Monotonicity of Bayesian confirmation measure in rule support and confidence

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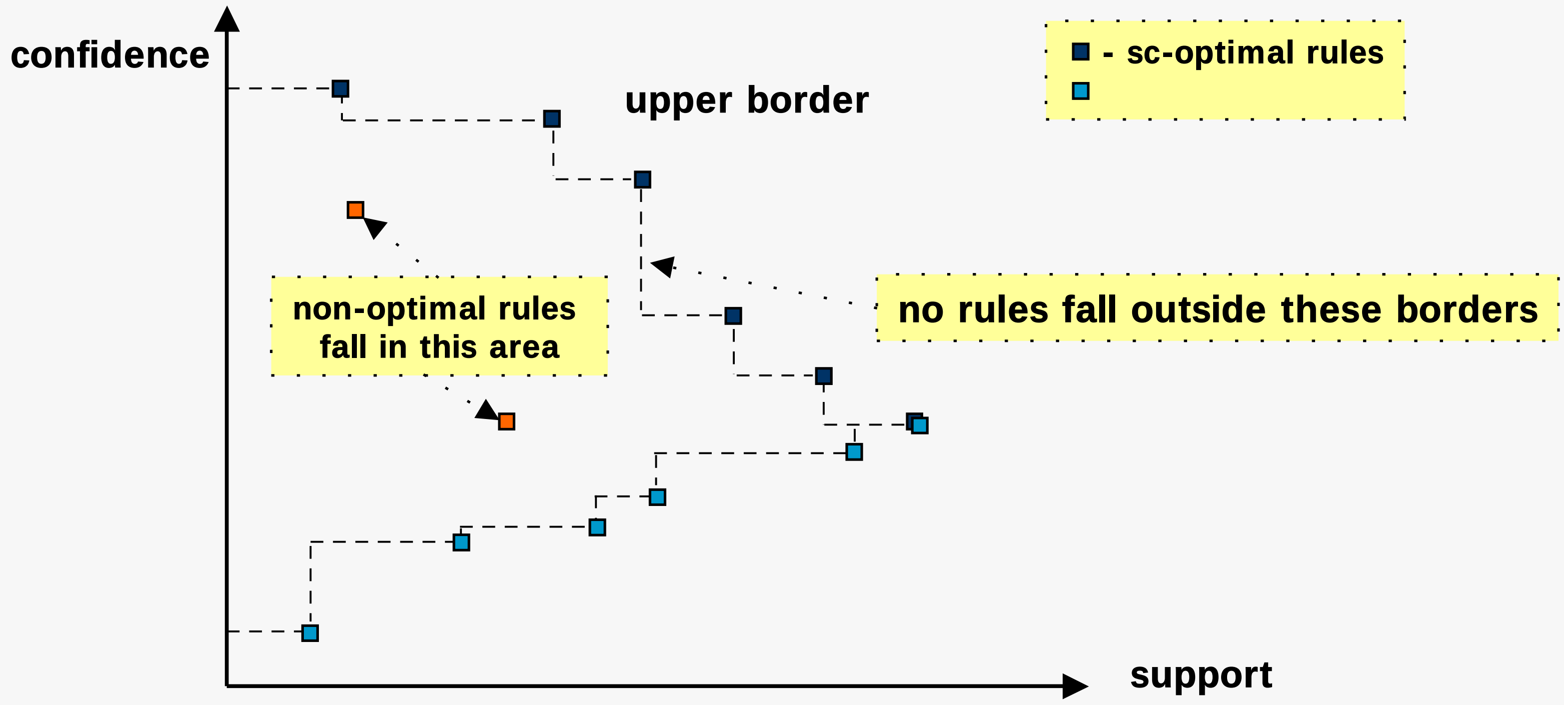
Monotoniczność Bayesowskiej miary konfirmacji względem współczynników zaufania i wsparcia reguł

Inspirations and motivations

- **The pareto optimal support-confidence border**
- **Bayesian confirmation measures**
- **The property of monotonicity (M)**

Motivations – pareto optimal support-confidence border (1)

- Mining **pareto optimal rule support-confidence border** identifies rules optimal with respect to measures such as: gain, p-s, lift, conviction etc.



Motivations - pareto optimal support-confidence border (2)

- The following conditions are sufficient for verifying whether rules optimal according to a measure $g(x)$ are included on the support-confidence pareto optimal border:

  1. $g(x)$ is monotone in support over rules with the same confidence, and
  2. $g(x)$ is monotone in confidence over rules with the same rule support.

- A function $g(x)$ is understood to be monotone in x, if $x_1 < x_2$ implies that $g(x_1) \leq g(x_2)$.

