Extreme Zero-Shot Learning (XZSL)

Krzysztof Dembczyński

Intelligent Decision Support Systems Laboratory (IDSS) Poznań University of Technology, Poland



Theoretical Foundations of Machine Learning Kraków, February 13, 2017















Alan Turing, 1912 births, 1954 deaths 20th-century mathematicians, 20th-century philosophers Academics of the University of Manchester Institute of Science and Technology Alumni of King's College, Cambridge Artificial intelligence researchers Atheist philosophers, Bayesian statisticians, British cryptographers, British logicians British long-distance runners, British male athletes, British people of World War II Computability theorists, Computer designers, English atheists English computer scientists, English inventors, English logicians English long-distance runners, English mathematicians English people of Scottish descent, English philosophers, Former Protestants Fellows of the Royal Society, Gay men Government Communications Headquarters people, History of artificial intelligence Inventors who committed suicide, LGBT scientists LGBT scientists from the United Kingdom. Male long-distance runners Mathematicians who committed suicide. Officers of the Order of the British Empire People associated with Bletchlev Park. People educated at Sherborne School People from Maida Vale, People from Wilmslow People prosecuted under anti-homosexuality laws. Philosophers of mind Philosophers who committed suicide. Princeton University alumni, 1930-39 Programmers who committed suicide, People who have received posthumous pardons Recipients of British royal pardons, Academics of the University of Manchester Suicides by cyanide poisoning. Suicides in England, Theoretical computer scientists



Alan Turing, 1912 births, 1954 deaths 20th-century mathematicians, 20th-century philosophers Academics of the University of Manchester Institute of Science and Technology Alumni of King's College, Cambridge Artificial intelligence researchers Atheist philosophers, Bayesian statisticians, British cryptographers, British logicians British long-distance runners, British male athletes, British people of World War II Computability theorists, Computer designers, English atheists English computer scientists. English inventors. English logicians English long-distance runners, English mathematicians English people of Scottish descent, English philosophers, Former Protestants Fellows of the Royal Society, Gay men Government Communications Headquarters people. History of artificial intelligence Inventors who committed suicide, LGBT scientists LGBT scientists from the United Kingdom. Male long-distance runners Mathematicians who committed suicide. Officers of the Order of the British Empire People associated with Bletchlev Park. People educated at Sherborne School People from Maida Vale, People from Wilmslow People prosecuted under anti-homosexuality laws. Philosophers of mind Philosophers who committed suicide. Princeton University alumni, 1930-39 Programmers who committed suicide, People who have received posthumous pardons Recipients of British royal pardons, Academics of the University of Manchester Suicides by cyanide poisoning. Suicides in England, Theoretical computer scientists

new tags: English machine learners, Polish machine learners, ...

Setting

• Multi-class classification:

$$\boldsymbol{x} = (x_1, x_2, \dots, x_d) \in \mathbb{R}^d \xrightarrow{h(\boldsymbol{x})} y \in \{1, \dots, m\}$$

	x_1	x_2	 x_d	y
\boldsymbol{x}	4.0	2.5	-1.5	5

Setting

$\boldsymbol{x} = (x_1, z)$	$= (x_1, x_2, .$		$(d) \in \mathbb{R}^d - \frac{h(\boldsymbol{x})}{d}$		$ ightarrow y \in$		
		x_1	x_2		x_d	y	
	\overline{x}	4.0	2.5		-1.5	5	

• Multi-class classification:

• Multi-label classification:

$$\boldsymbol{x} = (x_1, x_2, \dots, x_d) \in \mathbb{R}^d \xrightarrow{\boldsymbol{h}(\boldsymbol{x})} \boldsymbol{y} = (y_1, y_2, \dots, y_m) \in \{0, 1\}^m$$

	x_1	x_2	 x_d	y_1	y_2	 y_m
\boldsymbol{x}	4.0	2.5	-1.5	1	1	0

}

Extreme classification \Rightarrow a large number of labels $m (\geq 10^5)$

• Predictive performance:

- Predictive performance:
 - \blacktriangleright Learning theory for large m

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - ► Training and prediction under limited time and space budged

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - ► Learning with missing labels and positive-unlabeled learning

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - ► Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - ► Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score
 - Long-tail label distributions and zero-shot learning

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score
 - Long-tail label distributions and zero-shot learning
- Computational complexity:

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score
 - Long-tail label distributions and zero-shot learning
- Computational complexity:
 - ► time vs. space

Extreme classification \Rightarrow a large number of labels $m \ (\geq 10^5)$

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score
 - Long-tail label distributions and zero-shot learning

• Computational complexity:

- ► time vs. space
- ► #examples vs. #features vs. #labels

Extreme classification \Rightarrow a large number of labels $m \ (\geq 10^5)$

- Predictive performance:
 - \blacktriangleright Learning theory for large m
 - Training and prediction under limited time and space budged
 - Learning with missing labels and positive-unlabeled learning
 - ► Performance measures: Hamming loss, prec@k, NDCG@k, F-score
 - Long-tail label distributions and zero-shot learning

• Computational complexity:

- ► time vs. space
- ► #examples vs. #features vs. #labels
- training vs. validation vs. prediction

• Learning theory for an extremely large number of labels:

- Learning theory for an extremely large number of labels:
 - Statistical guarantees for the error rate that do not depend, or depend very weakly (sublinearly), on the total number of labels.

- Learning theory for an extremely large number of labels:
 - Statistical guarantees for the error rate that do not depend, or depend very weakly (sublinearly), on the total number of labels.
 - ► The **bound** on the error rate could be expressed in terms of the average number of **positive labels** (which is certainly much less than the total number of labels).

