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Finding meaningful Bayesian confirmation measures

Salvatore Greco
Department of Economics & Business
University of Catania, Italy
salgreco@unict.it
and Portsmouth Business School
Operations & Systems Management
University of Portsmouth, United Kingdom

Roman Słowiński *
Institute of Computing Science
Poznan University of Technology, Poland
Roman.Slowinski@cs.put.poznan.pl
and Systems Research Institute
Polish Academy of Sciences
Warsaw, Poland

Izabela Szczęch *
Institute of Computing Science
Poznan University of Technology, Poland
Izabela.Szczech@cs.put.poznan.pl

Abstract. The paper focuses on Bayesian confirmation measures used for evaluation of rules induced from data. To distinguish between many confirmation measures, their properties are analyzed. The article considers a group of symmetry properties. We demonstrate that the symmetry properties proposed in the literature focus on extreme cases corresponding to entailment or refutation of the rule's conclusion by its premise, forgetting intermediate cases. We conduct a thorough analysis of the symmetries regarding that the confirmation should express how much more probable the rule's hypothesis is when the premise is present rather than when the negation of the premise is present. As a result we point out which symmetries are desired for Bayesian confirmation measures. Next, we analyze a set of popular confirmation measures with respect to the symmetry properties and other valuable properties, being monotonicity M, Ex_1 and weak Ex_1 logicality L and weak L. Our work points out two measures to be the most meaningful ones regarding the considered properties.

Keywords: Bayesian confirmation measures, symmetry properties, rule evaluation

Address for correspondence: Roman.Slowinski@cs.put.poznan.pl
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1. Introduction

Discovering knowledge from data aims at finding "*valid, novel and potentially useful*" [9] patterns often expressed as "*if..., then...*" rules. To measure the relevance and utility of the discovered rules, quantitative measures, also known as interestingness or attractiveness measures, have been proposed and studied [13, 20, 31, 42]. Among these measures, an important role is played by Bayesian confirmation measures, which express in what degree a rule's premise confirms its conclusion [5, 10, 11, 17]. The discussion brought up in [2, 16, 42] showed the profits of using confirmation measures as interestingness measures for evaluation of rules. Such measures are extremely valuable due to the fact that they permit to filter rules which do not enhance sufficiently relationships between rule's premise and conclusion and should be discarded from further reasoning. In this context, a variety of non-equivalent confirmation measures should be regarded as a useful tool able to discriminate the most interesting rules discovered by induction from data.

To help to analyze measures and overcome the problem of their vast variety, some properties have been proposed. In general, properties group the measures according to similarities in their characteristics. Analysis of measures with respect to their properties is an important research area, because using the measures which satisfy the desirable properties one can better filter rules [43]. Among widely studied properties for Bayesian confirmation measures, there is a group of symmetry properties [3, 5, 8, 16], monotonicity properties [2, 16], weak Ex_1 and weak logicality L property [19, 43].

In this article, we focus on a thorough analysis of symmetry properties of some well-known Bayesian confirmation measures. It is an issue that has been taken up by many authors, e.g., Eells and Fitelson [8] or Crupi et al. [5]. However, our work brings new light to the topic, as we consider a different interpretation of the confirmation concept. Traditionally, a confirmation measure should give an account of the credibility that it is more probable to have the conclusion when the premise is present, rather than when the premise is absent. Instead, we consider the interpretation of a confirmation measure as giving an account of the credibility that it is more probable to have the conclusion when the premise is present, rather than when the negation of the premise is present. Following that interpretation we propose a new set of desirable symmetry properties for Bayesian confirmation measures. We enter into a detailed discussion with the works of Eells and Fitelson [8], as well as Crupi et al. [5], and explain why our results stand sometimes in opposition to theirs. We argue that an acceptable measure of Bayesian confirmation should satisfy evidence symmetry, hypothesis symmetry and a composition of those two, i.e. evidence-hypothesis sym-metry. It should not satisfy any other symmetries.

Having determined a set of desirable symmetries we provide an analysis of eight popular confirmation measures with respect to their symmetry properties. We prove that measures $S(H, E)$ [4] and $N(H, E)$ [34] are the best ones regarding the symmetries. Moreover, we evaluate those measures with respect to other valuable properties: monotonicity M, weak Ex_1 and weak L. We prove that measures $S(H, E)$ and $N(H, E)$ enjoy those properties. Thus, we argue that those two measures are a valuable and meaningful tool for assessing the quality of rules induced from data.