Motivations – Bayesian confirmation measures (1)

- Among widely studied interestingness measures, there is a group of **Bayesian confirmation measures**.
- They quantify the degree to which a piece of evidence built of the independent attributes provides "evidence for or against" or "support for or against" the hypothesis built of the dependent attributes
- Among the most well-known Bayesian confirmation measures proposed in the literature, an important role is played by a confirmation measure denoted by *f*, which has the property of hypothesis symmetry, property of monotonicity (M).

Presentation plan

- **Monotonicity of confirmation measure *f* in rule support and confidence**
- **Property of monotonicity (M)**
- **Rule support, confidence, gain measure and the property (M)**
- **Property (M) vs. monotonicity in rule support and confidence**
- **Further research plans**

Monotonicity of *f* in rule support and confidence (1)

- **Let us consider a Bayesian confirmation measure *f* defined as follows:**

$$f(\phi \to \psi) = \frac{conf(\psi \to \phi) - conf(\neg\psi \to \phi)}{conf(\psi \to \phi) + conf(\neg\psi \to \phi)}$$

- **Having observed that:**
  - $sup(\sim\phi \to \psi) + sup(\phi \to \psi) = sup(\psi)$,
  - $sup(\sim\phi) = |U| - sup(\phi)$,
  - $sup(\phi) = sup(\phi \to \psi) / conf(\phi \to \psi)$,
  - $sup(\sim\psi \to \phi) = sup(\phi) - sup(\phi \to \psi)$

$conf(\phi \to \psi)$

$sup(\phi \to \psi)$

we can transform *f* into such a form:

$$f(\phi \to \psi) = \frac{|U|\,conf(\phi \to \psi) - \sup(\psi)}{(|U| - 2\sup(\psi))\,conf(\phi \to \psi) + \sup(\psi)}$$

Monotonicity of *f* in rule support and confidence (2)

$$f(\phi \to \psi) = \frac{|U|\,conf(\phi \to \psi) - \sup(\psi)}{(|\mathrm{U}| - 2\sup(\psi))\,conf(\phi \to \psi) + \sup(\psi)}$$

- **we assume that |U| and *sup*(ψ) are constants as we consider only rules with a fixed conclusion (i.e. from one decision class)**

- **Let us verify whether *f* is**

1. **monotone in rule support for a fixed value of confidence, and**
2. **monotone in confidence for a fixed value of rule support.**

- **These are the Bayardo-Agrawal sufficient conditions for „laying on" support-confidence pareto border.**

Monotonicity of *f* in rule support for fixed confidence value

$$f(\phi \to \psi) = \frac{|U|\,conf(\phi \to \psi) - \sup(\psi)}{(|\mathrm{U}| - 2\sup(\psi))\,conf(\phi \to \psi) + \sup(\psi)}$$

- **Hypothesis: *f* is monotone in rule support for fixed confidence.**
- **Proof:**

f is independent of rule support $sup(\phi\to\psi)$, so for $conf(\phi\to\psi)=const$,

f is constant and thus monotone in rule support.

Monotonicity of *f* in confidence for fixed rule support

$$f(\phi \to \psi) = \frac{|U|\,conf(\phi \to \psi) - \sup(\psi)}{(|\mathrm{U}| - 2\sup(\psi))\,conf(\phi \to \psi) + \sup(\psi)}$$

- **Hypothesis: *f* is monotone in confidence for fixed rule support.**
- **Proof schema:**
  - **express *f* as a function of *conf*($\phi \to \psi$),**
  - **calcutate the derivative *f′* of *f* and verify its sing**
- **Conclusions:**

  since *f′* is always $\geq$ 0 then *f* is monotone in confidence.

Support-confidence monotonicity of *f* - conclusions

- **The Bayesian confirmation measure *f* is**

1. **independent of rule support and therefore monotone in rule support**
2. **and monotone in confidence.**

- **Rules optimal with respect to**

  ***f* lie on the support-confidence pareto border**

  (sic: we consider rules with fixed conclusion)

Utility of confidence vs. utility of confirmation *f* (1)

- **What's the use of looking for rules with optimal *f* since they lie on the pareto border?**
  - **The above result does not deny the interest of *f* in expressing the attractiveness of rules; it just states the monotonicity of *f* in confidence of rules for a fixed conclusion**
  - **This result does not refer, however, to utility of scales in which confirmation *f*($\phi \rightarrow \psi$) and confidence *conf*($\phi \rightarrow \psi$) are expressed**
  - **While the confidence *conf*($\phi \rightarrow \psi$) is the truth value of the knowledge pattern "*if* ϕ*, then* ψ", the confirmation measure *f*($\phi \rightarrow \psi$) says to what extend ψ is satisfied more frequently when ϕ is satisfied rather than when ϕ is not satisfied.**

Utility of confidence vs. utility of confirmation *f* (2)