- Learning theory for an extremely large number of labels:
 - Statistical guarantees for the error rate that do not depend, or depend very weakly (sublinearly), on the total number of labels.
 - ► The bound on the error rate could be expressed in terms of the average number of positive labels (which is certainly much less than the total number of labels).
 - Particular performance guarantees depend on the considered loss function.

- Learning theory for an extremely large number of labels:
 - Statistical guarantees for the error rate that do not depend, or depend very weakly (sublinearly), on the total number of labels.
 - ► The **bound** on the error rate could be expressed in terms of the average number of **positive labels** (which is certainly much less than the total number of labels).
 - Particular performance guarantees depend on the considered loss function.
 - Different theoretical settings: statistical learning theory, learning reductions, online learning.

• Training and prediction under limited time and space budget:

- Training and prediction under limited time and space budget:
 - Restricted computational resources (time and space) for both training and prediction.

- Training and prediction under limited time and space budget:
 - Restricted computational resources (time and space) for both training and prediction.
 - ► A trade-off between computational (time and space) complexity and the predictive performance.

- Training and prediction under limited time and space budget:
 - Restricted computational resources (time and space) for both training and prediction.
 - ► A trade-off between computational (time and space) complexity and the predictive performance.
 - By imposing hard constraints on time and space budget, the challenge is then to optimize the predictive performance of an algorithm under these constraints.

• Unreliable learning information:

- Unreliable learning information:
 - We cannot expect that all labels will be properly checked and assigned to training examples.

- Unreliable learning information:
 - We cannot expect that all labels will be properly checked and assigned to training examples.
 - Therefore we often deal with a problem of learning with missing labels or learning from positive and unlabeled examples.

• Performance measures:

- Performance measures:
 - Typical performance measures such as 0/1 or Hamming loss do not fit well to the extreme setting.
- Performance measures:
 - ➤ Typical performance measures such as 0/1 or Hamming loss do not fit well to the extreme setting.
 - Other measures are often used such as **precision@k** or the **F**-measure.

- Performance measures:
 - ➤ Typical performance measures such as 0/1 or Hamming loss do not fit well to the extreme setting.
 - Other measures are often used such as precision@k or the F-measure.
 - ► However, it remains an **open question** how to **design loss functions** suitable for extreme classification.

• Long-tail label distributions and zero-shot learning:

- Long-tail label distributions and zero-shot learning:
 - A close relation to the problem of estimating distributions over large alphabets.

- Long-tail label distributions and zero-shot learning:
 - A close relation to the problem of estimating distributions over large alphabets.
 - ► The distribution of label frequencies is often characterized by a **long-tail** for which proper **smoothing** (like add-constant or Good-Turing estimates) or **calibration** techniques (like isotonic regression or domain adaptation) have to be used.

- Long-tail label distributions and zero-shot learning:
 - A close relation to the problem of estimating distributions over large alphabets.
 - ► The distribution of label frequencies is often characterized by a **long-tail** for which proper **smoothing** (like add-constant or Good-Turing estimates) or **calibration** techniques (like isotonic regression or domain adaptation) have to be used.
 - ► In practical applications, learning algorithms run in rapidly changing environments: new labels may appear during testing/prediction phase (⇒ zero-shot learning)

- Long-tail label distributions and zero-shot learning:
 - Frequency of labels in the WikiLSHTC dataset:¹



▶ Many labels with only few examples (⇒ one-shot learning).

http://research.microsoft.com/en-us/um/people/manik/downloads/XC/ XMLRepository.html

- Long-tail label distributions and zero-shot learning:
 - ► Frequency of frequencies for the WikiLSHTC dataset:



► The missing mass obtained by the Good-Turing estimate: 0.014.

• Size of the problem:

- Size of the problem:
 - # examples: $n > 10^6$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

► Train time complexity:

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - # labels: $m > 10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

• Train time complexity: $> 10^{17}$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

- Train time complexity: $> 10^{17}$
- Space complexity:

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

- Train time complexity: $> 10^{17}$
- Space complexity: $> 10^{11}$

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - $\blacktriangleright~\#$ labels: $m>10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

- Train time complexity: $> 10^{17}$
- ► Space complexity: > 10¹¹
- Test time complexity:

- Size of the problem:
 - # examples: $n > 10^6$
 - # features: $d > 10^6$
 - # labels: $m > 10^5$
- Naive solution: A dense linear model for each label (1-vs-All):

$$\mathbf{X}^T \mathbf{W} = \hat{\mathbf{Y}}$$

- Train time complexity: $> 10^{17}$
- ► Space complexity: > 10¹¹
- ▶ Test time complexity: > 10¹¹

• It does not have to be so hard:

- It does not have to be so hard:
 - ► Large data → sparse data (sparse features and labels)

• It does not have to be so hard:

- ► Large data → sparse data (sparse features and labels)
- ► Fast learning algorithms for standard learning problems exist!

• It does not have to be so hard:

- ► Large data → sparse data (sparse features and labels)
- ► Fast learning algorithms for standard learning problems exist!
- High performance computing resources available!

