} is a cost-type measure, which means that value zero denotes full consistency and the greater the value, the less consistent is a given object.

In the following, we use denotation ϵ-VC-IRSA to underline the fact that in VC-IRSA we apply particular object consistency measure ϵ_{X_i}.

2.2 Basics of DRSA

In DRSA, condition attributes and decision classes are ordered. Then, it is often meaningful to consider *monotonicity constraints* (*monotonic relationships*) between ordered class labels and values of attributes expressed on ordinal or cardinal (numerical) scales [6, 13]. The constraints result from background knowledge, e.g., "the higher the service quality and the lower the price, the higher the customer satisfaction" [7]. Objects violating such constraints are called *inconsistent*.

When there exists a monotonic relationship between evaluation of objects on condition attributes and their class labels, then, in order to make a meaningful representation of classification decisions, one has to consider the *dominance relation* D in the P-evaluation space. Given $x, y \in U$, object y dominates object x, denoted by yDx, if and only if (iff) $y \succeq_{q_i} x$, for each $q_i \in P$, where $\succeq_{q_i}$ denotes weak preference relation over U confined to single attribute q_i. For any object $x \in U$, two dominance cones can be calculated in the P-evaluation space: positive dominance cone $D^+(x) = \{y \in U : yDx\}$, and negative dominance cone $D^-(x) = \{y \in U : xDy\}$.

The class labels are ordered, such that if $i < j$, then class X_i is considered to be worse than X_j. Moreover, rough approximations concern unions of decision classes: upward unions $X_i^{\geq} = \bigcup_{t \geq i} X_t$, and downward unions $X_i^{\leq} = \bigcup_{t \leq i} X_t$, where $i = 1, \ldots, n$ (technically, $X_1^{\geq}$, $X_n^{\leq}$ are not considered as $X_1^{\geq} = X_n^{\leq} = U$).

To simplify notation, where possible, we use a symbol X_i° to denote union of classes $X_i^{\geq}$ or $X_i^{\leq}$ (when both unions of classes are considered jointly). We denote by $\neg X_i^{\circ}$ the set $U \setminus X_i^{\circ}$. Moreover, we denote by $D^{\circ}(x)$ the dominance cone "concordant" with X_i°. Precisely, if in a given equation $X_i^{\geq}$ is substituted for X_i°, then $D^+(x)$ should be substituted for $D^{\circ}(x)$; if in the same equation $X_i^{\leq}$ is substituted for X_i°, then $D^-(x)$ should be substituted for $D^{\circ}(x)$.

CDRSA In CDRSA [6], lower approximation of union of classes X_i° is defined using strict inclusion relation between dominance cone $D^{\circ}(x)$ and approximated set X_i°:

$$\underline{X_i^{\circ}} = \{x \in U : D^{\circ}(x) \subseteq X_i^{\circ}\}. \tag{6}$$

VC-DRSA Definition (6) appears to be too restrictive in practical applications. It often leads to empty lower approximations of $X_i^{\geq}$ and $X_i^{\leq}$, preventing generalization of data in terms of decision rules. This explains the interest in VC-DRSA [2] which is a probabilistic extension of CDRSA. We use *object consistency measure* $\epsilon_{X_i^{\circ}} : U \rightarrow [0, 1]$, introduced in [2], defined as:

$$\epsilon_{X_i^{\circ}}(x) = \frac{|D^{\circ}(x) \cap \neg X_i^{\circ}|}{|\neg X_i^{\circ}|}. \tag{7}$$

Value $\epsilon_{X_i^{\circ}}(x)$ reflects the consistency of object x w.r.t. X_i° (or, the evidence for the membership of x to X_i°). $\epsilon_{X_i^{\circ}}$ is a cost-type measure, which means that value zero denotes full consistency and the greater the value, the less consistent is a given object. Then, the *probabilistic lower approximation* of union of classes X_i° is defined as:

$$\underline{X_i^{\circ}} = \{x \in X_i^{\circ} : \epsilon_{X_i^{\circ}}(x) \leq \theta_{X_i^{\circ}}\}, \tag{8}$$

where threshold $\theta_{X_i^{\circ}} \in [0, 1)$.

In the following, we use denotation ϵ-VC-DRSA to underline the fact that in VC-DRSA we apply particular object consistency measure $\epsilon_{X_i^{\circ}}$.

In [2], we introduced four *monotonicity properties* required from an object consistency measure: $(m1)$ – monotonicity w.r.t. growing set of attributes, $(m2)$ –

monotonicity w.r.t. growing union of classes, $(m3)$ – monotonicity w.r.t. super-union of classes, and $(m4)$ – monotonicity w.r.t. dominance relation. We also proved that $\epsilon_{X_i^\circ}$ has properties $(m1)$, $(m2)$, and $(m4)$, sufficient in practical applications.

