Surrogate Fitness via Factorization of Interaction Matrix

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Thought experiment

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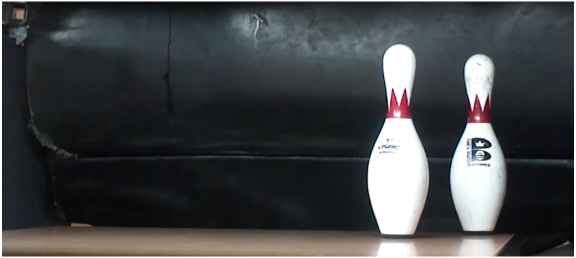


Fitness Evaluation

Observation 1

GP algorithms do not let programs know **which** tests they solve.

Typical fitness function in GP *aggregates* program's behavior on *tests* by

- counting the number of passed tests (discrete domains).

$$f(p) = |\{y_i \neq \hat{y}_i(p)\}|_i \tag{1}$$

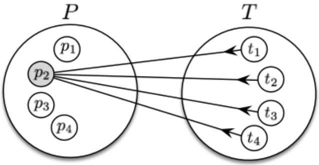
- summing the errors on individual tests (continuous domains).

$$f(p) = \sum_i (y_i - \hat{y}_i(p))^2 \tag{2}$$

Test-based Genetic Programming

Observation 2

Detailed information on particular interactions with **is available** in GP.



$g : P \times T \to [0,1]$

G	t_1	t_2	t_3	t_4
p_1	1	1	0	0
p_2	0	1	0	1
p_3	1	1	1	0
p_4	0	1	1	0

- P: set of m programs,
- T: set of n tests (fitness cases)
- $g(p,t)$: interaction function between $p \in P$ and $t \in T$
- G: $m \times n$ matrix of interaction outcomes between P and T
- Test-based problems (Pollack, Bucci, de Jong, Popovici)

Fitness Evaluation

Observation 3

Interaction outcomes for particular tests are *partially* dependent.

Can be used to derive **alternative search objectives** (*search drivers*):

- *Derivation of Search Objectives*
  (Krawiec & Liskowski, EuroGP 2015, ECJ 2016)

Fitness Evaluation

Observation 3

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Hypothesis

- An interaction outcome $g(p, t)$ can be reconstructed from other elements of G
- We may **reduce** so the number of program-test interactions.

The idea

Matrix factorization to estimate some program-test interactions in G.

SFIMX: Surrogate Fitness via Factorization of Interaction Matrix

Algorithm

1. Calculate **sparse** interaction matrix G between P and T
   - For each $p \in P$ draw a random subset of $\alpha|T|$ tests $T' \subset T$
   - Apply p to tests in T'
2. **Factorize** G into non-negative components W and H (rank $\leq k$)
3. **Reconstruct** the interaction outcomes by calculating $\hat{G} = WH$

- $\alpha \in (0, 1]$ - desired density of partial interaction matrix

Example: $P = \{p_1, \ldots, p_4\}$, $T = \{t_1, \ldots, t_5\}$, $\alpha = \frac{3}{5} = 0.6$

$$G = \begin{array}{c c} & \begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} & \begin{pmatrix} 2 & & 1 & 2 & \\ & 2 & 1 & & 1 \\ 1 & & & 2 & 2 \\ 2 & 1 & & & 1 \end{pmatrix} \end{array}$$

Missing outcomes due to $\alpha < 1$

Example

$$
G = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}
\begin{array}{c}
\begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\
\begin{pmatrix} 2 & & 1 & 2 & \\ & 2 & 1 & & 1 \\ 1 & & & 2 & 2 \\ 2 & 1 & & & 1 \end{pmatrix}
\end{array}
$$

