Analysis of monotonicity properties of some rule interestingness measures

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Presentation plan

- Introduction
- Basic quantitative characteristics of rules
- Properties of interestingness measures
- Results of the conducted analysis
- Application of the results
- Conclusions and lines of further investigation

Introduction - motivations

The number of rules
induced from datasets is usually quite large

- overwhelming for human comprehension,
- many rules are irrelevant or obvious (low practical value)

rule evaluation – interestingness (attractiveness) measures
(e.g. support, confidence, gain)

- each measure was proposed to capture different characteristics of rules
- the number of proposed measures is very large

need to analyze relationships between different measures

Introduction - motivations

The choice of interestingness measure for a certain application is a difficult problem

- **the users expectations vary,**
- **the number of proposed measures is overwhelming**

properties of interestingness measures, which reflect users′ expectations towards the behavior of measures in particular situations

need to analyze which measures have valuable and desired properties

Introduction – rule induction

- **Patterns in form of rules are induced from a data table**
- ***S*=⟨*U*, *A*⟩ – *data table*, where *U* and *A* are finite, non-empty sets**
  U* – *universe*; *A* – set of *attributes
- ***S*=⟨*U*, *C*, *D*⟩ – *decision table*, where C – set of *condition attributes*,**
  ***D* – set of *decision attributes*, *C*∩*D*=∅**

- ***Decision rule* or *association rule* induced from *S***
  is a *consequence relation*: f ® y ***read as if*** f ***then*** y
  where f **and** y **are condition and conclusion formulas**
  built from attribute-value pairs (*q*,*v*)

- **If the division into independent and dependent attributes is fixed, then rules are regarded as decision rules, otherwise as association rules.**

Introduction – rule induction

Characterization of nationalities

U	*Height*	*Hair*	*Eyes*	*Nationality*	Support
1	*tall*	*blond*	*blue*	*Swede*	270
2	*medium*	*dark*	*hazel*	*German*	90
3	*medium*	*blond*	*blue*	*Swede*	90
4	*tall*	*blond*	*blue*	*German*	360
5	*short*	*red*	*blue*	*German*	45
6	*medium*	*dark*	*hazel*	*Swede*	45

C ***D***

- **E.g. decision rules induced from „characterization of nationalities":**
  1) **If (*Height=tall*), then (*Nationality=Swede*)**
  2) **If (*Height=medium*) & (*Hair=dark*), then (*Nationality=German*)**

Introduction – interestingness measures

- To measure the relevance and utility of rules, quantitative measures called **attractiveness** or **interestingness measures**, have been proposed

  (e.g. support, confidence, lift, gain, conviction, Piatetsky-Shapiro,…)

- Unfortunately, there is no evidence which measure(s) is (are) the best

- Notation:
  - $sup(\mathbf{0})$ is the number of all objects from *U*, having property °

    e.g. $sup(\phi)$, $sup(\psi)$

Basic quantitative characteristics of rules

- **Support of rule $\phi \rightarrow \psi$ in *S*:**

$$sup(\phi \rightarrow \psi) = sup(\phi \wedge \psi)$$

- **Anti-support of rule $\phi \rightarrow \psi$ in *S*:**

$$anti\text{-}sup(\phi \rightarrow \psi) = sup(\phi \wedge \neg\psi)$$

Basic quantitative characteristics of rules

- **Rule Interest Function** **(Piatetsky-Shapiro 1991)**

$$RI(\phi \rightarrow \psi) = sup(\phi \rightarrow \psi) - \frac{sup(\psi)sup(\phi)}{|U|}$$

- **Gain measure** **(Fukuda et al. 1996)**

$$gain(\phi \rightarrow \psi) = sup(\phi \rightarrow \psi) - \Theta sup(\phi)$$

where Θ is a fraction constant between 0 and 1

- **Dependency Factor** **(Pawlak 2002)**

$$\eta(\phi \rightarrow \psi) = \frac{\frac{sup(\phi \rightarrow \psi)}{sup(\phi)} - \frac{sup(\psi)}{|U|}}{\frac{sup(\phi \rightarrow \psi)}{sup(\phi)} + \frac{sup(\psi)}{|U|}}$$

Property M

- **Property M** **(Greco, Pawlak, Słowiński 2004)**
- **An interestingness measure *I*(*a*, *b*, *c*, *d*) has the property M if it is a function non-decreasing with respect to *a* and *d* and non-increasing with respect to *b* and *c***

where:

a=$sup(\phi \rightarrow \psi)$

the number of objects in *U* for which ϕ and ψ hold together

b=$sup(\neg\phi \rightarrow \psi)$,

c=$sup\ (\phi \rightarrow \neg\psi)$,

d=$sup(\neg\phi \rightarrow \neg\psi)$

Interpretation of the property M

- **E.g. (Hempel) consider rule $\phi \rightarrow \psi$:**

  if x is a raven then x is black

- **ϕ is the property *to be a raven*, ψ is the property *to be black***
  - ***a* – the number of objects in *U* which are black ravens**