3 Different Ways of Handling Missing Values in IRSA and DRSA

In the following, a missing attribute value is denoted by $*$. Moreover, we use symbol X to denote an approximated set of objects. In case of IRSA, X denotes a single decision class $X_i \in U$, $i \in \{1,\ldots,n\}$. In case of DRSA, X denotes a single union of decision classes $X_i^\circ \in U$, $i \in \{1,\ldots,n\}$, $\circ \in \{\geq,\leq\}$.

3.1 Extensions of IRSA to handle missing values

The presence of missing values requires a proper adaptation of IRSA by redefinition of the indiscernibility relation I. Once we fix this definition, we can proceed in a "usual" way by calculating rough approximations of decision classes. In the literature concerning rough set approaches to handling missing attribute values in classification data (see, e.g., [8, 9]), one can find a proposal of a semantic distinction of missing values into "lost" and "do not care" values. The corresponding semantics is then used to define indiscernibility or similarity relation that is used to compare objects.

We consider two ways of redefining the indiscernibility relation I – we define indiscernibility relation I_d ("do not care" case) as

$$yI_dx \text{ iff for each } q_i \in P : q_i(x) \neq *, \text{ we have } q_i(y) = * \text{ or } q_i(y) = q_i(x), \tag{9}$$

and indiscernibility relation I_l ("lost" case) as

$$yI_lx \text{ iff for each } q_i \in P : q_i(x) \neq *, \text{ we have } q_i(y) \neq * \text{ and } q_i(y) = q_i(x), \tag{10}$$

where $x, y \in U$.

The approaches resulting from different definitions of the indiscernibility relation are denoted by CRSA-mv_i, VPRS-mv_i, and ϵ-VC-IRSA-mv_i (where $i \in \{d,l\}$, and the value of i depends on which indiscernibility relation – I_d or I_l – is employed). When these approaches are considered jointly, we use denotation IRSA-mv_i.

We consider the following desirable properties of IRSA-mv_i, $i \in \{d,l\}$:

1. Property S (reflecting symmetry of indiscernibility relation): IRSA-mv_i has property S iff $yI_ix \Leftrightarrow xI_iy$, for any $x, y \in U$.
2. Property R (reflecting reflexivity of indiscernibility relation): IRSA-mv_i has property R iff xI_ix, for any $x \in U$.
3. Property T (reflecting transitivity of indiscernibility relation): IRSA-mv_i has property T iff $yI_ix \wedge xI_iz \Rightarrow yI_iz$, for any $x, y, z \in U$.

4. Property P (reflecting precisiation of data): IRSA-mv_i has property P iff the lower approximation of any $X \subseteq U$ does not shrink when any missing attribute value is replaced by some non-missing value.
5. Property RI (rough inclusion): IRSA-mv_i has property RI iff $\underline{X} \subseteq X \subseteq \overline{X}$, for any $X \subseteq U$.
6. Property C (complementarity): IRSA-mv_i has property C iff $\underline{X} = U \setminus \overline{\neg X}$, for any $X \subseteq U$.
7. Property M_1 (monotonicity w.r.t. growing set of attributes): IRSA-mv_i has property M_1 iff the lower approximation of any $X \subseteq U$ does not shrink when set P is extended by new attributes.
8. Property M_2 (monotonicity w.r.t. growing set of objects): IRSA-mv_i has property M_2 iff the lower approximation of any $X \subseteq U$ does not shrink when this set is augmented by new objects.
9. Property MT (transitivity of membership to lower approximation): IRSA-mv_i has property MT iff for any $X \subseteq U$ and for any $x, y \in U$ it is true that $x \in \underline{X} \wedge y \in X \wedge yI_ix \Rightarrow y \in \underline{X}$.

The properties of CRSA-mv_i, VPRS-mv_i, and ϵ-VC-IRSA-mv_i, $i \in \{d, l\}$, are summarized in Table 1, where **T** and F denote presence and absence of a given property, respectively.