The paper extends the results presented at *The 2012 Joint Rough Set Symposium* [18] with the following material: details concerning derivation of desirable symmetry properties, analysis of a set of well-known Bayesian confirmation measures with respect to those symmetries and other properties being property M, Ex_1, weak Ex_1, logicality L and weak L. In the next section there are preliminaries on rules and their quantitative description. In Section 3, we discuss the concept of confirmation and its interpretation. A set of popular confirmation measures is presented. Section 4 focuses on the approaches to symmetry properties in the literature. Section 5 introduces a proposition of a new set of symmetry properties and provides a detailed analysis of all the symmetries. In Section 6 there is a thorough analysis of eight Bayesian confirmation measures with respect to the new symmetry properties, property of monotonicity M, weak Ex_1 and weak L properties. As a result, two measures, $S(H, E)$ and $N(H, E)$, are pointed out to be the best ones regarding the considered properties. Finally, Section 7 presents conclusions.

2. Preliminaries

A rule induced from a universe U shall be denoted by $E \to H$ (read as "if E, then H"). It consists of a premise (evidence) E and a conclusion (hypothesis) H. We shall define interestingness measures using the following notation corresponding to a 2x2 contingency table of the premise and the conclusion (Table (1)):

- a is the number of positive examples to the rule, i.e., the number of objects in U satisfying both the premise and the conclusion of the rule,
- b is the number of objects in U not satisfying the rule's premise, but satisfying its conclusion,
- c is the number of counterexamples, i.e. objects in U satisfying the premise but not the conclusion of the rule,
- d is the number of objects in U that do not satisfy neither the premise nor the conclusion of the rule.

Table 1. Contingency table of E and H for rule $E \to H$

	H	$\neg H$	Σ
E	a	c	$a+c$
$\neg E$	b	d	$b+d$
Σ	$a+b$	$c+d$	$\lvert U\rvert$

The cardinality of the universe U, denoted by $|U|$, is the sum of a, b, c and d. Reasoning in terms of a, b, c and d is natural and intuitive for data mining techniques since all observations are gathered in some kind of an information table describing each object by a set of attributes. However, a, b, c and d can also be regarded as frequencies that can be used to estimate probabilities: e.g., the probability of the premise is expressed as $Pr(E) = (a + c)/|U|$, and the probability of the conclusion as $Pr(H) = (a + b)/|U|$. Moreover, conditional probability of the conclusion given the premise is $Pr(H|E) = a/(a + c)$.

3. Bayesian confirmation measures

Generally speaking, measures possessing the property of Bayesian confirmation (also referred to as Bayesian confirmation measures or simply confirmation measures) are expected to obtain:

- values greater than 0 when the premise of a rule confirms the conclusion of a rule,
- values equal to 0 when the rule's premise and conclusion are neutral to each other,
- and finally, values smaller than 0 when the premise disconfirms the conclusion.

Unlike popular support or confidence measures used mostly for association rules [1], confirmation measures give a clear information as to when the rule is completely misleading due to disconfirmation of the conclusion by the rule's premise. Moreover, using the quantitative confirmation theory for data analysis allows to benefit from the ideas of such prominent philosophers of science as Carnap [3], Hempel [22] and Popper [37].

Formally, an interestingness measure $c(H, E)$ has the property of Bayesian confirmation if and only if it satisfies the following conditions:

$$(BC) \qquad c(H,E)\begin{cases} > 0 & if \quad Pr(H|E) > Pr(H), \\ = 0 & if \quad Pr(H|E) = Pr(H), \\ < 0 & if \quad Pr(H|E) < Pr(H). \end{cases} \tag{1}$$