    //the more black ravens we observe, the more credible becomes the rule
  - ***b* – the no. of objects in *U* which are black non-ravens**
  - ***c* – the no. of objects in *U* which are non-black ravens**
  - ***d* – the no. of objects in *U* which are non-black non-ravens**

Property of hypothesis symmetry

- **Property of hypothesis symmetry (HS)** (Carnap `62, Eells, Fitelson ′02)
- **An interestingness measure $c(\phi \rightarrow \psi)$ has the property HS if**

$$c(\phi \rightarrow \psi) = -c(\phi \rightarrow \neg\psi)$$

- **Interpretation:** the impact of ϕ on ψ should be of the same strength, but of the opposite sign as the impact of ϕ on $\neg\psi$

- **Example:** We draw cards from a standard deck.
  Let ϕ: the drawn card is *the seven of spades* and ψ: *the card is black.*
  ϕ is conclusive for ψ and negatively conclusive for $\neg\psi$

Aim and scope of the conducted analysis

Interestingness measure	Analyzed properties	
	Property M	Property HS
Rule Interest Function		
Gain		
Dependency Factor		

Results of the analysis with respect to property M

- ***Theorem:***

  Rule Interest Function has the property M

- ***Theorem:***

  Gain measure has the property M

- ***Theorem:***

  Dependency factor does not have the property M

Results of the analysis with respect to property M

- ***Theorem:*** **Rule Interest Function has the property M**

- ***Proof outline:***

Notation *a*=$sup(\phi\rightarrow\psi)$**, *b*=**$sup(\neg\phi\rightarrow\psi)$**, *c*=**$sup(\phi\rightarrow\neg\psi)$**, *d*=**$sup(\neg\phi\rightarrow\neg\psi)$

$$RI(\phi \rightarrow \psi) = \frac{ad - bc}{a+b+c+d}$$

To prove the monotonicity of *RI* wrt *a* we have to show that if *a* increases by Δ>0, then *RI* does not decrease i.e.

$$\frac{(a+\Delta)d - bc}{(a+\Delta)+b+c+d} - \frac{ad - bc}{a+b+c+d} \geq 0$$

Analogous steps wrt *b*, *c* and *d*.