Table 1. Properties of CRSA-mv_i, VPRS-mv_i, and ϵ-VC-IRSA-mv_i, $i \in \{d, l\}$

Property / Approach	CRSA-mv_d	CRSA-mv_l	VPRS-mv_d	VPRS-mv_l	ϵ-VC-IRSA-mv_d	ϵ-VC-IRSA-mv_l
S	**T**	F	**T**	F	**T**	F
R	**T**	**T**	**T**	**T**	**T**	**T**
T	F	**T**	F	**T**	F	**T**
P	**T**	F	F	F	**T**	F
RI	**T**	**T**	F	F	**T**	**T**
C	**T**	F	**T**	F	**T**	**T**
M_1	**T**	**T**	F	F	**T**	**T**
M_2	**T**	**T**	**T**	**T**	**T**	**T**
MT	F	**T**	F	**T**	F	**T**

According to Table 1, CRSA-mv_l is dominated by ϵ-VC-IRSA-mv_l and dominates VPRS-mv_l. Moreover, CRSA-mv_d and ϵ-VC-IRSA-mv_d satisfy the same properties, and they both dominate VPRS-mv_d. Thus, taking into account considered desirable properties, we can conclude that there are three non-dominated approaches – CRSA-mv_d, ϵ-VC-IRSA-mv_d, and ϵ-VC-IRSA-mv_l.

3.2 Extensions of DRSA to handle missing values

The presence of missing values requires a proper adaptation of DRSA by redefinition of the dominance relation D. Once we fix this definition, we can proceed in a "usual" way by calculating rough approximations of unions of decision classes. We review some ways of redefining the dominance relation which are known from literature and we discuss a few other possibilities. In the following, we denote considered dominance relations by D_i, where $i \in \{1,\ldots,5\}$ stands for the version id.

Although it would be possible to adapt the distinction between "lost" and "do not care" values to the case of ordinal classification problems with monotonicity constraints, this would require additional knowledge about the nature of missing values in each particular problem. In this paper, instead of distinguishing a priori the semantics of the missing values, we propose to consider some desirable properties that dominance-based rough set approaches should have when handling missing values of any origins.

The approaches, resulting from different definitions of the dominance relation, are denoted by CDRSA-mv_i and ϵ-VC-DRSA-mv_i, where $i \in \{1,\ldots,5\}$. When these approaches are considered jointly, we use denotation DRSA-mv_i.

We consider the following desirable properties of DRSA-mv_i, $i \in \{1,\ldots,5\}$:

1. Property S (reflecting a specific kind of symmetry): DRSA-mv_i has property S iff $y \in D_i^+(x) \Leftrightarrow x \in D_i^-(y)$, for any $x, y \in U$.
2. Property R (reflecting reflexivity of dominance relation): DRSA-mv_i has property R iff xD_ix, for any $x \in U$.
3. Property T (reflecting transitivity of dominance relation): DRSA-mv_i has property T iff $yD_ix \wedge xD_iz \Rightarrow yD_iz$, for any $x, y, z \in U$.
4. Property P (reflecting precisiation of data): DRSA-mv_i has property P iff the lower approximation of any $X \subseteq U$ does not shrink when any missing attribute value is replaced by some non-missing value.
5. Property RI (rough inclusion): DRSA-mv_i has property RI iff $\underline{X} \subseteq X \subseteq \overline{X}$, for any $X \subseteq U$.
6. Property C (complementarity): DRSA-mv_i has property C iff $\underline{X} = U \setminus \overline{\neg X}$, for any $X \subseteq U$.
7. Property M_1 (monotonicity w.r.t. growing set of attributes): DRSA-mv_i has property M_1 iff the lower approximation any $X \subseteq U$ does not shrink when set P is extended by new attributes.
8. Property M_2 (monotonicity w.r.t. growing union of classes): DRSA-mv_i has property M_2 iff for any $X \subseteq U$, the lower approximation of X does not shrink when this set is augmented by new objects.
9. Property M_3 (monotonicity w.r.t. super-union of classes): DRSA-mv_i has property M_3 iff given any two upward union of classes $X_i^{\geq}, X_j^{\geq}$, with $1 \leq i < j \leq n$, there is $\underline{X_i^{\geq}} \supseteq \underline{X_j^{\geq}}$, and, moreover, given any two downward union of classes $X_i^{\leq}, X_j^{\leq}$, with $1 \leq i < j \leq n$, there is $\underline{X_i^{\leq}} \subseteq \underline{X_j^{\leq}}$.
10. Property M_4 (monotonicity w.r.t. dominance relation): DRSA-mv_i has property M_4 iff for any upward and downward unions of classes $X_i^{\geq}, X_j^{\leq} \subseteq U$, with $i, j \in \{1,\ldots,n\}$, and for any $x, y \in U$ such that yD_ix, it is true that $\Big((x \in \underline{X_i^{\geq}} \wedge y \in X_i^{\geq} \Rightarrow y \in \underline{X_i^{\geq}})$ and $(y \in \underline{X_j^{\leq}} \wedge x \in X_j^{\leq} \Rightarrow x \in \underline{X_j^{\leq}})\Big)$.

