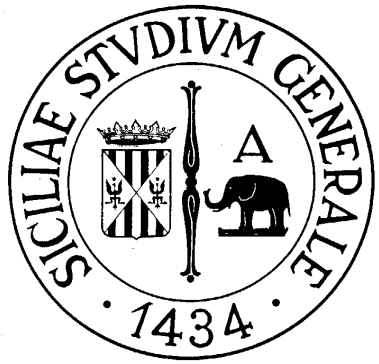

Mining Association Rules with respect to Support and Anti-support - Experimental results

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Presentation plan

- Introduction
- Basic quantitative characteristics of rules

- Rule evaluation with respect to support and confidence
- Confirmation Perspective on the support-confidence evaluations

- Rule evaluation with respect to support and anti-support
- Confirmation Perspective on the support-confidence evaluations

Introduction – rule induction

- Patterns in form of rules are induced from a data table
- $S=\langle U, A\rangle$ – *data table*, where U and A are finite, non-empty sets
  U – *universe*; A – set of *attributes*
- $S=\langle U, C, D\rangle$ – *decision table*, where C – set of *condition attributes*,
  D – set of *decision attributes*, $C\cap D=\varnothing$

- *Decision rule* or *association rule* induced from S
  is a *consequence relation*: $\phi\rightarrow\psi$ *read as* ***if*** ϕ ***then*** ψ
  where ϕ and ψ are condition and conclusion formulas
  built from attribute-value pairs (q,v)

- **In this work we consider association rules with a fixed conclusion**

Introduction – attractiveness measures

- To measure the relevance and utility of rules, quantitative measures called attractiveness or interestingness measures, have been proposed

  (e.g. support, confidence, lift, gain, conviction, Piatetsky-Shapiro,…)

- Unfortunately, there is no evidence which measure(s) is (are) the best

- Notation:
  - $sup(\circ)$ is the number of all objects from *U*, having property $\circ$

    e.g. $sup(\phi)$, $sup(\psi)$

Basic quantitative characteristics of rules

- Basic quantitative characteristics of rules
  - *Support* of rule $\phi \rightarrow \psi$ in *S*:

$$sup(\phi \rightarrow \psi) = sup(\phi \wedge \psi)$$

  - *Confidence* (called also *certainty factor*) of rule $\phi \rightarrow \psi$ in *S*:

$$conf(\phi \rightarrow \psi) = \frac{sup(\phi \rightarrow \psi)}{sup(\phi)}$$

  - *Anti-support* of rule $\phi \rightarrow \psi$ in *S*:

$$anti\text{-}sup(\phi \rightarrow \psi) = sup(\phi \wedge \neg\psi)$$

Bayesian confirmation measures

- An attractiveness measure *c* has the property of confirmation if is satisfies the following condition:

$$c(\phi,\psi)\begin{cases} > 0 \;\; if \;\; Pr(\psi|\phi) > Pr(\psi) \\ = 0 \;\; if \;\; Pr(\psi|\phi) = Pr(\psi) \\ < 0 \;\; if \;\; Pr(\psi|\phi) < Pr(\psi) \end{cases}$$

- Measures of confirmation quantify the strength of confirmation that premise ϕ gives to conclusion ψ
- „ψ is verified more often, when ϕ is verified, rather than when ϕ is not verified”

Bayesian confirmation property - interpretation

- $c(\phi, \psi)>0$ means that property ψ is satisfied more frequently when ϕ is satisfied, rather than generically in S (where the frequency is $Pr(\psi)$)

- $c(\phi, \psi)=0$ means that property ψ is satisfied with the same frequency whether ϕ is satisfied or not

- $c(\phi, \psi)<0$ means that property ψ is satisfied less frequently when ϕ is satisfied, rather than generically

Confirmation measure *f*

- Measures of confirmation quantify the strength of confirmation that premise ϕ gives to conclusion ψ

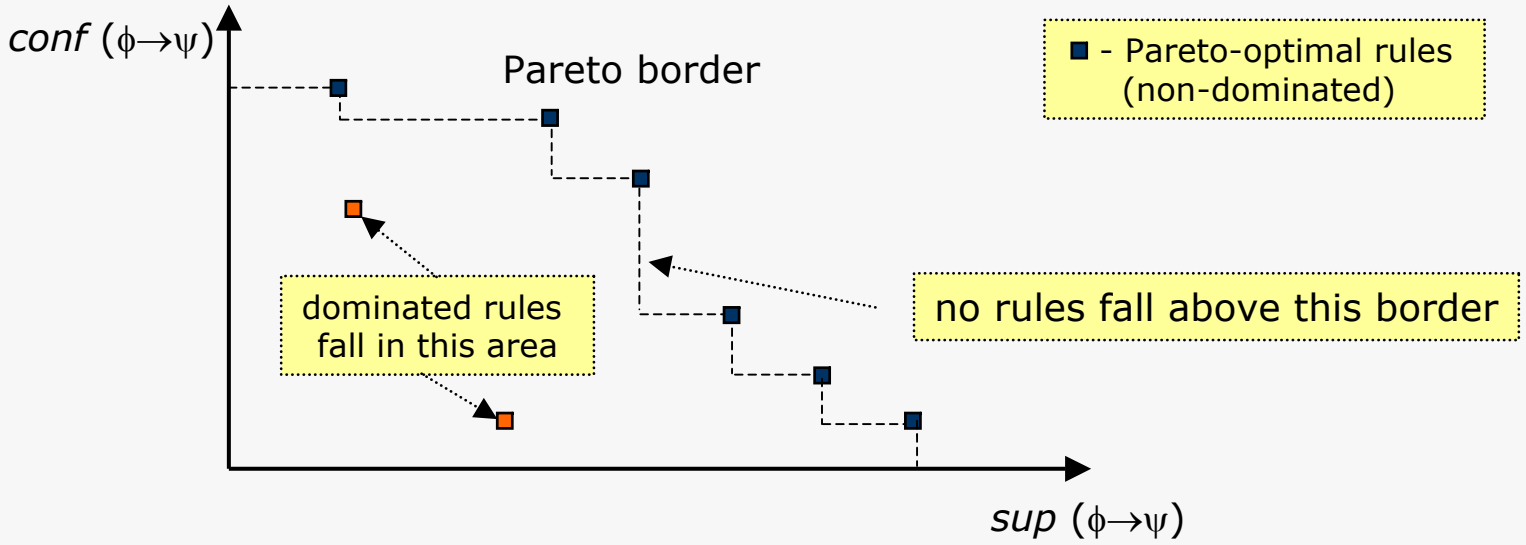
- Confirmation measure *f* (Good 1984, Heckerman 1988, Pearl 1988, Fitelson 2001)

$$f(\phi \rightarrow \psi) = \frac{conf(\psi \rightarrow \phi) - conf(\neg\psi \rightarrow \phi)}{conf(\psi \rightarrow \phi) + conf(\neg\psi \rightarrow \phi)}$$

Support-confidence evaluation

Support-confidence Pareto border

- Support-confidence Pareto border is the set of non-dominated, Pareto-optimal rules with respect to both rule support and confidence



- Mining the border identifies rules optimal with respect to measures such as: *lift*, *gain*, *conviction*, *Piatetsky-Shapiro*,…

Confirmation perspective on support-confidence space

- Is there a curve separating rules with negative value of any measure with the confirmation property in the support-confidence space?