Results of the analysis with respect to property HS

- ***Theorem:***

  Rule Interest Function has the property HS

- ***Theorem:***

  Gain measure has the property HS iff $\Theta=1/2$

- ***Theorem:***

  Dependency factor does not have the property M

Application of the results

Support - Anti-support Pareto border

- ***Theorem:***

  For a set of rules with the same conclusion,

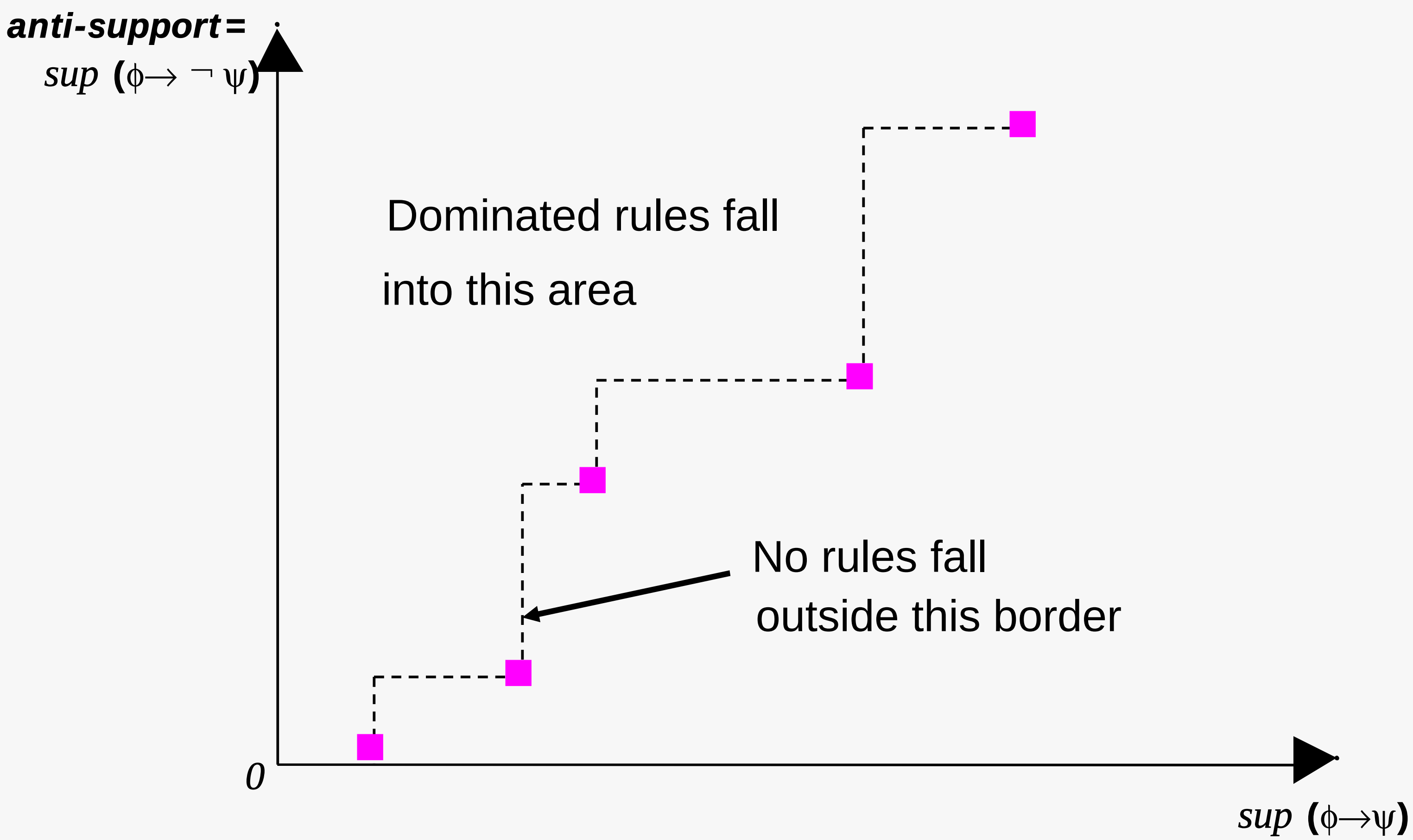
  due to (anti) monotonic dependencies between
  measures of support and anti-support on one hand
  and any interestingness measure with property M on the other hand

  the best rules according to any measure with the property M must reside on the support - anti-support Pareto optimal border

- The support – anti-support Pareto border is a set of non-dominated rules with respect to those measures,
  i.e. the set of rules for which there is no other rule
  with greater support and smaller anti-support

Brzezińska I., Greco S., Słowiński R.: Mining Pareto-Optimal Rules with Respect to Support and Confirmation or Support and Anti-Support (EAAI Journal, 2007)

Support - Anti-support Pareto border



The best rules according to any measure with the property M must reside on the support - anti-support Pareto border