Terminal				a 🔊	🖇 🚎 📢 ১ গ্ৰহণ	Lativization 🕸	
0.912227	0.905463	22	22.0	1.0000	-0.1043	87	
0.861865	0.811503	44	44.0	-1.0000	-0.0604	65	
0.823944	0.785142	87	87.0	1.0000	-0.2309	60	
0.766675	0.709405	174	174.0	1.0000	0.0754	25	
0.642809	0.518943	348	348.0	1.0000	0.3440	47	
0.540082	0.437356	696	696.0	1.0000	0.9767	24	
0.450636	0.361190	1392	1392.0	1.0000	0.6204	181	
0.376935	0.303234	2784	2784.0	1.0000	0.4380	50	
0.320936	0.264938	5568	5568.0	-1.0000	-0.9257	89	8 601.0 1,0200 0,5300 000
0.281048	0.241153	11135	11135.0	1.0000	1.0000	62	
0.249233	0.217415	22269	22269.0	1.0000	1.0000	140	
0.221765	0.194296	44537	44537.0	1.0000	1.0000	41	
0.201490	0.181213	89073	89073.0	-1.0000	-1.0000	27	State Law Law
0.187823	0.174157	178146	178146.0	1.0000	1.0000	49	
0.176267	0.164711	356291	356291.0	-1.0000	-1.0000	100	
0.165728	0.155188	712582	712582.0	-1.0000	-1.0000	69	
finished r	run						A 215 System Store CPL B-CPD Control (3)
number of	examples =	781265					
weighted example sum - 7.813e+05							
weighted label sum = - A 018+00							
average loss = 0.1645							
bost const		51/12					
total foat	tuco pumboc	- 50036400					
WW -C FCW	1 train tyt	1 /6c user @	21c system	199% CDU	0 993 +0+	- a1	
Science 1 of 2 of							
Stephine [jt/ttypts/16]							
II							

Figure: Vowpal Wabbit² at a lecture of John Langford³

² Vowpal Wabbit, http://hunch.net/~vw

³ http://cilvr.cs.nyu.edu/doku.php?id=courses:bigdata:slides:start

Fast binary classification⁴

- Data set: RCV1
- Predicted category: CCAT
- # training examples: 781 265
- # features: 60M
- Size: 1.1 GB
- Command line: time vw -sgd rcv1.train.txt -c
- Learning time: 1-3 secs on a laptop.

⁴ http://cilvr.cs.nyu.edu/doku.php?id=courses:bigdata:slides:start

Computational challenges

How can we reduce computational (time and space) costs of the naive solution?

• Fast training by least squares:⁵

⁵ T. Hastie, R. Tibshirani, and J.H. Friedman. *Elements of Statistical Learning: Data Mining, Inference, and Prediction.* Springer, second edition, 2009

• Fast training by least squares:⁵

$$\mathbf{W} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$$

⁵ T. Hastie, R. Tibshirani, and J.H. Friedman. *Elements of Statistical Learning: Data Mining, Inference, and Prediction.* Springer, second edition, 2009

• Fast training by least squares:⁵

$$\mathbf{W} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$$

- Works well in low dimensional feature spaces.
- Does not realy improve space and test time complexity.

⁵ T. Hastie, R. Tibshirani, and J.H. Friedman. *Elements of Statistical Learning: Data Mining, Inference, and Prediction.* Springer, second edition, 2009

• Training time complexity:

- ⁶ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer
- ⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008
- ⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009
- ⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008
- ¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009
- ¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - ► Stochastic gradient descent⁶ or coordinate gradient descent⁷

⁶ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer

⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008

⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009

⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008

¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009

¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec - distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)

⁶ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer

⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008

⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009

⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008

¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009

¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec - distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)
 - Negative sampling.⁹

⁶ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer

⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008

⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009

⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008

¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009

¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec - distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - ► Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)
 - Negative sampling.⁹
- Space complexity:

^b L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer

⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008

⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009

⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008

¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009

¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec - distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - ► Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)
 - Negative sampling.⁹
- Space complexity:
 - Proper regularization: L_1 vs L_2 .

⁶ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer

⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008

⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009

⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008

¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009

¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec - distributed sparse machines for extreme multilabel classification. CoRR, 2016

- Training time complexity:
 - Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)
 - Negative sampling.⁹
- Space complexity:
 - Proper regularization: L_1 vs L_2 .
 - ► Feature hashing.¹⁰
- ^b L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer
- ⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008
- ⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009
- ⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008
- ¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009
- ¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec distributed sparse machines for extreme multilabel classification. CoRR, 2016
Linear models

- Training time complexity:
 - Stochastic gradient descent⁶ or coordinate gradient descent⁷
 - ► Sparse feature vectors (e.g., sparse updates in SGD⁸)
 - Negative sampling.⁹
- Space complexity:
 - Proper regularization: L_1 vs L_2 .
 - ► Feature hashing.¹⁰
 - Removing small weights.¹¹
- ⁵ L. Bottou. Large-scale machine learning with stochastic gradient descent. In Yves Lechevallier and Gilbert Saporta, editors, COMPSTAT, pages 177–187, Paris, France, August 2010. Springer
- ⁷ R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin. LIBLINEAR: A library for large linear classification. *Journal of Machine Learning Research*, 9:1871–1874, 2008
- ⁸ John Duchi and Yoram Singer. Efficient online and batch learning using forward backward splitting. JMLR, 10:2899–2934, 2009
- ⁹ Ronan Collobert and Jason Weston. A unified architecture for natural language processing: Deep neural networks with multitask learning. In *ICML*, pages 160–167, 2008
- ¹⁰ K.Q. Weinberger, A. Dasgupta, J. Langford, A. Smola, and J. Attenberg. Feature hashing for large scale multitask learning. In *ICML*, pages 1113–1120. ACM, 2009
- ¹¹ Rohit Babbar and Bernhard Schölkopf. Dismec distributed sparse machines for extreme multilabel classification. CoRR, 2016

Linear models

• Low-dimensional representation of $\mathbf{X},\,\mathbf{W},\,\mathbf{Y}:$

 $\mathbf{Y}=\mathbf{U}^{\dagger}\mathbf{V}\mathbf{X}$

- ▶ feature space: PCA on X.
- ▶ label space: PCA on Y,¹² compressed sensing,¹³ etc.
- \blacktriangleright both spaces: CCA on both ${\bf X}$ and ${\bf Y},^{14}$ etc.
- ▶ matrix factorization of W.¹⁵
- A kind of lossy compression/embedding methods.