- *Theorem:*
  Due to monotonicity of confidence in *c*,

$$c(\phi \rightarrow \psi) \geq 0 \quad \Leftrightarrow \quad conf\,(\phi \rightarrow \psi) \geq sup(\psi)/|U|$$

  Thus, rules lying below a constant:

$$sup(\psi)/|U|$$

  have a negative value of any confirmation measure.
  For those rules, the premise only disconfirms the conclusion!

- $sup(\psi)/|U|$ is a constant expressing what percentage of the whole data set is taken by considered class ψ

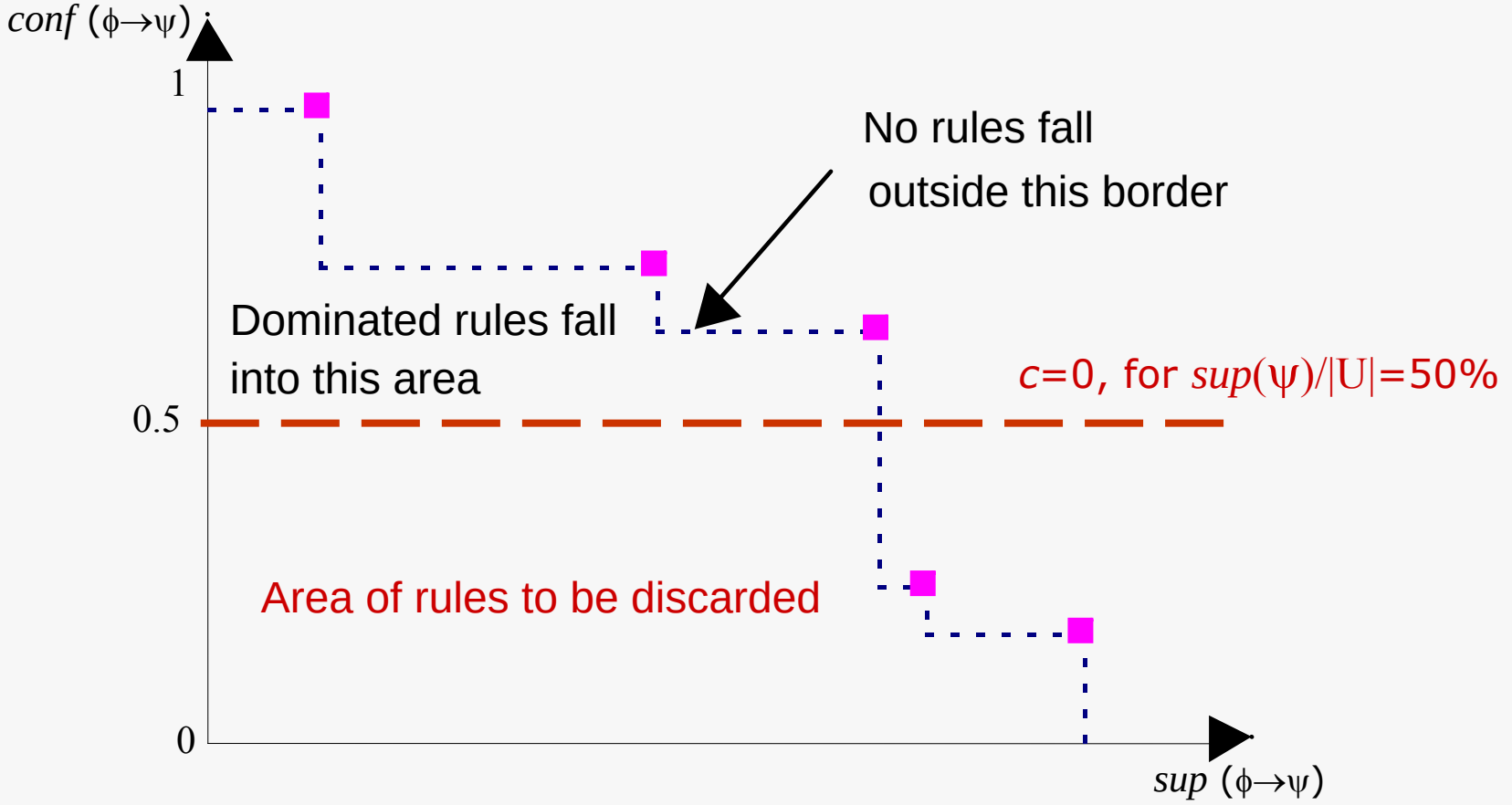
Confirmation perspective on support-confidence space

- A more general condition $c(\phi, \psi) \geq k$, $k \geq 0$ for some specific confirmation measure, $f(\phi, \psi)$, was also investigated.

- *Theorem:*

$$f(\phi \rightarrow \psi) \geq k \Leftrightarrow conf(\phi \rightarrow \psi) \geq sup(\psi)(k+1) \,/\, [|U| - k(|U| - 2sup(\psi))]$$