Missing outcomes due to $\alpha < 1$

$$
W = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}
\begin{array}{c}
\begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\
\begin{pmatrix} 0.46 & 1.96 & 0.6 \\ 1.27 & 0.1 & 0.95 \\ 1.37 & 0.02 & 2.83 \\ 0.4 & 1.86 & 1.60 \end{pmatrix}
\end{array},
\quad
H = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array}
\begin{array}{c}
\begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\
\begin{pmatrix} 0.48 & 1.50 & 0.01 & 0.41 & 0.41 \\ 0.87 & 0.14 & 0.19 & 0.77 & 0.01 \\ 0.11 & 0.09 & 1.02 & 0.50 & 0.51 \end{pmatrix}
\end{array}
$$

$$
\hat{G} = WH = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}
\begin{array}{c}
\begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\
\begin{pmatrix} 2 & 1.02 & 1 & 2 & 0.52 \\ 0.8 & 2 & 1 & 1.07 & 1 \\ 1 & 2.31 & 2.1 & 2 & 2 \\ 2 & 1 & 2.01 & 2.4 & 1 \end{pmatrix}
\end{array}
\xrightarrow{f(p_i) = \sum_{j=1}^{n} g_{ij}}
\begin{array}{l} f(p_1) = 6.54 \\ f(p_2) = 5.87 \\ f(p_3) = 9.41 \\ f(p_4) = 8.41 \end{array}
$$

- Sizes of W and H controlled by parameter $k \geq 1$ (here: $k = 3$)
- Technical realization: multiplicative update rule.
- Cost of evaluation **reduced** $\frac{1}{\alpha}$ **times.**

Experiment

- GP: 'vanila' GP, $|P| = 1000$
- SFIMX: $|P|$ increased $(1 - \alpha)$ times $\implies$ same budget
  - FULL: $k = |T|$,
  - HALF: $k = \frac{|T|}{2}$,
  - LOG: $k = log_2|T|$
- RSS: Calculates fitness using $\alpha|T|$ random tests (same budget)
- $\alpha \in 0.1, 0.2, \ldots, 1.0$

Domain	Instruction set	Problem	Variables	#tests
Boolean	and, nand or, nor	Cmp6	6	64
		Cmp8	8	256
		Par5	5	32
		Mux6	6	64
		Maj6	6	64
Algebras	$a_i(x, y)$	Disc-a1...a5	3	27
	$a_i(x, y)$	Malcev-a1...a5	3	15

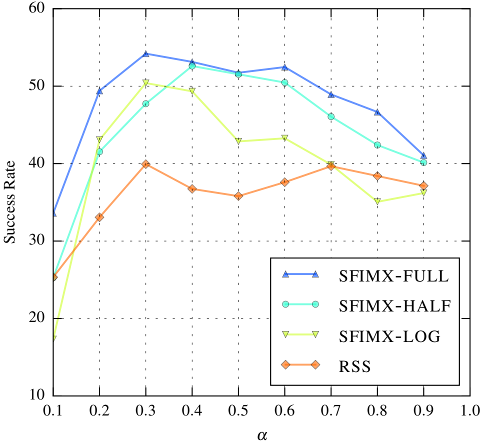
Experiment

Average ranks on success rate (Friedman's $p \ll 0.001$)

	SFIMX			GP	RSS
	HALF	FULL	LOG		
All problems	2.07	2.13	2.67	3.90	4.23
Boolean	2.4	1.7	3.3	3.2	4.4
Categorical	1.90	2.35	2.35	4.25	4.15

- Best results for $\alpha = 0.3$ and $\alpha = 0.4$
- Roughly the same performance as GP using only 10% of interactions
- LOG variant $\rightarrow$ high compression without affecting the performance
  - $\Longrightarrow$ Interaction outcomes are indeed corrrelated.
- Low k $\rightarrow$ low computational overhead of factorization
  - For SFIMX-LOG: only 6% of the total cost of $1,000|T|$ interactions

Impact of α on success rates



Success rates improve as sparsity in G increases up to $\alpha = 0.3$

Conclusions

- SFIMX = well-informed and scalable surrogate fitness.
- Target domains:
  - Problems with expensive interaction functions
  - Problems with large numbers of tests
  - Evolving controllers, two-player games, image analysis, ...
- Replaces a discrete fitness function with a continuous one.
- Ongoing work: continous domains

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Thank You