According to (BC) conditions, the confirmation of the conclusion provided by the premise should be identified with an increase in the probability of the conclusion H provided by the premise E. The lack of influence of the premise E on the probability of conclusion H is interpreted as neutrality between the premise and conclusion. Finally, the disconfirmation of the conclusion by the premise is understood as a decrease of probability of the conclusion H imposed by the premise E [11, 30]. It is important to note that condition $Pr(H|E) > Pr(H)$ is not the only way of expressing that E confirms H. Among logically equivalent formulations there is ([11, 30]):

$$Pr(H|E) > Pr(H|\neg E).$$

The equivalence of the above condition allows us to express the (BC) conditions as:

$$(BC') \qquad c(H,E)\begin{cases} > 0 & if \quad Pr(H|E) > Pr(H|\neg E), \\ = 0 & if \quad Pr(H|E) = Pr(H|\neg E), \\ < 0 & if \quad Pr(H|E) < Pr(H|\neg E). \end{cases} \tag{2}$$

According to (BC'), E confirms H when the probability of H given E is higher than the probability of H given $\neg E$ (neutrality and disconfirmation can be defined analogously). Consequently, for a given rule $E \to H$, interestingness measures with the property of confirmation express the credibility of the following proposition: *H is satisfied more frequently when E is satisfied, rather than when the negation of E is satisfied.*

Let us stress that the (BC) conditions (or (BC') equivalently) do not impose any constraints on the confirmation measures except for requiring when the measures should obtain positive or negative values. As a result many alternative, non-equivalent measures of confirmation have been proposed [5, 10]. The most commonly used ones are gathered in Table 2.

Measure $D(H, E)$ has been supported by Earman [6], Eells [7], Gillies [14] and Jeffrey [25] and Rosenkrantz [39]. Measure $S(H, E)$ has been proposed by Christensen [4] and Joyce [26]. Measure

Table 2. Popular confirmation measures

$D(H,E) = P(H\|E) - P(H) = \dfrac{a}{a+c} - \dfrac{a+b}{\|U\|}$
$S(H,E) = P(H\|E) - P(H\|\neg E) = \dfrac{a}{a+c} - \dfrac{b}{b+d}$
$M(H,E) = P(E\|H) - P(E) = \dfrac{a}{a+b} - \dfrac{a+c}{\|U\|}$
$N(H,E) = P(E\|H) - P(E\|\neg H) = \dfrac{a}{a+b} - \dfrac{c}{c+d}$
$C(H,E) = P(E \wedge H) - P(E)P(H) = \dfrac{a}{\|U\|} - \dfrac{(a+c)(a+b)}{\|U\|^2}$
$R(H,E) = \dfrac{P(H\|E)}{P(H)} - 1 = \dfrac{a\|U\|}{(a+c)(a+b)} - 1$
$G(H,E) = 1 - \dfrac{P(\neg H\|E)}{P(\neg H)} = 1 - \dfrac{c\|U\|}{(a+c)(c+d)}$
$F(H,E) = \dfrac{P(E\|H) - P(E\|\neg H)}{P(E\|H) + P(E\|\neg H)} = \dfrac{ad - bc}{ad + bc + 2ac}$

$M(H,E)$ has been considered by Mortimer [33] and measure $N(H,E)$ by Nozick [34]. Measure $C(H,E)$ has been introduced by Carnap [3]. Measure $R(H,E)$ has been defended by Horwich [24], Keynes [28], Mackie [29], Milne [32], Schlesinger [40] and Pollard [36]. Measure $G(H,E)$ has been considered by Rips [38]. Measure $F(H,E)$ has been supported by Kemeny and Oppenheim [27], Good [15], Heckerman [21], Horvitz and Heckerman [23], Pearl [35] and Schum [41]. Fitelson [11] has also advocated for measure $F(H,E)$.

To help to handle the plurality of Bayesian confirmation measures, many authors have considered properties of such measures. Analysis of measures with respect to their properties is a way to distinguish measures that behave according to user's expectations. Among desirable properties of confirmation measures there are properties of monotonicity M [16], properties of logicality L [3, 12], weak logicality [19], properties Ex_1 and weak Ex_1 concerning conclusively confirmatory rules [5, 19, 43].

An important group of properties constitute symmetry properties considered by many authors, e.g., [3, 5, 8]. In the next section we concentrate on the symmetry properties and analyze them from the viewpoint of the (BC') interpretation of confirmation stating that the hypothesis H is satisfied more frequently when E is satisfied, rather than when $\neg E$ is satisfied. We propose a modification of the symmetry properties so that they would deploy such concept of confirmation.