Confirmation perspective on support-confidence space

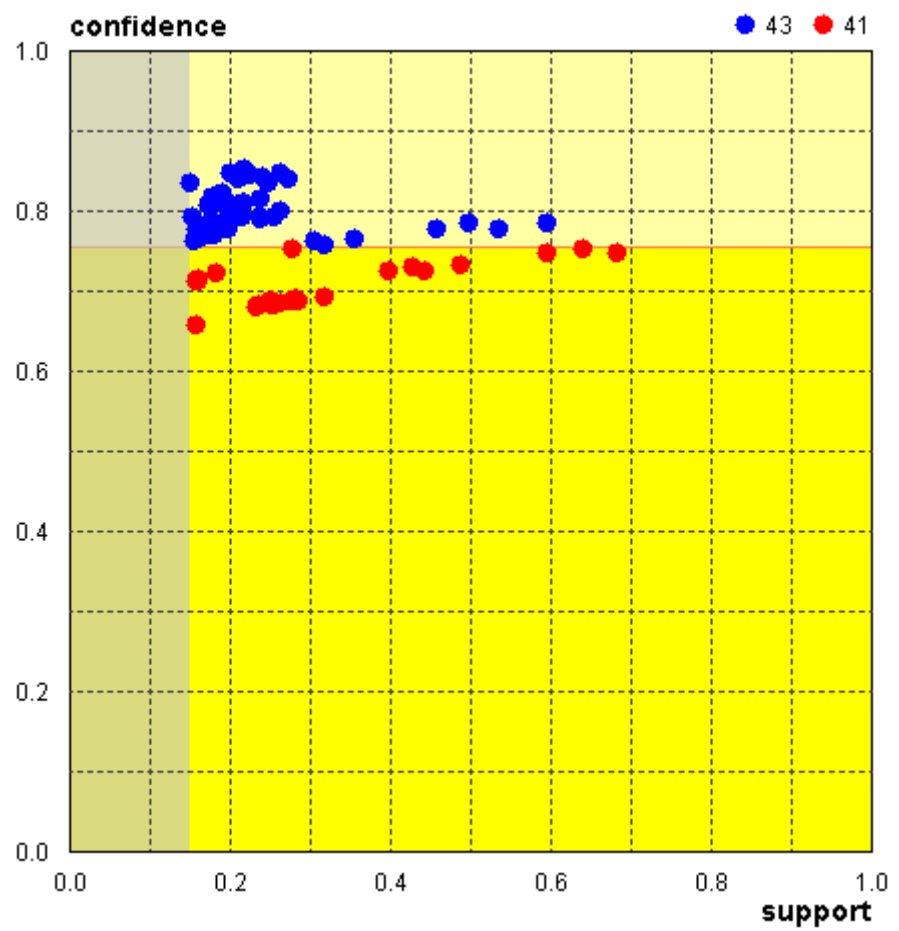


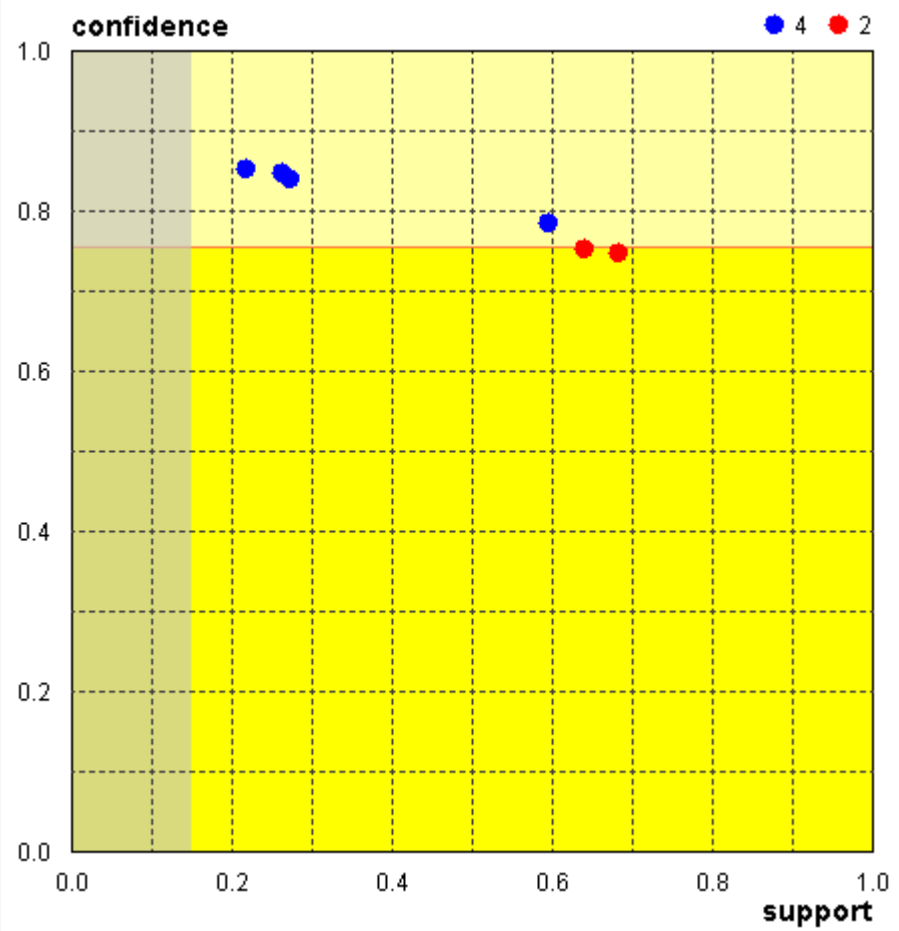
For rules lying below the curve for which $c=0$
the premise only disconfirms the conclusion

General info about the dataset

- Dataset ***adult***, created in ’96 by B. Becker/R. Kohavi from census database
- **32 561 instances**
- 9 nominal attributes
  - workclass: Private, Local-gov, etc.;
  - education: Bachelors, Some-college, etc.;
  - marital-status: Married, Divorced, Never-married, et.;
  - occupation: Tech-support, Craft-repair, etc.;
  - relationship: Wife, Own-child, Husband, etc.;
  - race: White, Asian-Pac-Islander, etc.;
  - sex: Female, Male;
  - native-country: United-States, Cambodia, England, etc.;
  - salary: >50K, <=50K
- **throughout the experiment, *sup*($\phi \rightarrow \psi$) is denoted as „support” and expressed as a relative rule support [0-1]**

Support-confidence (workclass=Private)





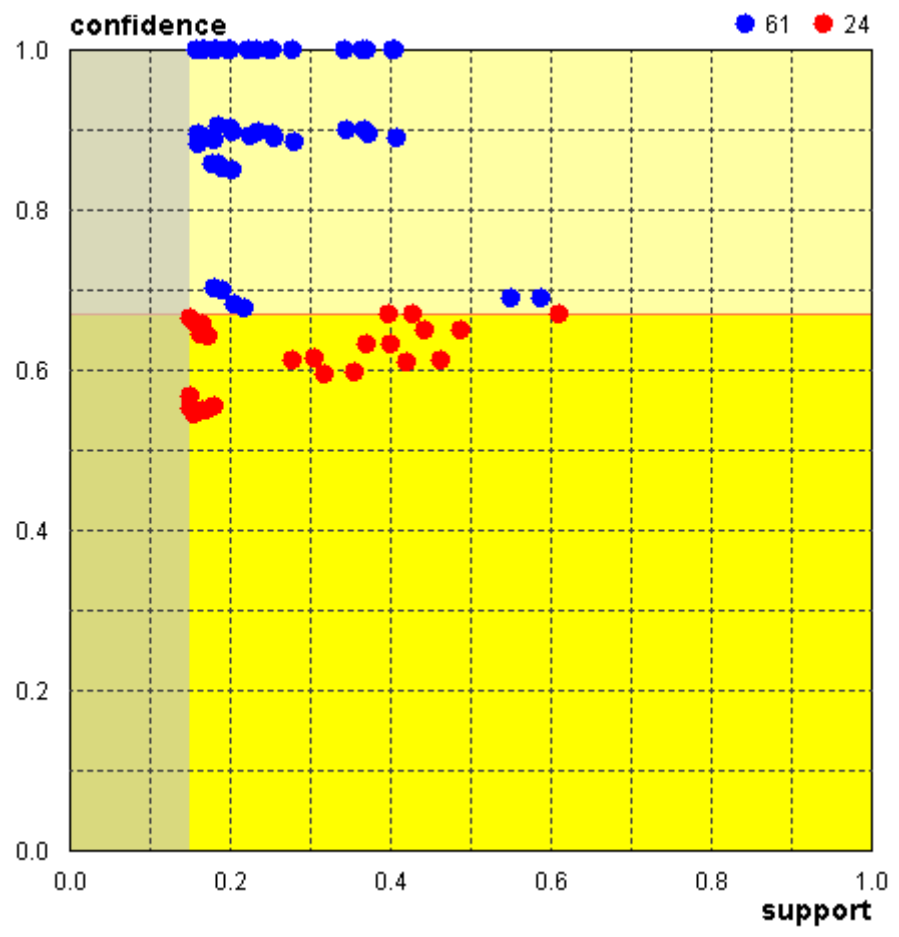
- ● indicates rules with negative confirmation
- the class constitutes over 70% of the whole dataset
- rules with high confidence can be disconfirming
- even some rules from the Pareto border need to be discarded