- ¹³ D. Hsu, S. Kakade, J. Langford, and T. Zhang. Multi-label prediction via compressed sensing. In NIPS, 2009
- ¹⁴ Yao-Nan Chen and Hsuan-Tien Lin. Feature-aware label space dimension reduction for multilabel classification. In NIPS, pages 1529–1537. Curran Associates, Inc., 2012
- ¹⁵ Hsiang-Fu Yu, Prateek Jain, Purushottam Kar, and Inderjit S. Dhillon. Large-scale Multi-label Learning with Missing Labels. In *ICML*, 2014

¹² F. Tai and H.-T. Lin. Multi-label classification with principal label space transformation. In *Neural Computat.*, volume 9, pages 2508–2542, 2012

Computational challenges

• Prediction time is still linear in the number of labels!

Computational challenges

• Prediction time is still linear in the number of labels!

How to reduce the test time complexity?

Test time complexity for linear models

• Classification of a test example in case of linear models can be formulated as:

$$i^* = \operatorname*{arg\,max}_{i \in \{1,\dots,m\}} \boldsymbol{w}_i \boldsymbol{x} \,,$$

i.e., the problem of maximum inner product search (MIPS).

• Exact solution: the threshold algorithm¹⁶

¹⁶ Ronald Fagin, Amnon Lotem, and Moni Naor. Optimal aggregation algorithms for middleware. In PODS '01, pages 102–113. ACM, New York, NY, USA, 2001

MIPS vs. nearest neighbors

• MIPS is similar, but not the same, to nearest neighbor search under the square or cosine distance:

$$egin{array}{rcl} i^{*} &=& rgmin_{i\in\{1,...,m\}} \|m{w}_{i}-m{x}\|_{2}^{2} =rgmax_{i\in\{1,...,m\}} m{w}_{i}m{x} - rac{\|m{w}_{i}\|_{2}^{2}}{2} \ i^{*} &=& rgmax_{i\in\{1,...,m\}} rac{m{w}_{i}m{x}}{\|m{w}_{i}\|\|m{x}\|} = rgmax_{i\in\{1,...,m\}} rac{m{w}_{i}m{x}}{\|m{w}_{i}\|} \ \end{array}$$

• Some tricks are used to treat MIPS as nearest neighbor search.¹⁷

 ¹⁷ A. Shrivastava and P. Li. Improved asymmetric locality sensitive hashing (ALSH) for maximum inner product search (mips). In UAI, 2015
Sudheendra Vijayanarasimhan, Jonathon Shlens, Rajat Monga, and Jay Yagnik. Deep networks with large output spaces. CoRR, abs/1412.7479, 2014
J. Yagnik, D. Strelow, D. A. Ross, and R. s. Lin. The power of comparative reasoning. In International Conference on Computer Vision, pages 2431–2438, Nov 2011

Decision trees

- Fast prediction: logarithmic in n
- Training can be expensive: computation of split criterion
- Two new algorithms: LomTree¹⁸ and FastXML¹⁹

¹⁸ Anna Choromanska and John Langford. Logarithmic time online multiclass prediction. In NIPS 29, 2015

¹⁹ Yashoteja Prabhu and Manik Varma. Fastxml: A fast, accurate and stable tree-classifier for extreme multi-label learning. In *KDD*, pages 263–272. ACM, 2014

FastXML

- Uses an **ensemble** of standard decision trees.
- Sparse linear classifiers trained in internal nodes.
- Very efficient training procedure.
- Empirical distributions in leaves.
- A test example passes one path from the root to a leaf.



FastXML

- Uses an **ensemble** of standard decision trees.
- Sparse linear classifiers trained in internal nodes.
- Very efficient training procedure.
- Empirical distributions in leaves.
- A test example passes one path from the root to a leaf.



Label trees

• Organize classifiers in a tree structure (one leaf \Leftrightarrow one label).²⁰



- Structure of the tree can be given or trained.
- Different training and test procedures for multi-class and multi-label classification.

²⁰ S. Bengio, J. Weston, and D. Grangier. Label embedding trees for large multi-class tasks. In *NIPS*, pages 163–171. Curran Associates, Inc., 2010

Probabilistic label trees (PLT)²¹

• PLT are based on *b*-ary label trees.



- Probabilistic classifiers in all nodes of the tree.
- Internal node classifier decides whether to go down the tree.
- A test example may follow many paths from the root to leaves.

²¹ K. Jasinska, K. Dembczynski, R. Busa-Fekete, K. Pfannschmidt, T. Klerx, and E. Hüllermeier. Extreme F-measure maximization using sparse probability estimates. In *ICML*, 2016

• Class probability estimators in nodes for estimating $\mathbf{P}(y_i = 1 | \boldsymbol{x})$.



• Using the chain rule of probability

$$\mathbf{P}(y_i = 1 \,|\, \boldsymbol{x}) = \eta(\boldsymbol{x}, i) = \prod_{t \in \text{Path}(i)} \eta_T(\boldsymbol{x}, t) \,,$$

where $\eta_T(\boldsymbol{x},t) = \begin{cases} \mathbf{P}(z_t = 1 | \boldsymbol{x}) & \text{if } t \text{ is root,} \\ \mathbf{P}(z_t = 1 | z_{\text{pa}(t)} = 1, \boldsymbol{x}) & \text{otherwise.} \end{cases}$

• Class probability estimators in nodes for estimating $\mathbf{P}(y_i = 1 | \boldsymbol{x})$.