4. Symmetry properties

Inspired by the work of Carnap [3], Eells and Fitelson have analysed in [8] a set of well-known confirmation measures from the viewpoint of the following four properties of symmetry:

- evidence symmetry ES: $c(H,E) = -c(H,\neg E)$
- hypothesis symmetry HS: $c(H,E) = -c(\neg H,E)$
- inversion (commutativity) symmetry IS: $c(H,E) = c(E,H)$
- evidence-hypothesis (total) symmetry EHS: $c(H,E) = c(\neg H,\neg E)$

On the basis of examples of drawing cards from a standard deck, Eells and Fitelson concluded in [8] that, in fact, only hypothesis symmetry HS is a desirable property, while evidence symmetry ES, commutativity symmetry IS and total symmetry EHS are not.

To illustrate their approach let us recall their counterexample to evidence symmetry ES [8]. A card is randomly drawn from a standard deck. Let the evidence E be that *the card is the seven of spades*, and let the hypothesis H be that *the card is black*. Having drawn *the seven of spades* is a strong evidence confirming that *the card is black*. On the other hand, however, $\neg E$ i.e. evidence that the drawn *card is not the seven of spades*, does not refute conclusively the hypothesis, as such evidence is almost "informationless" with regard to the colour of the card. Thus, *seven of spades* confirms that *the card is black* to a greater extent than *not-seven of spades* disconfirms the same hypothesis. As a result, according to Eells and Fitelson, the equality in evidence symmetry is found unattractive, and an acceptable measure of Bayesian confirmation should not satisfy the evidence symmetry (i.e. for some situation $c(H,E) \neq -c(H,\neg E)$). The above reasoning seems convincing in extreme cases corresponding to entailment or refutation represented by rules. One may doubt it, however, when some intermediate cases are considered (see Section 5).

Moreover, recently, Crupi et at. [5] have argued for an extended and systematic treatment of the issue of symmetry properties. They propose to analyse a confirmation measure $c(H,E)$ with respect to seven symmetries being all combinations obtained by applying the negation operator to the premise E, the hypothesis H or both, and/or by inverting E and H:

- $ES(H,E) : c(H,E) = -c(H,\neg E)$
- $HS(H,E) : c(H,E) = -c(\neg H,E)$
- $EIS(H,E) : c(H,E) = -c(\neg E,H)$
- $HIS(H,E) : c(H,E) = -c(E,\neg H)$
- $IS(H,E) : c(H,E) = c(E,H)$
- $EHS(H,E) : c(H,E) = c(\neg H,\neg E)$
- $EHIS(H,E) : c(H,E) = c(\neg E,\neg H)$

The extension of the set of symmetry properties considered by Crupi et al. [5] goes even further, as the analysis is conducted separately for the case of confirmation (i.e. when $Pr(H|E) > Pr(H)$), and for the case of disconfirmation (i.e. when $Pr(H|E) < Pr(H)$). This way we obtain 14 symmetry properties. Using examples (analogical to Eells and Fitelson) of drawing cards from a standard deck, Crupi et al. point out which of the symmetries are desired and which are definitely unwanted. For instance, the inversion symmetry IS is undesired in case of confirmation as for a rule: *if Jack was drawn, then the card is a face*, the *face* does not confirm *Jack* with the same strength as *Jack* confirms *face*, i.e. $c(H,E) \neq c(E,H)$. On the other hand, symmetry IS is desirable in case of disconfirmation, as for an exemplary rule: *if the drawn card is an ace, then it is a face*, the strength with which *an ace* disconfirms face is the same as the strength with which *the face* disconfirms *an ace*, i.e. $c(H,E) = c(E,H)$.