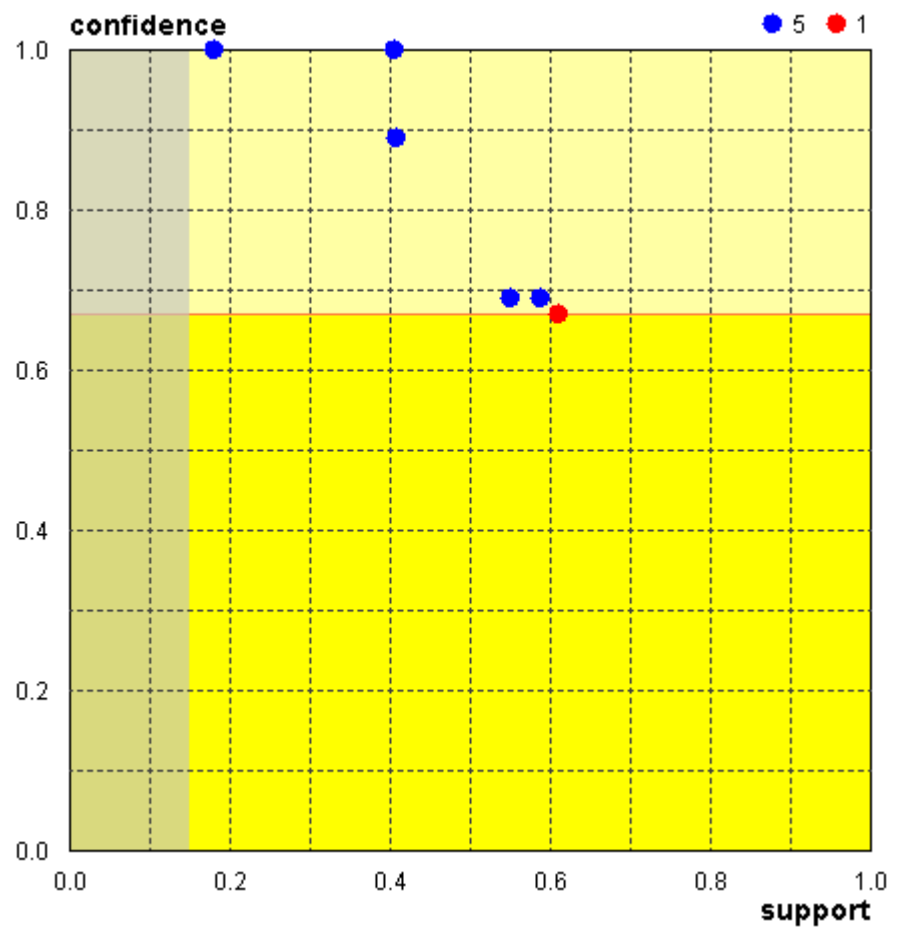
Few rules describing class: workclass=Private

premise	conclusion	supp	conf	s	f	a-supp
education is HS-grad **and race is** White **and native-country is** United-States	**workclass is** Private	0.20	0.79	0.05	0.10	0.06
education is HS-grad **and sex is** Male **and native-country is** United-States	**workclass is** Private	0.16	0.76	0.01	0.02	0.05
education is HS-grad **and native-country is** United-States	**workclass is** Private	0.24	0.79	0.05	0.10	0.06
education is Some-college **and native-country is** United-States	**workclass is** Private	0.16	0.77	0.02	0.03	0.05
marital-status is Married-civ-spouse **and relationship is** Husband **and race is** White	**workclass is** Private	0.25	0.69	-0.11	-0.17	0.12
relationship is Husband **and race is** White **and sex is** Male	**workclass is** Private	0.25	0.69	-0.11	-0.17	0.12
relationship is Husband **and sex is** Male **and native-country is** United-States	**workclass is** Private	0.25	0.68	-0.12	-0.18	0.12
race is White **and sex is** Male	**workclass is** Private	0.43	0.73	-0.06	-0.06	0.16
sex is Male **and native-country is** United-States	**workclass is** Private	0.44	0.72	-0.07	-0.07	0.17

- the table contains few examples of rules with the conclusion workclass=Private

Support-confidence (sex=Male)