Note that there is a relationship between the above properties M_1, M_2, M_3, and M_4, concerning lower approximations of unions of classes, and monotonicity properties (m1), (m2), (m3), and (m4), introduced in [2], concerning object consistency measures used in VC-DRSA. However, when the dominance relation used in VC-DRSA is redefined, this relationship is no longer one to one – for some $j \in \{1,\ldots,4\}$, (mj) may be satisfied while M_j is not satisfied.

DRSA-mv_1 [4, 5] considers dominance relation to be a directional statement where a subject is compared to a referent which cannot have missing values. Object y dominates referent x iff for each $q_i \in P$, $y \succeq_{q_i} x$ or $q_i(y) = *$; y is dominated by referent x iff for each $q_i \in P$, $x \succeq_{q_i} y$ or $q_i(y) = *$. Note that DRSA-mv_1 fails when all (or most of the) objects have a missing value. Moreover, dominance cones are defined only for objects without missing values. Thus, approximations of unions of classes do not contain objects with missing values.

DRSA-mv_2 was first proposed in [4, 5], and then extended in [3] to handle imprecise evaluations on attributes and imprecise assignments to decision classes, both modeled by intervals. When considering missing values only, each object is assigned to a single decision class, and each missing attribute value corresponds to the interval spanning entire value set of this attribute. This results in the following definition of so-called *possible dominance relation*: object y dominates object x iff for each $q_i \in P$, $y \succeq_{q_i} x$, or $q_i(y) = *$, or $q_i(x) = *$.

In DRSA-mv_3, object y dominates object x iff for each $q_i \in P$ such that $q_i(x) \neq *$, we have $q_i(y) \neq *$ and $y \succeq_{q_i} x$. Object y is dominated by object x iff for each $q_i \in P$ such that $q_i(x) \neq *$, we have $q_i(y) \neq *$ and $x \succeq_{q_i} y$.

DRSA-mv_4 (DRSA-mv_5) uses the *lower(upper)-end dominance relation* introduced in [3]. It boils down to treating each missing attribute value as the worst (best) value in the value set of this attribute. Then, the definition of dominance relation is the same as in the case without missing values.

The properties of DRSA-mv_i, $i = 1,\ldots,5$, are summarized in Table 2, where **T** and F denote presence and absence of a given property, respectively. Moreover, in case of two symbols $\cdot/\cdot$, the first one reflects only CDRSA-mv_i while the second one reflects only ϵ-VC-DRSA-mv_i.

Table 2. Properties of DRSA-mv_i, $i = 1,\ldots,5$

Property / Approach	DRSA-mv_1	DRSA-mv_2	DRSA-mv_3	DRSA-mv_4	DRSA-mv_5
S	F	**T**	F	**T**	**T**
R	F	**T**	**T**	**T**	**T**
T	**T**	F	**T**	**T**	**T**
P	**T**	**T**	F	F	F
RI	F	**T**	**T**	**T**	**T**
C	F	**T**	F/**T**	**T**	**T**
M_1	F	**T**	**T**	**T**	**T**
M_2	**T**	**T**	**T**	**T**	**T**
M_3	**T**/F	**T**/F	**T**/F	**T**/F	**T**/F
M_4	**T**	F	**T**	**T**	**T**

According to Table 2, DRSA-mv_1 is the least attractive due to lack of many important properties (like, e.g., RI and M_1). DRSA-mv_3 is dominated by DRSA-mv_4, and by DRSA-mv_5. However, the choice from among DRSA-mv_2, DRSA-mv_4, and DRSA-mv_5, depends on a particular application.

Taking into account the semantics of missing values considered in [8, 9], it can be said that DRSA-mv_2 treats missing values as “do not care” values while DRSA-mv_3 treats missing values as “lost” values.

4 Conclusions

We considered different ways of dealing with missing attribute values in ordinal and non-ordinal classification data when analyzed using Indiscernibility-based Rough Set Approach (IRSA) or Dominance-based Rough Set Approach (DRSA). Moreover, we proposed some desirable properties for IRSA and DRSA that a rough set approach capable of dealing with missing attribute values should possess. We analyzed which of these properties are satisfied by the considered rough set approaches resulting from different definition of indiscernibility or dominance relation. Based on this analysis, we indicated non-dominated indiscernibility-based and dominance-based rough set approaches.

Acknowledgment The 1st and 3rd author acknowledge financial support from the Poznań University of Technology, grant no. 91-516/DS-MLODA KADRA. The 2nd author acknowledges financial support from the Polish National Science Centre, grant no. DEC-2011/01/B/ST6/07318.

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