• Training: reduced complexity by the conditions used in the nodes.

• Class probability estimators in nodes for estimating $\mathbf{P}(y_i = 1 | \boldsymbol{x})$.



- Training: reduced complexity by the conditions used in the nodes.
- Prediction: priority queue search or branch and bound.

- The same idea under different names:
 - Conditional probability trees²²
 - ► Probabilistic classifier chains²³
 - Hierarchical softmax²⁴
 - ► Homer²⁵
 - Nested dichotomies²⁶
 - Multi-stage classification²⁷

²⁴ Frederic Morin and Yoshua Bengio. Hierarchical probabilistic neural network language model. In AISTATS, pages 246–252, 2005

²² A. Beygelzimer, J. Langford, Y. Lifshits, G. B. Sorkin, and A. L. Strehl. Conditional probability tree estimation analysis and algorithms. In UAI, pages 51–58, 2009

²³ K. Dembczyński, W. Cheng, and E. Hüllermeier. Bayes optimal multilabel classification via probabilistic classifier chains. In *ICML*, pages 279–286. Omnipress, 2010

²⁵ G. Tsoumakas, I. Katakis, and I. Vlahavas. Effective and efficient multilabel classification in domains with large number of labels. In Proc. ECML/PKDD 2008 Workshop on Mining Multidimensional Data, 2008

²⁶ J. Fox. Applied regression analysis, linear models, and related methods. Sage, 1997

²⁷ Marek Kurzynski. On the multistage bayes classifier. *Pattern Recognition*, 21(4):355–365, 1988

FastXML vs. PLT

	FastXML	PLT
tree structure	\checkmark	\checkmark
structure learning	\checkmark	×
number of trees	≥ 1	1
number of leaves	linear in $\#$ examples	m
internal nodes models	linear	linear
leaves models	empirical distribution	linear
visited paths during prediction	1 per tree	several
sparse probability estimation	\checkmark	\checkmark

	#labels	#features	#test	#train	inst./lab.	lab./inst.
RCV1	2456	47236	155962	623847	1218.56	4.79
AmazonCat	13330	203882	306782	1186239	448.57	5.04
Wiki10	30938	101938	6616	14146	8.52	18.64
Delicious	205443	782585	100095	196606	72.29	75.54
WikiLSHTC	325056	1617899	587084	1778351	17.46	3.19
Amazon	670091	135909	153025	490449	3.99	5.45

Table: Datasets from the Extreme Classification repository.²⁸

²⁸ http://research.microsoft.com/en-us/um/people/manik/downloads/XC/ XMLRepository.html

		PLT		FastXML		
	P@1	P@3	P@5	P@1	P@3	P@5
RCV1	90.46	72.4	51.86	91.13	73.35	52.67
AmazonCat	91.47	75.84	61.02	92.95	77.5	62.51
Wiki10	84.34	72.34	62.72	81.71	66.67	56.70
Delicious	45.37	38.94	35.88	42.81	38.76	36.34
WikiLSHTC	45.67	29.13	21.95	49.35	32.69	24.03
Amazon	36.65	32.12	28.85	34.24	29.3	26.12

	PLT					FastXML			
	train [min]	test [ms]	b	depth	#calls	train [min]	test [ms]	depth	#calls
RCV1	64	0.22	32	2,25	43,46	78	0.96	14.95	747
AmazonCat	96	0.17	16	3,43	54,39	561	1.14	17.44	871
Wiki10	290	2.66	32	2,98	121,98	16	3.00	10.83	541
Delicious	1327	32.97	2	17,69	11779,65	458	4.01	14.79	739
WikiLSHTC	653	3.00	32	3,66	622,27	724	1.17	18.01	900
Amazon	54	0.99	8	6,45	374,30	422	1.39	15.92	796

• Reduce extreme classification to structured output prediction:

²⁹ Joint work with Kalina Jasinska and Nikos Karampatziakis

- Reduce extreme classification to structured output prediction:
 - encode labels by sequences of bits,

²⁹ Joint work with Kalina Jasinska and Nikos Karampatziakis

- Reduce extreme classification to structured output prediction:
 - encode labels by sequences of bits,
 - choose a proper dependence structure between bits,

²⁹ Joint work with Kalina Jasinska and Nikos Karampatziakis

- Reduce extreme classification to structured output prediction:
 - encode labels by sequences of bits,
 - choose a proper dependence structure between bits,
 - use appropriate **training** and **inference** methods.

²⁹ Joint work with Kalina Jasinska and Nikos Karampatziakis

- Reduce extreme classification to structured output prediction:
 - encode labels by sequences of bits,
 - choose a proper dependence structure between bits,
 - use appropriate **training** and **inference** methods.
- Enables to work under limited time and space budget.

²⁹ Joint work with Kalina Jasinska and Nikos Karampatziakis



• LTLS³⁰ encodes labels as paths in a trellis of width 2.

³⁰ Kalina Jasinska and Nikos Karampatziakis. Log-time and log-space extreme classication. In Extreme Classification workshop at NIPS, 2016



- LTLS³⁰ encodes labels as paths in a **trellis** of width 2.
- Each **path** corresponds to one and only one label.

³⁰ Kalina Jasinska and Nikos Karampatziakis. Log-time and log-space extreme classification. In Extreme Classification workshop at NIPS, 2016



- LTLS³⁰ encodes labels as paths in a **trellis** of width 2.
- Each **path** corresponds to one and only one label.
- Training concerns models on edges.