- indicates rules with negative confirmation

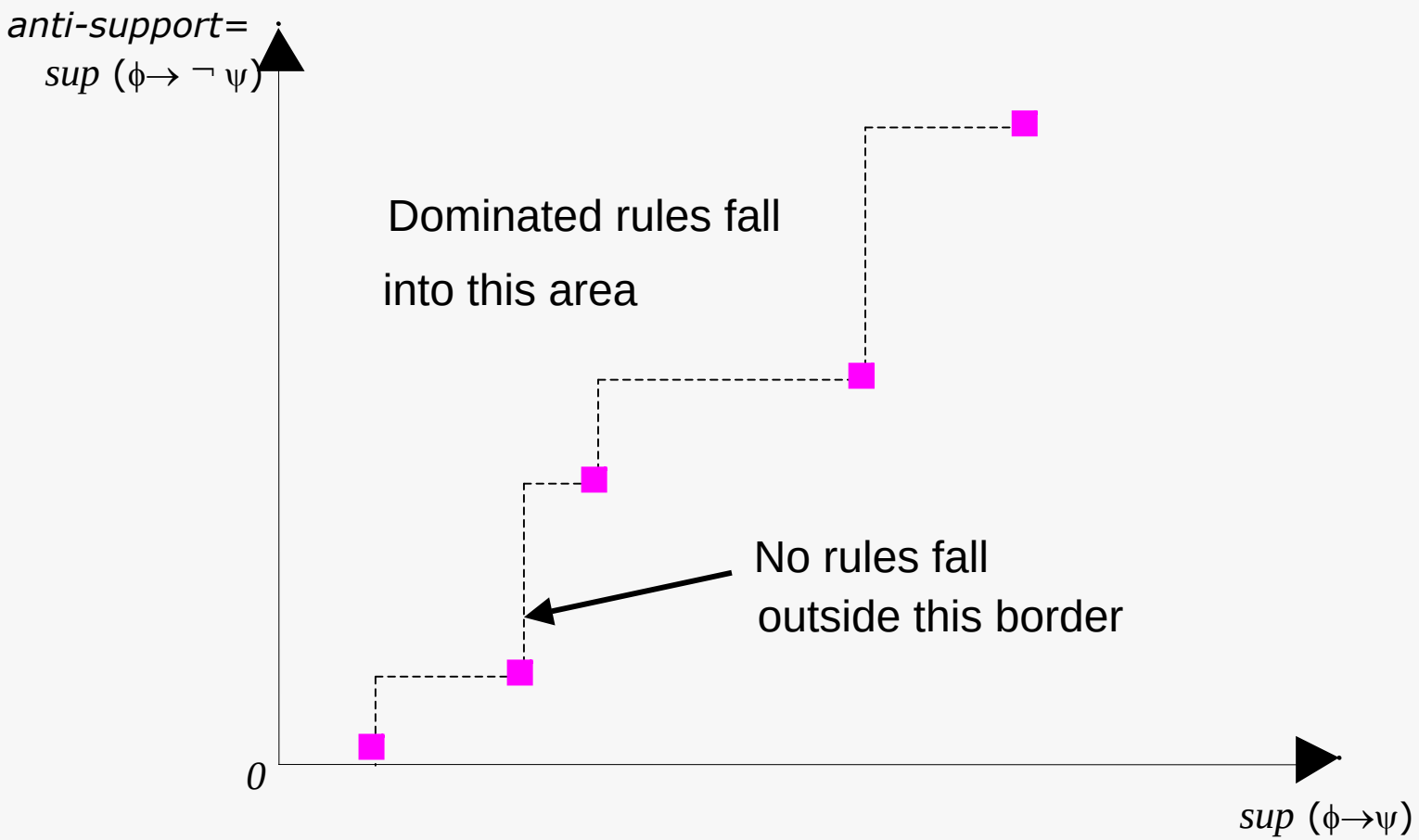
Support-confidence - summary

Considered conclusion	No. of all rules	No. of rules with non–positive confirm.	Reduction percentage
workclass='Private'	84	41	49%
sex=Male	85	24	28%
income<=50kUSD	87	43	49%

Considered conclusion	No. of all rules on Pareto border	No. of rules with non–positive confirm.	Reduction percentage
workclass='Private'	6	2	33%
sex=Male	6	1	17%
income<=50 kUSD	5	1	20%

Support - anti-support evaluation

Support - anti-support Pareto border



Theorem: The best rules according to any measure with the property M must reside on the support - anti-support Pareto border

Brzezińska I., Greco S., Słowiński R.: Mining Pareto-Optimal Rules with Respect to Support and Confirmation or Support and Anti-Support (EAAI Journal, 2007)

Confirmation perspective on support - anti-support space

- Is there a curve separating rules with negative value of any confirmation measure in the support-anti-support space?

- *Theorem:*

  Due to anti-monotonicity of anti-support in *c*,

$$c(\phi \rightarrow \psi) \geq 0 \quad \Leftrightarrow \quad anti\text{-}sup(\phi \rightarrow \psi) \leq sup(\phi \rightarrow \psi)[|U|/sup(\psi)\text{-}1]$$

  Thus, rules lying above a linear function:

$$sup(\phi \rightarrow \psi)[|U|/sup(\psi)\text{-}1]$$

  have a negative value of any confirmation measure.

  For those rules, the premise only disconfirms the conclusion!

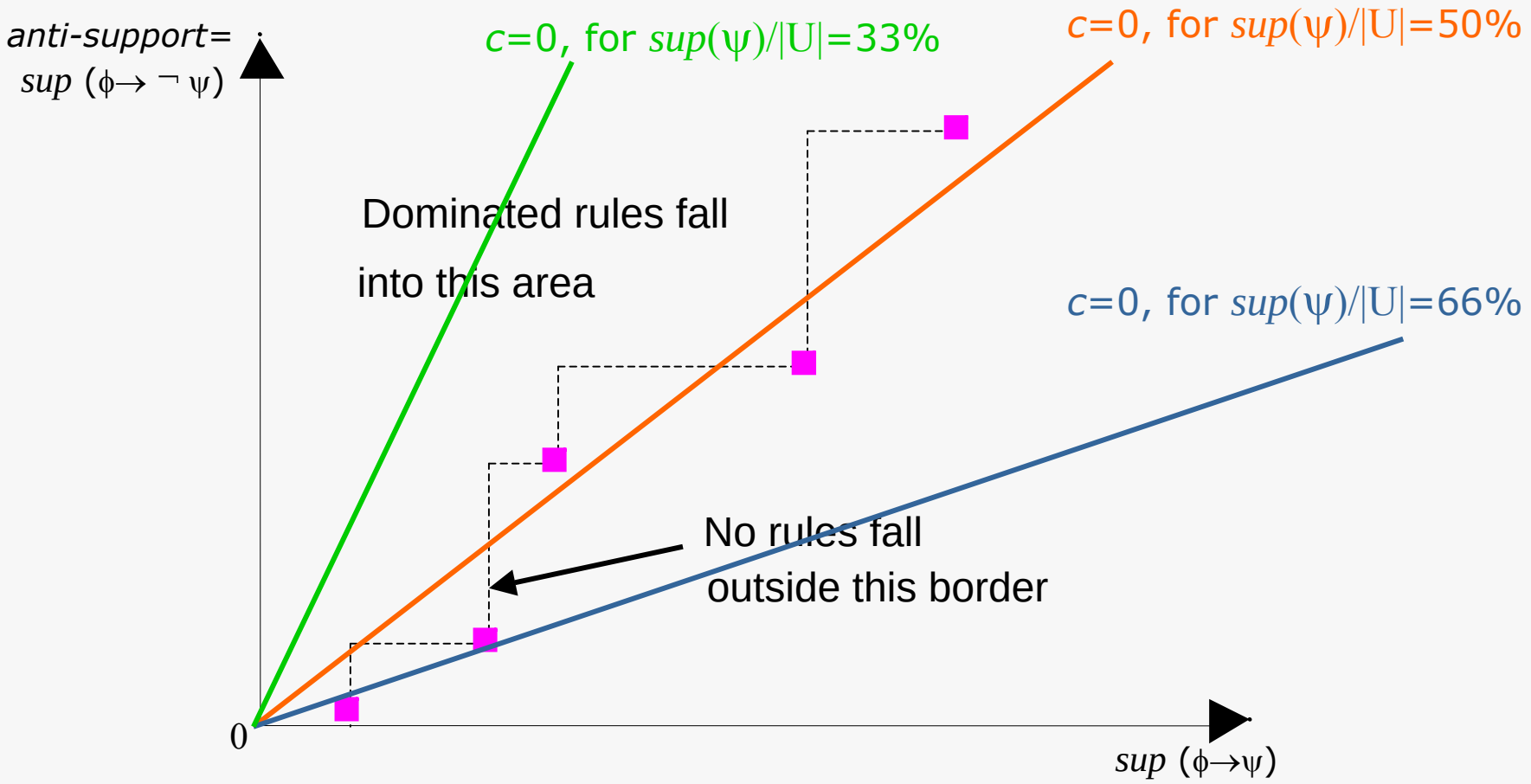
Confirmation perspective on support - anti-support space

- A more general condition $c(\phi, \psi) \geq k$, $k \geq 0$ for some specific confirmation measure, $f(\phi, \psi)$, was also investigated.

