

Appendix

Theorem 3. The only normalizations that transform all of the analyzed measures into measures Z and A satisfying the symmetries of Ex_2 are the normalization of Crupi et al. and its likelihoodist counterpart. Moreover, measures D , R and G normalized using the Bayesian approach satisfy all symmetries gathered up in Ex_2 .

Proof:

Considering the formulation of a confirmation measure in terms of quantities a, b, c, d , and denoting by $f^+(a,b,c,d)$ the formulation of the confirmation measure in case of confirmation, and by $f^-(a,b,c,d)$ the formulation of the confirmation measure in case of disconfirmation, the Ex_2 conditions of symmetry properties can be expressed as follows:

$$\begin{array}{ll}
 (E^+) & f^+(a,b,c,d) \neq -f^-(b,a,d,c), & (H^+) & f^+(a,b,c,d) = -f^-(c,d,a,b), \\
 (EI^+) & f^+(a,b,c,d) \neq -f^-(b,d,a,c), & (HI^+) & f^+(a,b,c,d) = -f^-(c,a,d,b), \\
 (I^+) & f^+(a,b,c,d) \neq f^+(a,c,b,d), & (EH^+) & f^+(a,b,c,d) \neq f^+(d,c,b,a), \\
 (EHI^+) & f^+(a,b,c,d) = f^+(d,b,c,a), & (E^-) & f^-(a,b,c,d) \neq -f^+(b,a,d,c), \\
 (H^-) & f^-(a,b,c,d) = -f^+(c,d,a,b), & (EI^-) & f^-(a,b,c,d) = -f^+(b,d,a,c), \\
 (HI^-) & f^-(a,b,c,d) \neq -f^+(c,a,d,b), & (I^-) & f^-(a,b,c,d) = f^-(a,c,b,d), \\
 (EH^-) & f^-(a,b,c,d) \neq f^-(d,c,b,a), & (EHI^-) & f^-(a,b,c,d) \neq f^-(d,b,c,a).
 \end{array}$$

Let us observe that in the above formulation, “=” means that the corresponding symmetry is desirable, while “ \neq ” means that the corresponding symmetry is undesirable. To verify if a desirable symmetry holds, we have to prove that it is true for all combinations of a, b, c, d values. In case of an undesirable symmetry, it is enough to prove that it does not hold for at least one combination of a, b, c, d values.

The Tables A-D contain the results of verification of the above symmetries for measures normalized in different ways. Some of the symmetries hold unconditionally, like symmetry H^+ for D_N measure. Such cases are marked as “always” in Tables A-D. Measures that do not always satisfy symmetries EHI^+ , H^- , H^+ , HI^+ , EI^- , I^- do not fulfil Ex_2 conditions as a whole.

As for undesirable symmetries E^+ , EI^+ , I^+ , HI^- , EH^- , EH^+ , E^- , EHI^- , however, we are looking for at least one case of a, b, c, d values in which the equation describing them is not satisfied. Thus, measures for which such a combination never occurs (like in case of symmetry I^+ for measure M_N) also do not satisfy Ex_2 .

Looking at the results gathered in Tables A-D, we can conclude that the normalization of Crupi et al. and its likelihoodist counterpart, transform all of the analyzed measures into measures Z and A (respectively) satisfying the symmetries of Ex_2 . Moreover, measures D , R and G normalized using the Bayesian approach satisfy all symmetries gathered up in Ex_2 .

Table A. Symmetries for measures normalized using the approach inspired by Nicod

		D_N	S_N	M_N	N_N	C_N	R_N	G_N
E^+	$c(H, E) \neq -c(H, \neg E)$	iff $ab \neq cd$	iff $ab \neq cd$	iff $b \neq 0$	iff $b^2 + bd \neq c^2 + ac$	iff $ab \neq cd$	iff $b \neq 0$	iff $c \neq 0$
H^+	$c(H, E) = -c(\neg H, E)$	always	always	iff $c=0$	always	always	iff $c=0$	iff $c=0$
EI^+	$c(H, E) \neq -c(\neg E, H)$	iff $b \neq c$	iff $b \neq c$	iff $b \neq 0$	iff $b \neq c$	iff $b \neq c$	iff $b \neq 0$	iff $c \neq 0$
HI^+	$c(H, E) = -c(E, \neg H)$	iff $a=d$	iff $a=d$	iff $c=0$	iff $a=d$	iff $a=d$	iff $c=0$	iff $c=0$
Γ^+	$c(H, E) \neq c(E, H)$	iff $b \neq c$	iff $b \neq c$	never	iff $b \neq c$	iff $b \neq c$	never	iff $b \neq c$
EH^+	$c(H, E) \neq c(\neg H, \neg E)$	iff $ab \neq cd$	iff $ab \neq cd$	*)	iff $b^2 + bd \neq c^2 + ac$	iff $ab \neq cd$	*)	iff $b \neq c$
EHI^+	$c(H, E) = c(\neg E, \neg H)$	iff $a=d$	iff $a=d$	**)	iff $a=d$	iff $a=d$	**)	always
E^-	$c(H, E) \neq -c(H, \neg E)$	iff $ab \neq cd$	iff $ab \neq cd$	iff $a \neq 0$	iff $d^2 + bd \neq a^2 + ac$	iff $ab \neq cd$	iff $a \neq 0$	iff $d \neq 0$
H^-	$c(H, E) = -c(\neg H, E)$	always	always	iff $a=0$	always	always	iff $a=0$	iff $a=0$
EI^-	$c(H, E) = -c(\neg E, H)$	iff $b=c$	iff $b=c$	iff $a=0$	iff $b=c$	iff $b=c$	iff $a=0$	iff $a=0$
HI^-	$c(H, E) \neq -c(E, \neg H)$	iff $a \neq d$	iff $a \neq d$	iff $a \neq 0$	iff $a \neq d$	iff $a \neq d$	iff $a \neq 0$	iff $d \neq 0$
Γ^-	$c(H, E) = c(E, H)$	iff $b=c$	iff $b=c$	always	iff $b=c$	iff $b=c$	always	***)
EH^-	$c(H, E) \neq c(\neg H, \neg E)$	iff $ab \neq cd$	iff $ab \neq cd$	iff $a \neq d$	iff $d^2 + bd \neq a^2 + ac$	iff $ab \neq cd$	iff $a \neq d$	****)
EHI^-	$c(H, E) \neq c(\neg E, \neg H)$	iff $a \neq d$	iff $a \neq d$	iff $a \neq d$	iff $a \neq d$	iff $a \neq d$	iff $a \neq d$	never

*) iff $a^2bc + ab^2c + abc^2 \neq b^2cd + bc^2d + bcd^2$,

i.e. iff $a \neq d$ and $b \neq 0$ and $c \neq 0$

**) iff $a^2bc + ab^2c + abc^2 = b^2cd + bc^2d + bcd^2$,

i.e. iff $a = d$ or $b = 0$ or $c = 0$

***) iff $a^2cd + ac^2d + acd^2 = a^2bd + ab^2d + abd^2$,

i.e. iff $b = c$ or $a = 0$ or $d = 0$

****) iff $a^2cd + ac^2d + acd^2 \neq a^2bd + ab^2d + abd^2$,

i.e. iff $b \neq c$ and $a \neq 0$ and $d \neq 0$