- **Consider the possible result of rolling a die: 1,2,3,4,5,6, and let the conclusion ψ="the result is 6".**
  - **ϕ_1="the result is divisible by 3",** ***conf*($\phi_1 \rightarrow \psi$)=1/2, *f*($\phi_1 \rightarrow \psi$)=2/3,**
  - **ϕ_2="the result is divisible by 2",** ***conf*($\phi_2 \rightarrow \psi$)=1/3, *f*($\phi_2 \rightarrow \psi$)=3/7,**
  - **ϕ_3="the result is divisible by 1",** ***conf*($\phi_3 \rightarrow \psi$)=1/6, *f*($\phi_3 \rightarrow \psi$)=0.**
- **This example acknowledges the monotonicity of confirmation in confidence, it clearly shows that the value of *f* has a more useful interpretation than *conf*,**
- **In particular, in case of rule $\phi_3 \rightarrow \psi$, which can also be read as "in any case, the result is 6"; indeed, the "any case" does not add any information which could confirm that the result is 6, and this fact is expressed by *f*($\phi_3 \rightarrow \psi$)=0.**

Utility of confidence vs. utility of confirmation *f* (3)

- Consider the possible result of rolling a die: 1,2,3,4,5,6, and let the premise be kept fixed at ϕ="the result is divisible by 2"
  - ψ_1="the result is 6" , *conf*($\phi\rightarrow\psi_1$)=1/3, *f*($\phi\rightarrow\psi_1$)=3/7
  - ψ_2="the result is *not* 6" *conf*($\phi\rightarrow\psi_2$)=2/3, *f*($\phi\rightarrow\psi_2$)=–3/7.
- In this example, rule $\phi\rightarrow\psi_2$ has greater confidence than rule $\phi\rightarrow\psi_1$
- However, rule $\phi\rightarrow\psi_2$ is less interesting than rule $\phi\rightarrow\psi_1$ because premise ϕ reduces the probability of conclusion ψ_2 from 5/6=*sup*(ψ_2) to 2/3= *conf*($\phi\rightarrow\psi_2$), while it augments the probability of conclusion ψ_1 from 1/6=*sup*(ψ_1) to 1/3= *conf*($\phi\rightarrow\psi_1$).
- In consequence, premise ϕ disconfirms conclusion ψ_2, which is expressed by a negative value of *f*($\phi\rightarrow\psi_2$)=–3/7, and it confirms conclusion ψ_1, which is expressed by a positive value of *f*($\phi\rightarrow\psi_1$)=3/7.

Property of monotonicity (M)

n **The property of monotonicity [proposed by Greco et al.]**

(M) $c(\phi,\psi) = F\,[sup(\phi\rightarrow\psi),\ sup(\sim\phi\rightarrow\psi),\ sup(\phi\rightarrow\ \sim\psi),\ sup(\sim\phi\rightarrow\ \sim\psi)]$

is a function non-decreasing with respect to $sup(\phi\rightarrow\psi)$ **and** $sup(\sim\phi\rightarrow\ \sim\psi)$**,**

and non-increasing with respect to $sup(\sim\phi\rightarrow\psi)$ **and** $sup(\phi\rightarrow\ \sim\psi)$**.**

n **Notation (for simpllicity)**

$a = sup(\phi\rightarrow\psi)$, ã

$b = sup(\sim\phi\rightarrow\psi)$, ä

$c = sup(\phi\rightarrow\ \sim\psi)$, ä

$d = sup(\sim\phi\rightarrow\ \sim\psi)$. ã

Verification whether *f(x)* satisfies the property (M)

- **In order to verify whether a measure *f(x)* has the property of monotonicity (M) we must check if all of the following conditions are satisfied:**

1. **the increase of *a* must not result in decrease of *f(x)*,** ã
2. **the increase of *b* must not result in increase of *f(x)*,** ä
3. **the increase of *c* must not result in increase of *f(x)*,** ä
4. **the increase of *d* must not result in decrease of *f(x)*.** ã

Rule support has the property of monotonicity (M)?

- **Rule support** is defined as the number of objects in *U* having both property ϕ and ψ.

  sup*($\phi\rightarrow\psi$) = *a

- **Verification:**

1. ***a*** ã **=> *sup*($\phi\rightarrow\psi$)** ã **(non-decreasing)** **P**
2. ***b*** ã **=> *sup*($\phi\rightarrow\psi$) = const** **(non-increasing)** **P**
3. ***c*** ã **=> *sup*($\phi\rightarrow\psi$) = const** **(non-increasing)** **P**
4. ***d*** ã **=> *sup*($\phi\rightarrow\psi$) = const** **(non-decreasing)** **P**

- **Conclusions:**

  Rule support has the property (M)

Confidence has the property of monotonicity (M)? (1)

n **Confidence is defined as:**

***conf*(φ→ψ) = *sup*(φ→ψ)/*sup*(φ) = *a*/(*a*+*c*)**

n **Verification:**

1. ***a*** ã **=> *conf*(φ→ψ) ?**

Let us assume that Δ>0 is a number by which we shall increase a.