- *Theorem:*

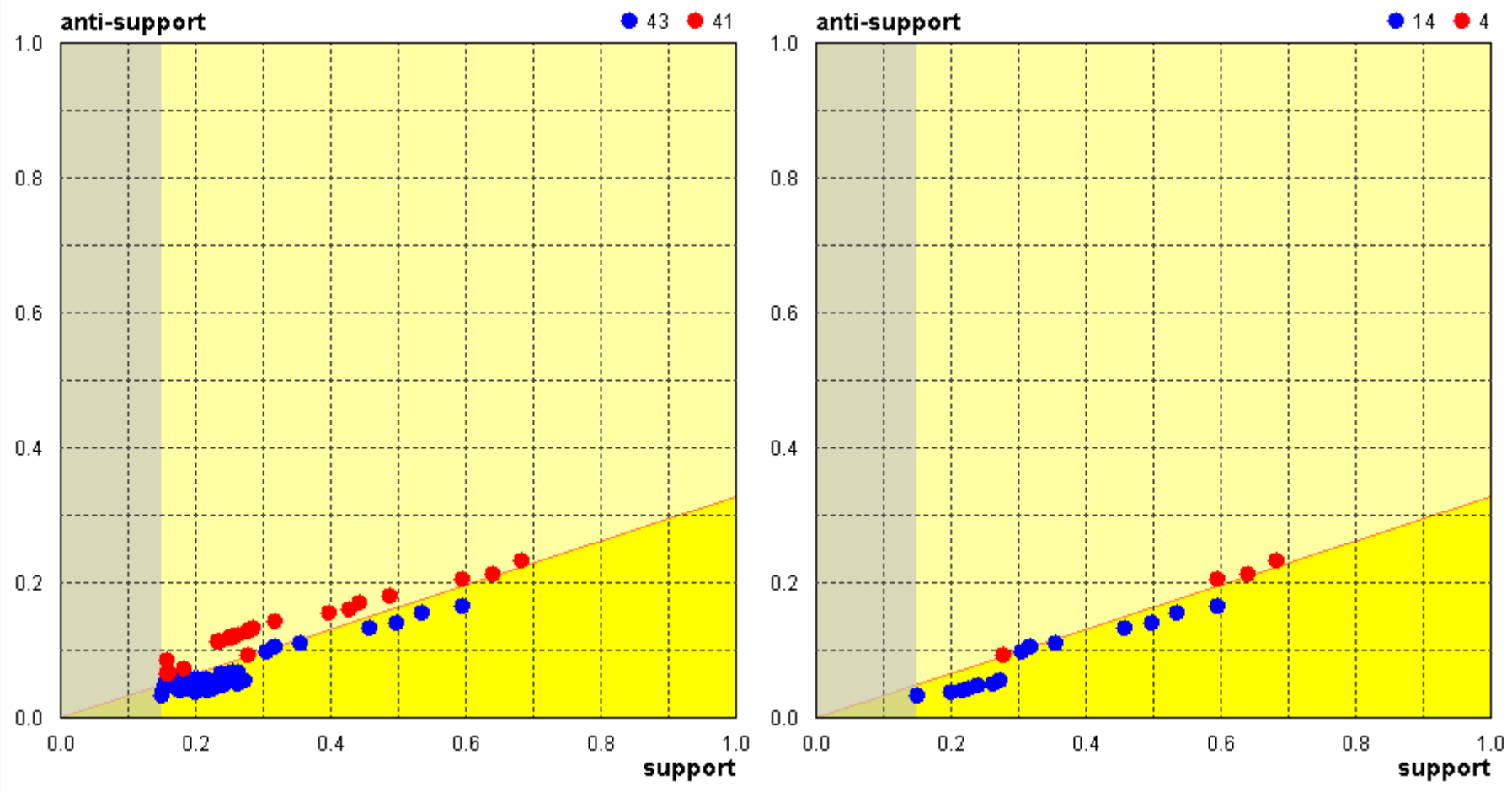
$$f(\phi \rightarrow \psi) \geq k \Leftrightarrow \textit{anti-sup}(\phi \rightarrow \psi) \leq \textit{sup}(\phi \rightarrow \psi)[|U|/\textit{sup}(\psi)\text{-}1]$$

Confirmation perspective on support - anti-support border



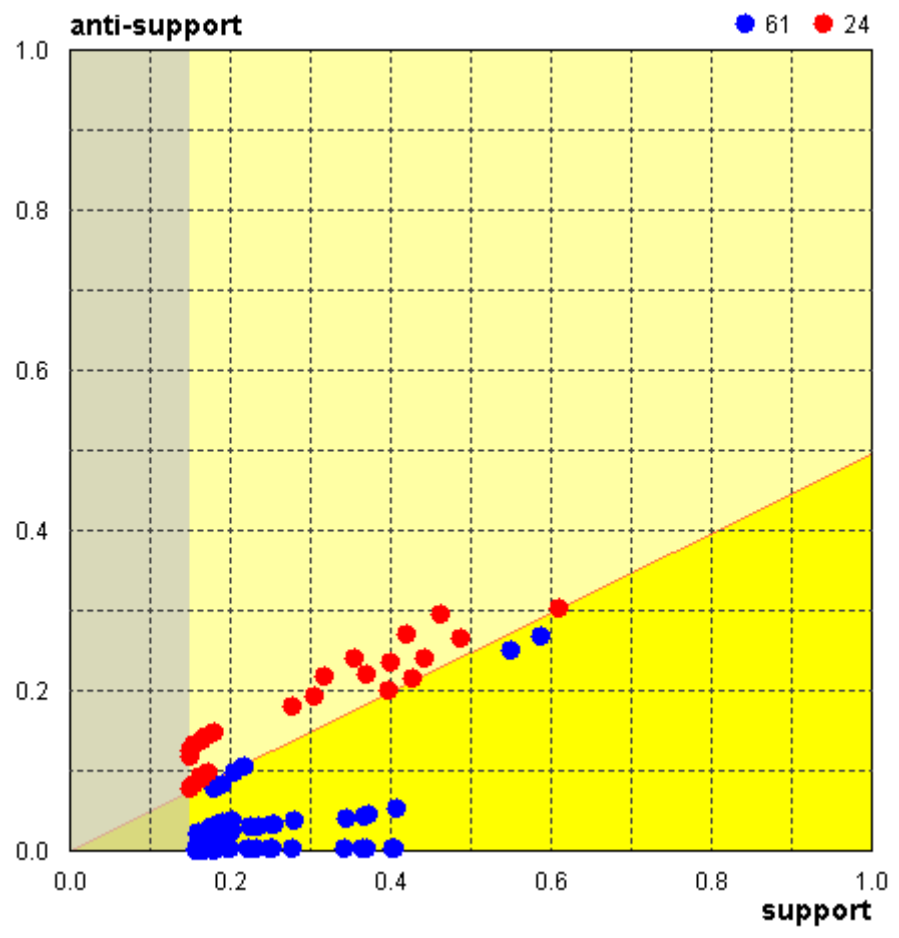
For rules lying above the curve for which $c=0$
the premise only disconfirms the conclusion