³⁰ Kalina Jasinska and Nikos Karampatziakis. Log-time and log-space extreme classification. In Extreme Classification workshop at NIPS, 2016



- LTLS³⁰ encodes labels as paths in a **trellis** of width 2.
- Each **path** corresponds to one and only one label.
- Training concerns models on edges.
- Prediction of the most probable labels corresponds to **finding** the most probable paths.

³⁰ Kalina Jasinska and Nikos Karampatziakis. Log-time and log-space extreme classification. In Extreme Classification workshop at NIPS, 2016

• Can be trained with logistic loss or a variant of structured hinge loss³¹

³¹ Ian En-Hsu Yen, Xiangru Huang, Pradeep Ravikumar, Kai Zhong, and Inderjit Dhillon. Pd-sparse : A primal and dual sparse approach to extreme multiclass and multilabel classification. In International Conference on Machine Learning, 2016

- Can be trained with logistic loss or a variant of structured hinge loss³¹
- Can work with any number of labels (not only the powers of 2).

³¹ Ian En-Hsu Yen, Xiangru Huang, Pradeep Ravikumar, Kai Zhong, and Inderjit Dhillon. Pd-sparse : A primal and dual sparse approach to extreme multiclass and multilabel classification. In International Conference on Machine Learning, 2016

- Can be trained with logistic loss or a variant of structured hinge loss³¹
- Can work with any number of labels (not only the powers of 2).
- The number of edges is upperbounded by $5\lceil \log_2 m\rceil + 1$

³¹ Ian En-Hsu Yen, Xiangru Huang, Pradeep Ravikumar, Kai Zhong, and Inderjit Dhillon. Pd-sparse : A primal and dual sparse approach to extreme multiclass and multilabel classification. In International Conference on Machine Learning, 2016

- Can be trained with logistic loss or a variant of structured hinge loss³¹
- Can work with any number of labels (not only the powers of 2).
- The number of edges is upperbounded by $5\lceil \log_2 m\rceil + 1$
- The inference can be efficiently performed by the Viterbi algorithm.

³¹ Ian En-Hsu Yen, Xiangru Huang, Pradeep Ravikumar, Kai Zhong, and Inderjit Dhillon. Pd-sparse : A primal and dual sparse approach to extreme multiclass and multilabel classification. In *International Conference on Machine Learning*, 2016

- Can be trained with logistic loss or a variant of structured hinge loss³¹
- Can work with any number of labels (not only the powers of 2).
- The number of edges is upperbounded by $5\lceil \log_2 m\rceil + 1$
- The inference can be efficiently performed by the Viterbi algorithm.
- Therefore, the space and time complexity of training and testing is **logarithmic** in the number of labels.

³¹ Ian En-Hsu Yen, Xiangru Huang, Pradeep Ravikumar, Kai Zhong, and Inderjit Dhillon. Pdsparse : A primal and dual sparse approach to extreme multiclass and multilabel classification. In *International Conference on Machine Learning*, 2016

		LTLS-LR	PLT	LOMtree	FastXML
sector	P@1 model size	0.8616 12.06	0.8730 16.00	0.8210 17.00	0.8490 7.00
aloi.bin	P@1 model size	0.8128	0.9088 128	0.8947 106	0.9550 992
Dmoz	P@1 model size	0.2082	0.3263 2048	0.2127 1800	0.3840 1500
LSHTC1	precision@1 model size	0.0950	0.1524 1024	0.1056 744	0.2166 308

Extreme Zero-Shot Learning³²

- In traditional setting the target variable is a binary indicator.
- We replace the binary indicator by a richer representation.
- We use a textual description of a label.

³² Joint work with Marcin Elantkowski and Moustapha Cisse

Extreme Zero-Shot Learning

 \bullet Consider a multi-label dataset with n training examples and m labels of the form

 $\{(\boldsymbol{x}_i,\mathcal{Y}_i)\}_{i=1}^n$
• Consider a multi-label dataset with \boldsymbol{n} training examples and \boldsymbol{m} labels of the form

 $\{(\boldsymbol{x}_i,\mathcal{Y}_i)\}_{i=1}^n$

• $\mathcal{Y}_i = \{\ell_1^i, \ell_2^i, \dots, \ell_{k_i}^i\}$ is a set of labels relevant for the *i*-th example, with $0 \leq k_i \leq m$ and $\ell_j^i \in \mathcal{Y}_s$, where $\mathcal{Y}_s = \{1, 2, \dots, m\}$ is a set of all labels in the training data.

• Consider a multi-label dataset with \boldsymbol{n} training examples and \boldsymbol{m} labels of the form

 $\{(\boldsymbol{x}_i,\mathcal{Y}_i)\}_{i=1}^n$

- $\mathcal{Y}_i = \{\ell_1^i, \ell_2^i, \dots, \ell_{k_i}^i\}$ is a set of labels relevant for the *i*-th example, with $0 \leq k_i \leq m$ and $\ell_j^i \in \mathcal{Y}_s$, where $\mathcal{Y}_s = \{1, 2, \dots, m\}$ is a set of all labels in the training data.
- $\bar{\mathcal{Y}}_i = \mathcal{Y}_s \setminus \mathcal{Y}_i$ is a set of labels irrelevant for the *i*-th example.