Condition 1 will be satisfied if and only if

$$conf\ (\phi \rightarrow \psi) = \frac{a}{a+c} \leq conf\ ^{\prime}(\phi \rightarrow \psi) = \frac{(a+\Delta)}{(a+\Delta)+c}$$

$$\Leftrightarrow\ c\Delta \geq 0$$

Since c, *Δ>0 we have:*

a ã **=> *conf*(φ→ψ)** ã **(non-decreasing)** **P**

Confidence has the property of monotonicity (M)? (2)

n **Verification:**

1. ***a*** ã **=>** ***conf*(**$\phi\to\psi$**)** ã **(non-decreasing)** **P**
2. ***b*** ã **=>** ***conf*(**$\phi\to\psi$**) = const** **(non-increasing)** **P**
3. ***c*** ã **=>** ***conf*(**$\phi\to\psi$**)** ä **(non-increasing)** **P**
4. ***d*** ã **=>** ***conf*(**$\phi\to\psi$**) = const** **(non-decreasing)** **P**

n **Conclusions:**

Confidence has the property (M)

Gain measure has the property of monotonicity (M)?

n **Gain measure is defined as:**

$gain(\phi \rightarrow \psi) = sup(\phi \rightarrow \psi) - \Theta sup(\phi) = a - \Theta(a+c)$

where Θ is a fractional constant between 0 and 1.

n **Verification:**

1. ***a*** ã **=>** ***gain***$(\phi \rightarrow \psi)$ ã **(non-decreasing)** **P**
2. ***b*** ã **=>** ***gain***$(\phi \rightarrow \psi)$ **= const (non-increasing)** **P**
3. ***c*** ã **=>** ***gain***$(\phi \rightarrow \psi)$ ä **(non-increasing)** **P**
4. ***d*** ã **=>** ***gain***$(\phi \rightarrow \psi)$ **= const (non-decreasing)** **P**

n **Conclusions:**

Gain measure has the property (M),

Piatetsky-Shapiro measure also has the property (M).

Property (M) vs. monotonicity in rule support and confidence

- **Many measures (*sup*, *conf*, *gain*, *p-s*, *f* etc.) having the property (M) are also:**
  - **monotone in rule support for fixed confidence and**
  - **monotone in confidence for fixed rule support value**

  B / A-property

- **Hypothesis 1:**

  If a measure has the property of monotonicity (M) (i.e., satisfies the four conditions concerning *a*, *b*, *c*, *d*), then it must also satisfy the two conditions of monotonicity in confidence for a fixed rule support and monotonicity in rule support for fixed value of confidence.

Counterexample for Hypothesis 1 (1)

- Let us consider a **Bayesian confirmation measure *s*** defined as follows:

  $s(\phi \rightarrow \psi) = conf(\phi \rightarrow \psi) - conf(\sim\phi \rightarrow \psi)$.

- It has been proved by Greco et al. that *s* has the property (M)

- Let us verify whether *s* is

1. monotone in rule support for a fixed confidence value
2. monotone in confidence for a fixed value of rule support

Counterexample for Hypothesis 1 (2)

- **We can transform *s* to the following form:**

$$s = \frac{|U|\,conf^{2}(\phi \to \psi) - conf(\phi \to \psi)sup(\psi)}{|U|\,conf(\phi \to \psi) - sup(\phi \to \psi)}$$

- **we assume that |U| and *sup*(ψ) are constants**
- **Verification of the derivatives of**

1. ***s*(*sup*($\phi \to \psi$)) for *conf*($\phi \to \psi$)=const**
2. ***s*(*conf*($\phi \to \psi$)) for *sup*($\phi \to \psi$)=const**

has proved that *s* is monotone in rule support but not in confidence for fixed values of *conf*($\phi \to \psi$) and *sup*($\phi \to \psi$) respectively.

- **Thus, Hypothesis 1 is not true.**

Property (M) vs. monotonicity in rule support and confidence

- **Proving that the four-condition property of monotonicity (M) implies the rule support-confidence monotonicity is not possible as those problems are orthogonal.**
- **Verifying whether a measure has the property of monotonicity (M) requires violation of the conditions: $|U|$, $sup(\psi) = const$, which must be satisfied in order to prove that a measure is monotone in rule support when confidence is held fixed, and in confidence for a fixed value of rule support.**
- **Knowing that a measure has the property (M), we still know nothing about the relationship between that measure and confidence (or rule support).**

Further research plans

- **Developing effective algorithms inducing rules optimal with respect to the confirmation measure *f*.**

- **Developing algorithms looking for pareto optimal border with respect to *a*, *b*, *c* and *d*.**

References

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