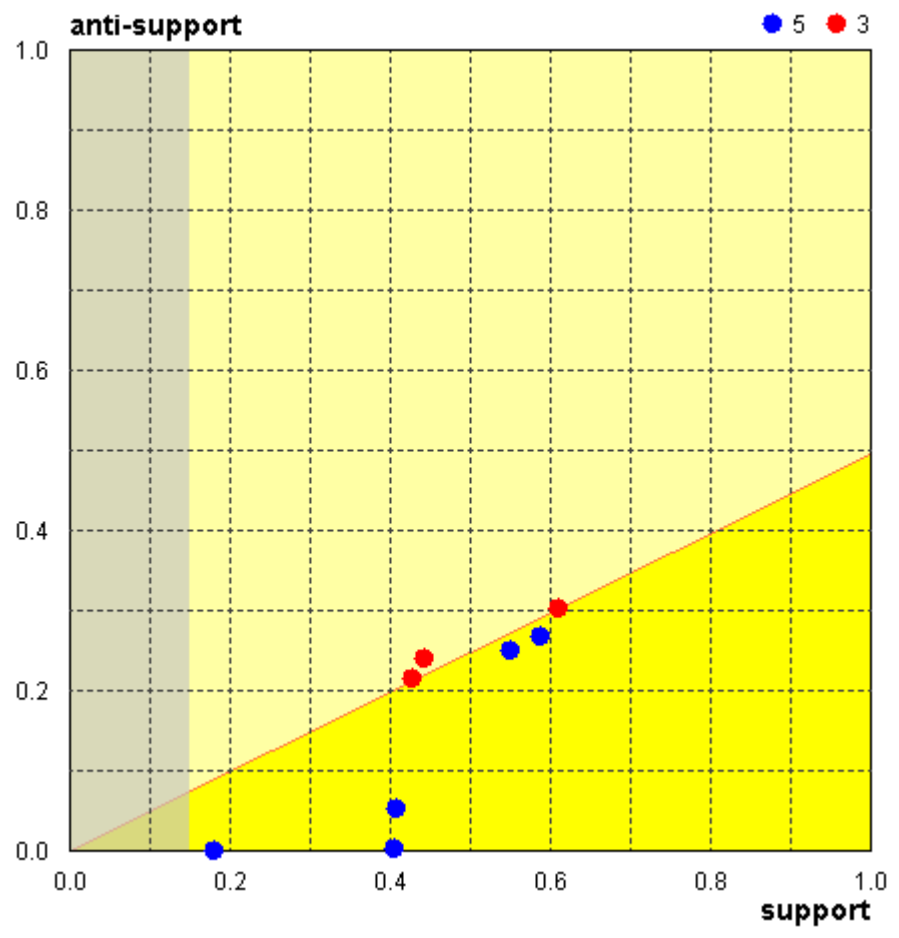
Support - anti-support (workclass=Private)



- ● indicates rules with negative confirmation
- even some rules from the Pareto border need to be discarded

Support-anti-support (sex=Male)





- ● indicates rules with negative confirmation

Support – anti-support - summary

Considered conclusion	No. of all rules	No. of rules with non–positive confirm.	Reduction percentage
workclass='Private'	84	41	49%
sex=Male	85	24	28%
income<=50kUSD	87	43	49%

Considered conclusion	No. of all rules on Pareto border	No. of rules with non–positive confirm.	Reduction percentage
workclass='Private'	18	4	22%
sex=Male	8	3	38%
income<=50 kUSD	15	4	27%