Table 3. Desirable (YES) and undesirable (NO) symmetry properties by Crupi et al. [5]

	in case of confirmation	**in case of disconfirmation**
ES	NO: *for some* (H,E) $c(H,E) \neq -c(H,\neg E)$	
HS	YES: *for any* (H,E) $c(H,E) = -c(\neg H,E)$	
EIS	NO: *for some* (H,E) $c(H,E) \neq -c(\neg E,H)$	YES: *for any* (H,E) $c(H,E) = -c(\neg E,H)$
HIS	YES: *for any* (H,E) $c(H,E) = -c(E,\neg H)$	NO: *for some* (H,E) $c(H,E) \neq -c(E,\neg H)$
IS	NO: *for some* (H,E) $c(H,E) \neq c(E,H)$	YES: *for any* (H,E) $c(H,E) = c(E,H)$
EHS	NO: *for some* (H,E) $c(H,E) \neq c(\neg H,\neg E)$	
$EHIS$	YES: *for any* (H,E) $c(H,E) = c(\neg E,\neg H)$	NO: *for some* (H,E) $c(H,E) \neq c(\neg E,\neg H)$

Table 3 [5] reports the outcomes of such analysis for all 14 symmetry properties. The results obtained by Crupi et al. concur with Eells and Fitelson with respect to ES, IS, HS and EHS in case of confirmation.

5. A new set of symmetry properties

Let us observe that Eells and Fitelson [8], as well as Crupi et al. [5], concentrate on entailment and refutation of the hypothesis by the premise. The first is regarded as the highest confirmation and the latter as the highest disconfirmation. This, however, boils the concept of confirmation down only to situations where there are no counter-examples (entailment) and where there are no positive examples to a rule (refutation). Rules for which the premise implies the conclusion are seen as rules with the highest possible confirmation, and rules for which the premise refutes the conclusion obtain the least possible confirmation, i.e. the highest possible disconfirmation.

In our opinion, however, the concept of confirmation is much broader than a simple analysis whether there are counterexamples to a rule or not. In fact, according to the (BC') interpretation of the confirmation concept, a confirmation measure should give an account of the credibility that it is *more probable to have the conclusion (H) when the premise is present (E), rather than when the negation of the premise is present ($\neg E$).* This means that we should look at confirmation from the perspective of passing from a situation where the premise is absent ($\neg E$) to the situation where the premise is present. Then, the increase of confirmation (i.e. the difference in conditional probabilities $Pr(H|E)$ and $Pr(H|\neg E)$) becomes important, not just the absence or presence of counterexamples. Let us illustrate our point of view using the following exemplary scenario α, where the values from contingency table of E and H are: $a = 100$, $b = 99$, $c = 0$, $d = 1$, $|U| = 200$. For scenario α, the values of conditional probabilities are: $Pr(H|E) = a/(a+c) = 1$ and $Pr(H|\neg E) = b/(b+d) = 0.99$, thus passing from the situation where the premise is absent to the situation where the premise is present, we have only a 1% increase, which is not a big value at all, despite the fact that we get to the point where there are no counterexamples ($Pr(H|E) = 1$).

Analogically, for disconfirmation a confirmation measure $c(H, E)$ should express *how much it is less probable to have* H *when* E *is present rather than when* $\neg E$ *is present*. Again, we should, thus, pass from the situation where the premise is absent to the situation where the premise is present.

Let us observe that in this perspective both confirmation and disconfirmation are based on the consideration of $Pr(H|E)$ and $Pr(H|\neg E)$. The highest confirmation is obtained when $Pr(H|\neg E) = 0$ and $Pr(H|E) = 1$, i.e. when the passing from situation where the premise is absent to the situation where the premise is present, we get a 100% increase. The highest disconfirmation, on the other hand, occurs when $Pr(H|\neg E) = 1$ and $Pr(H|E) = 0$, i.e. when the passing from situation where the premise is absent to the situation where the premise is present, we get a 100% decrease. On the basis of these observations we postulate to consider the symmetry properties together for cases of confirmation and disconfirmation. There is no need to treat them differently as they both consider passing from $Pr(H|\neg E)$ to $Pr(H|E)$.

Now, let us conduct a thorough analysis in order to verify which of the seven symmetries: ES, HS, EIS, HIS, IS, EHS, $EHIS$ are desirable and which are unattractive from the view point of (BC') interpretation of confirmation. Let us use again the exemplary scenario α, where the values from contingency table of E and H are: $a = 100$, $b = 99$, $c = 0$, $d = 1$, $|U| = 200$.