Application of the result

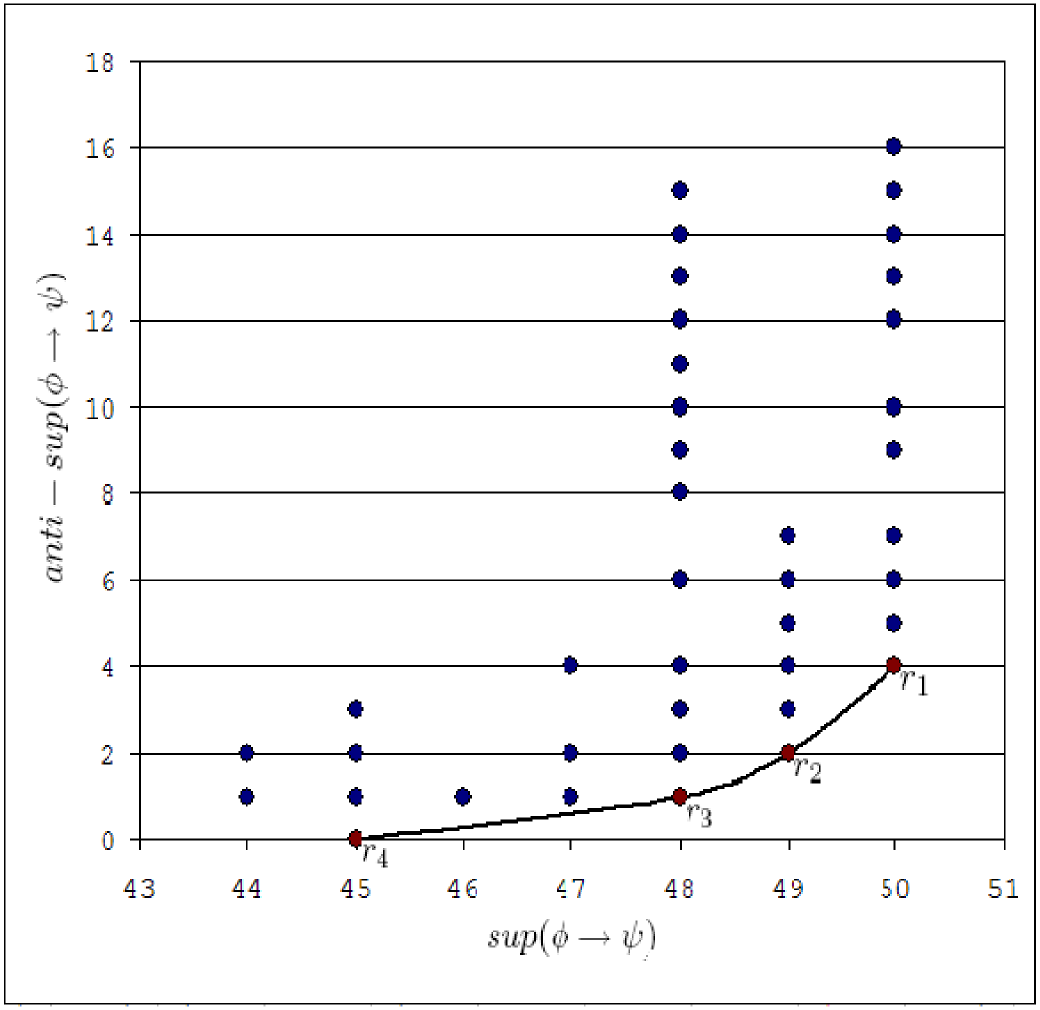
- **Since *RI* and *Gain* satisfy the property M we can conclude that rules optimal with respect to them will be found in the set of rules non-dominated according to support and anti-support.**
  (considering rules with the same conclusion)

- **Experiments illustrating the result:**

  Dataset: *busses*, containing info. about technical state of buses

  Set of 85 rules with the same conclusion

Application of the result



Legend

dominated rules

Pareto-optimal border

r_1 represents rules optimal with respect to gain for $\Theta = 0.33$
and with respect to gain for $\Theta = 0.66$

r_2 represents rules optimal with respect to gain for $\Theta = 0.5$

r_3 represents rules optimal with respect to gain for $\Theta = 0.66$
and with respect to RI

r_4 represents rules optimal with respect to dependency factor

Conclusions

Conclusions

Interestingness measure	Properties	
	Property M	Property HS
Rule Interest Function	YES	YES
Gain	YES	iff $Q = 1/2$
Dependency Factor	NO	NO

Conclusions

- **Properties explain how the measures behave in certain situations and thus, group them helping the user choose the measure relevant for his expectations**

  e.g. we know that *RI* is monotonically dependent on
  the number of objects supporting the rule
  or the number of objects supporting neither premise nor conclusion

Conclusions

- **Possession of property M implies potential efficiency improvement:**
  - **we can concentrate on mining only the support – anti-support Pareto set instead of conducting rule evaluation separately wrt to *RI*, Gain, or any other measure with property M**
  - **rules optimal wrt to *RI*, Gain or any other measure with property M can be mined from the support – anti-support Pareto set instead of searching the set of all rules**
  - **due to relationship between anti-support and any measure with property M the rule order wrt anti-support is the same for any other measure with M**

Lines of further investigation

- Analysis of properties M and Hypothesis Symmetry with respect to other interestingness measures
- Analysis of other properties, eg. property of confirmation

Thank you!

Knowledge representation semantics – computational experiment*

- Decision rules were generated from lower approximations of preference-ordered decision classes defined according to Variable-consistency Dominance-based Rough Set Approach (VC-DRSA) (Greco, Matarazzo, Slowinski, Stefanowski 2001)

File	objects	atr+crit	classes	rules (alg)	consistency	length	coverage
Buses	76	0+8	3	266 (all)	$\geq$ 0.75	$\leq$ 3	$\geq$ 0.9
Nativity	342	0+33	2	64 (mc)	$\geq$ 0.75	no limit	no limit
Urology	500	18+9	3	186 (mc)	$\geq$ 0.96	no limit	no limit

Rule induction algorithms: „all" = all rules algorithm (DOMAPRIORI)

„mc" = minimal-cover algorithm (DOMLEM)

*by Iza Brzezińska