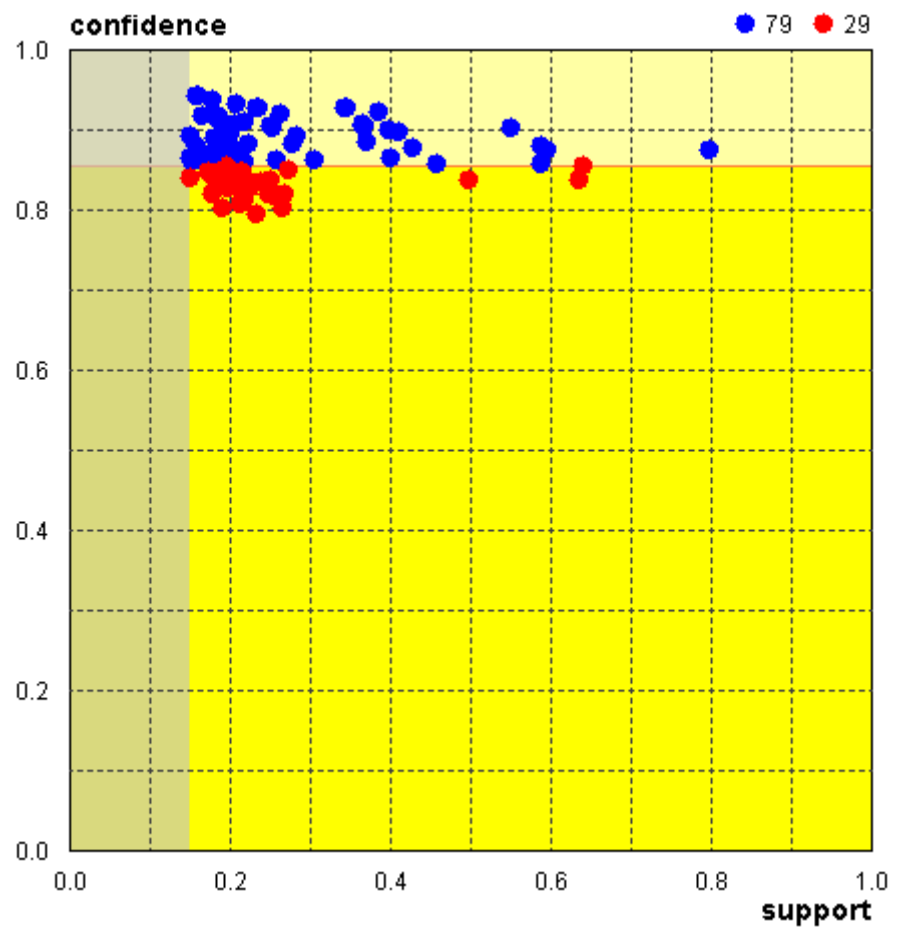
Summary

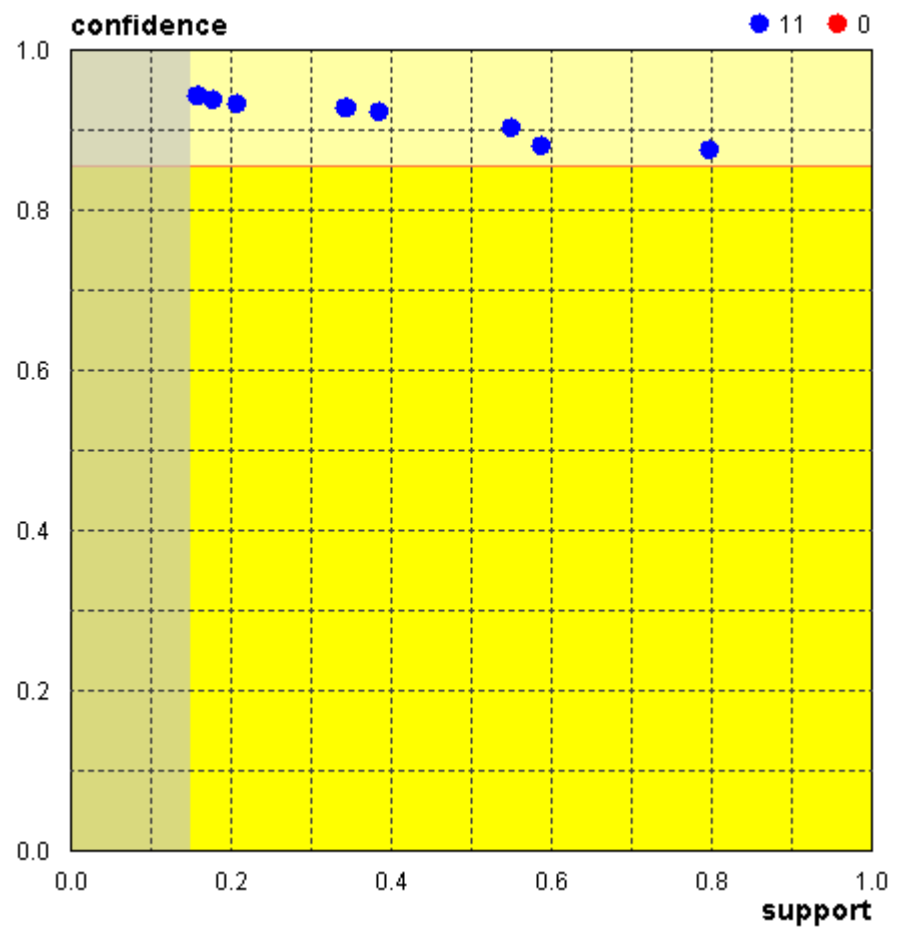
Summary

- The support-confidence and support – anti-support Pareto-optimal borders are characterized by valuable features.
- Inspired by the strength of the semantics of the family of confirmation measures, we have shown that it is reasonable to eliminate rules with non-positive or small values of confirmation.
- We have shown analytically that simple linear functions imposed on the two-dimensional spaces limit the set of induced rules to rules for which the premise confirms the conclusion.
- Experimental results show how big the reduction of a rule set can be.

Thank you!

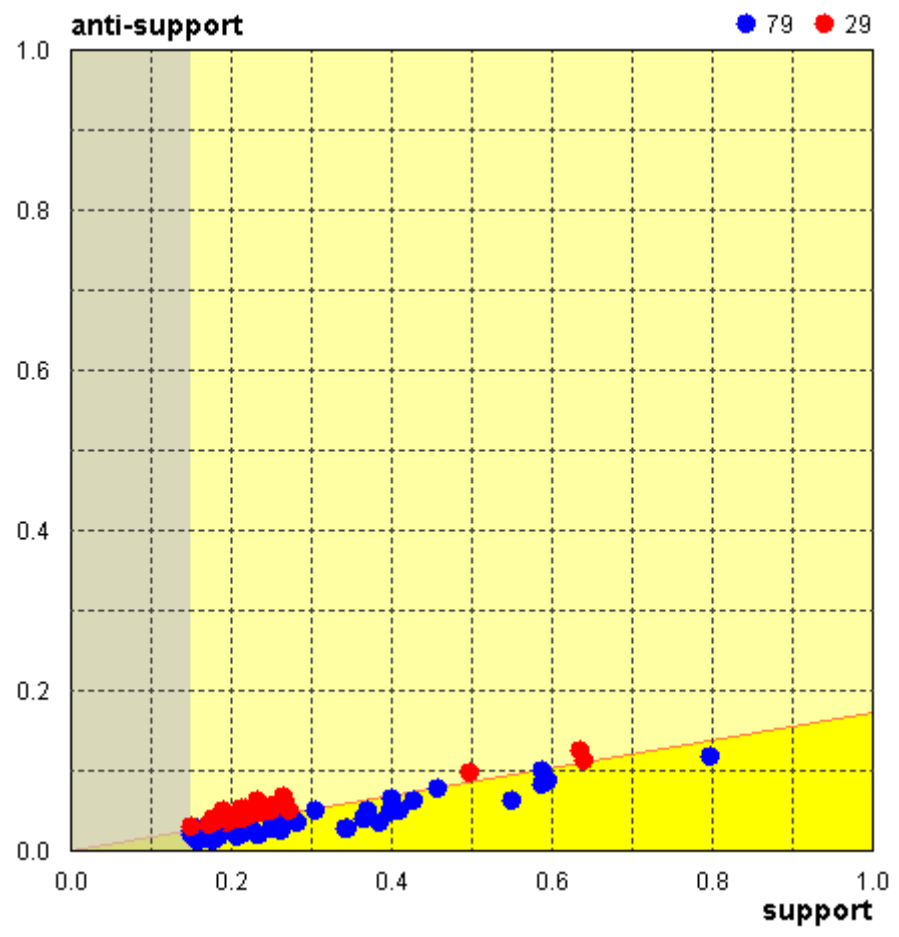
Support-confidence (race=White)

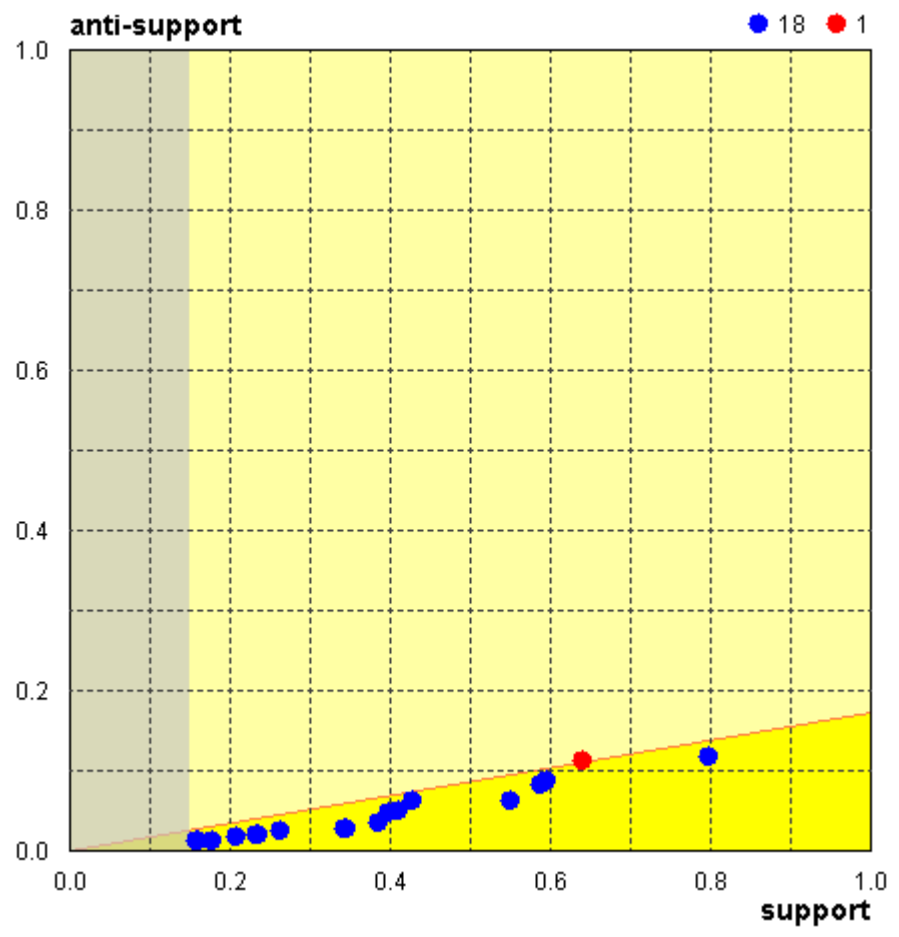




- indicates rules with negative confirmation

Support-anti-support (race=White)





- ● indicates rules with negative confirmation