• Consider a multi-label dataset with \boldsymbol{n} training examples and \boldsymbol{m} labels of the form

 $\{(\boldsymbol{x}_i,\mathcal{Y}_i)\}_{i=1}^n$

- $\mathcal{Y}_i = \{\ell_1^i, \ell_2^i, \dots, \ell_{k_i}^i\}$ is a set of labels relevant for the *i*-th example, with $0 \leq k_i \leq m$ and $\ell_j^i \in \mathcal{Y}_s$, where $\mathcal{Y}_s = \{1, 2, \dots, m\}$ is a set of all labels in the training data.
- $\bar{\mathcal{Y}}_i = \mathcal{Y}_s \setminus \mathcal{Y}_i$ is a set of labels irrelevant for the *i*-th example.
- x_i is a sparse vector of features (e.g., word indices) from a set $\mathcal{V}_{\mathcal{X}}$, represented as a sequence of features

$$\boldsymbol{x}_i = \{x_1^i, x_2^i, \dots, x_{d_i}^i\}, \quad x_j^i \in \mathcal{V}_{\mathcal{X}},$$

where d_i denotes the number of non-zero features for the *i*-th example (e.g., a number of words in a document).

- At test time, we are given a set of m^\prime additional labels

$$\mathcal{Y}_u = \{m+1, m+2, \dots, m+m'\}$$

that can be assigned to test examples.

- At test time, we are given a set of m^\prime additional labels

$$\mathcal{Y}_u = \{m+1, m+2, \dots, m+m'\}$$

that can be assigned to test examples.

• These labels have not been observed during training, i.e. $\mathcal{Y}_s \cap \mathcal{Y}_u = \emptyset$.

- At test time, we are given a set of m^\prime additional labels

$$\mathcal{Y}_u = \{m+1, m+2, \dots, m+m'\}$$

that can be assigned to test examples.

- These labels have not been observed during training, i.e. $\mathcal{Y}_s \cap \mathcal{Y}_u = \emptyset$.
- Additionally, for each label $\ell \in \{1, 2, ..., m + m'\}$ we are given its description (e.g., textual description from a dictionary) of the form

$$\boldsymbol{t}_{\ell} = \{t_1^{\ell}, t_2^{\ell}, \dots, t_{m_{\ell}}^{\ell}\}, t_j^{\ell} \in \mathcal{V}_{\mathcal{Y}}$$

where $\mathcal{V}_\mathcal{Y}$ is a separate feature space describing labels, possibly different from $\mathcal{V}_\mathcal{X}.$

• We are interested in finding a scoring function

 $f:({\bm x}_i,\ell)\to \mathbb{R}\,,$ such that $f({\bm x}_i,\ell)>f({\bm x}_i,\hat\ell)$ for all $\ell\in\mathcal{Y}_i,\,\hat\ell\in\bar{\mathcal{Y}}_i.$

• We are interested in finding a scoring function

$$f:(\boldsymbol{x}_i,\ell)\to\mathbb{R},$$

such that $f(\boldsymbol{x}_i, \ell) > f(\boldsymbol{x}_i, \hat{\ell})$ for all $\ell \in \mathcal{Y}_i$, $\hat{\ell} \in \overline{\mathcal{Y}}_i$.

• To leverage the provided label description we use:

$$f:(\boldsymbol{x}_i,\boldsymbol{t}_\ell)\to\mathbb{R},$$

i.e., the label descriptions instead of raw labels.

• To obtain the scoring function we learn a mapping of both:

³³ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁴ Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In IJCAI, pages 2764–2770, 2011

- To obtain the scoring function we learn a mapping of both:
 - feature spaces and

³³ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁴ Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In IJCAI, pages 2764–2770, 2011

• To obtain the scoring function we learn a mapping of both:

- feature spaces and
- label descriptions

³³ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁴ Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In *IJCAI*, pages 2764–2770, 2011

- To obtain the scoring function we learn a mapping of both:
 - feature spaces and
 - label descriptions

into a common embedding space, in which the dot product between both representations is maximized.

³³ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁴ Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In *IJCAI*, pages 2764–2770, 2011

- To obtain the scoring function we learn a mapping of both:
 - feature spaces and
 - label descriptions

into a common embedding space, in which the dot product between both representations is maximized.

- While these mappings are different, they are learned jointly to optimize the loss function of choice:
 - ▶ logistic loss (like in FastText³³) or
 - ▶ WARP loss (like in Wsabie³⁴)

³³ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁴ Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In IJCAI, pages 2764–2770, 2011



• More formally, we define

$$\phi_{\mathcal{X}}(\boldsymbol{x}_i): \mathbb{R}^{|\mathcal{V}_{\mathcal{X}}|} \to \mathbb{R}^p \quad \phi_{\mathcal{Y}}(\boldsymbol{t}_{\ell}): \mathbb{R}^{|\mathcal{V}_{\mathcal{Y}}|} \to \mathbb{R}^p,$$

and consider scoring function $f(\cdot)$ of the form:

$$f(\boldsymbol{x}_i, \boldsymbol{t}_\ell) = \phi_{\mathcal{X}}(\boldsymbol{x}_i)^\top \phi_{\mathcal{Y}}(\boldsymbol{t}_\ell).$$

• More formally, we define

$$\phi_{\mathcal{X}}(\boldsymbol{x}_i): \mathbb{R}^{|\mathcal{V}_{\mathcal{X}}|} \to \mathbb{R}^p \quad \phi_{\mathcal{Y}}(\boldsymbol{t}_{\ell}): \mathbb{R}^{|\mathcal{V}_{\mathcal{Y}}|} \to \mathbb{R}^p,$$

and consider scoring function $f(\cdot)$ of the form:

$$f(\boldsymbol{x}_i, \boldsymbol{t}_\ell) = \phi_{\mathcal{X}}(\boldsymbol{x}_i)^\top \phi_{\mathcal{Y}}(\boldsymbol{t}_\ell).$$

• To obtain representation $\phi(\cdot)$ we use a weight matrix $W \in \mathbb{R}^{|\mathcal{V}| \times p}$ which acts as a look-up table over the features (e.g., words).

• More formally, we define

$$\phi_{\mathcal{X}}(\boldsymbol{x}_i): \mathbb{R}^{|\mathcal{V}_{\mathcal{X}}|} \to \mathbb{R}^p \quad \phi_{\mathcal{Y}}(\boldsymbol{t}_{\ell}): \mathbb{R}^{|\mathcal{V}_{\mathcal{Y}}|} \to \mathbb{R}^p,$$

and consider scoring function $f(\cdot)$ of the form:

$$f(\boldsymbol{x}_i, \boldsymbol{t}_\ell) = \phi_{\mathcal{X}}(\boldsymbol{x}_i)^\top \phi_{\mathcal{Y}}(\boldsymbol{t}_\ell).$$

- To obtain representation $\phi(\cdot)$ we use a weight matrix $W \in \mathbb{R}^{|\mathcal{V}| \times p}$ which acts as a look-up table over the features (e.g., words).
- Embedding of an example or a label is then obtained by averaging representations of individual features within it.

• More formally, we define

$$\phi_{\mathcal{X}}(\boldsymbol{x}_i): \mathbb{R}^{|\mathcal{V}_{\mathcal{X}}|} \to \mathbb{R}^p \quad \phi_{\mathcal{Y}}(\boldsymbol{t}_\ell): \mathbb{R}^{|\mathcal{V}_{\mathcal{Y}}|} \to \mathbb{R}^p,$$

and consider scoring function $f(\cdot)$ of the form:

$$f(\boldsymbol{x}_i, \boldsymbol{t}_\ell) = \phi_{\mathcal{X}}(\boldsymbol{x}_i)^\top \phi_{\mathcal{Y}}(\boldsymbol{t}_\ell).$$

- To obtain representation $\phi(\cdot)$ we use a weight matrix $W \in \mathbb{R}^{|\mathcal{V}| \times p}$ which acts as a look-up table over the features (e.g., words).
- Embedding of an example or a label is then obtained by averaging representations of individual features within it.
- We maintain two separate weight matrices, $W_{\mathcal{X}}$ and $W_{\mathcal{Y}}$, for $\phi_{\mathcal{X}}(\cdot)$ and $\phi_{\mathcal{Y}}(\cdot)$, respectively.

- Learning in the extreme setting:
 - Negative sampling
- Testing in the extreme setting:
 - ► Fast nearest neighbor search in the embedding space
- Problems with unseen labels:
 - ► Labels can be described by features not seen during training.

XZSL – Experimental results

- We compare the above approach against:
 - ► *k*-NN in TF-IDF space
 - Label name in example.
 - ► fastText³⁵
 - ► All-in-Text³⁶
 - ► FastXML³⁷

³⁵ Armand Joulin, Edouard Grave, Piotr Bojanowski, and Tomas Mikolov. Bag of tricks for efficient text classification. CoRR, abs/1607.01759, 2016

³⁶ Jinseok Nam, Eneldo Loza Mencía, and Johannes Fürnkranz. All-in text: Learning document, label, and word representations jointly. In AAAI Conference on Artificial Intelligence, pages 1948–1954, 2016

³⁷ Yashoteja Prabhu and Manik Varma. Fastxml: A fast, accurate and stable tree-classifier for extreme multi-label learning. In KDD, pages 263–272. ACM, 2014

XZSL – Experimental results

• Statistics of the BioASQ dataset

6,792,815
4,912,719
23,669
2,435
10.83
528,156
39,958

XZSL – Experimental results

	TF-IDF	Label in Exp	FastText	All in Text	XZSL
P@1	0.143	0.050	0.875	0.74	0.812
rank loss	0.278	0.50	0.012	0.035	0.013
avg. prec.	0.040	0.017	0.461	0.327	0.37
P@1 (zsl)	0.084	0.048	-	0.013	0.046
rank loss (zsl)	0.217	0.299	-	0.216	0.095
avg. prec. (zsl)	0.142	0.083	-	0.026	0.1

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction
 - Statistical challenges:

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction
 - Statistical challenges:
 - Is learning possible in the extreme setting?

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction
 - Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction
 - Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,

- Take-away message:
 - ► Extreme classification: #examples, #features, #labels
 - ► Complexity: time vs. space, training vs. validation vs. prediction
 - Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,
 - Unreliable learning information,

- ► Extreme classification: #examples, #features, #labels
- ► Complexity: time vs. space, training vs. validation vs. prediction
- Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,
 - Unreliable learning information,
 - Long-tail label distributions,

- ► Extreme classification: #examples, #features, #labels
- ► Complexity: time vs. space, training vs. validation vs. prediction
- Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,
 - Unreliable learning information,
 - Long-tail label distributions,
 - Zero-shot learning.

- ► Extreme classification: #examples, #features, #labels
- ► Complexity: time vs. space, training vs. validation vs. prediction
- Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,
 - Unreliable learning information,
 - Long-tail label distributions,
 - Zero-shot learning.
- For more check:

- ► Extreme classification: #examples, #features, #labels
- ► Complexity: time vs. space, training vs. validation vs. prediction
- Statistical challenges:
 - Is learning possible in the extreme setting?
 - Training and prediction under limited time and space budged.
 - Performance measures,
 - Unreliable learning information,
 - Long-tail label distributions,
 - Zero-shot learning.
- For more check:
 - http://www.cs.put.poznan.pl/kdembczynski