5.1. Evidence symmetry (ES)

Analyzing ES we need to verify whether the equation $c(H, E) = -c(H, \neg E)$ is desirable or not. Let us examine both sides of this equation using scenario α. Let us observe, that for $c(H, E)$ we have that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$, which gives us a 1% increase of confirmation. On the other hand, for $c(H, \neg E)$ we get exactly the same components but the other way around: $Pr(H|E) = 1$ and $Pr(H|\neg E) = 0.99$, which results in 1% decrease of confirmation. Thus, clearly the confirmation of a rule $E \rightarrow H$ should be of the same value but of the opposite sign as the confirmation of a $\neg E \rightarrow H$ rule. Therefore, we can conclude that the evidence symmetry is desirable.

This result is contrary to what Eells and Fitelson [8], and Crupi et al. [5] advocated for. This is due to the fact that they treat the entailment of the conclusion by the premise (i.e. situation where there are no counterexamples to the rule) as the maximal confirmation, whereas we consider the increase of confirmation when passing from the absence of the premise ($\neg E$) to its presence (E). For the exemplary rule of Eells and Fitelson: *if the drawn card is the seven of spades then the card is black*, the conditional probabilities are the following $Pr(H|E) = 1$ and $Pr(H|\neg E) = 0.49$. They claim that because $Pr(H|E) > Pr(H|\neg E)$, we should regard the ES as unattractive. However, if we interpret the concept of confirmation as expressing how much more it is probable to have the *black card* when we have drawn the *seven of spades* than when we have drawn *not-the seven of spades*, we see that the confirmation is the same (but of the opposite sign) for both of the rules: $E \rightarrow H$ and $\neg E \rightarrow H$. Both of the rules are characterized by a confirmation having an absolute value of 51%, however in the first case, it has a positive sign and therefore there is a confirmation, and in the latter a negative sign and therefore there is disconfirmation. Thus, we claim that using the (BC') interpretation of the confirmation concept, evidence symmetry is a desirable property for Bayesian confirmation measures.

5.2. Hypothesis symmetry (HS)

In case of the hypothesis symmetry HS we are concordant with Eells and Fitelson, and Crupi et al. claiming that it is a desirable property, i.e. $c(H, E) = -c(\neg H, E)$. Reaching to the scenario α, we can calculate that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$ (increase of 1%), whereas $Pr(\neg H|\neg E) = 0.01$ and $Pr(\neg H|E) = 0$ (decrease of 1%). This supports the conclusion that confirmation for rules $E \rightarrow H$ and $E \rightarrow \neg H$ is the same but of the opposite sign.

5.3. Evidence-inversion symmetry (EIS)

According to Crupi et al. [5] (the EIS was not analyzed by Eells and Fitelson) the EIS is an unattractive property in case of confirmation. Our analysis concurs with that result. Crupi et al. call up the counterexample for EIS, where $c(face, Jack) > -c(not - Jack, face)$. Using the scenario α we have that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$, whereas $Pr(\neg E|\neg H) = 1$ and $Pr(\neg E|H) = 99/199$. Thus, it also is a EIS counterexample since 1% increase in confirmation is not equal to over 50% decrease. So generally, $c(H, E) \neq -c(\neg E, H)$.

Crupi et al. conducted also the analysis of EIS in case of disconfirmation. This time however, they found it as a desirable property, since "*E refutes H iff H implies $\neg E$*". Let us recall, that disconfirmation should express how much it is less probable to have H when we have E rather than when we have $\neg E$. Following that interpretation, let us use scenario β where $a = 0$, $b = 1$, $c = 100$, $d = 99$ and $|U| = 200$ (scenario β can be regarded as a "disconfirmational" counterpart of scenario α). We obtain $Pr(H|\neg E) = 1/100$ and $Pr(H|E) = 0$, whereas $Pr(\neg E|\neg H) = 99/199$ and $Pr(\neg E|H) = 1$. Thus, we have a 1% decrease in confirmation (i.e. 1% increase in disconfirmation) for $E \rightarrow H$ rule and an over 50% increase in confirmation for rule $H \rightarrow \neg E$. The EIS property is therefore unacceptable also in case of disconfirmation (contrary to results of Crupi et al.).

Scenario β was used to show the way to discuss contrary results concerning symmetries in case of disconfirmation. However, as we have argued at the beginning of Section 5, there is no need to discuss the symmetry properties separately for the case of confirmation and disconfirmation. In the rest of the paper, we shall only provide analysis for symmetry properties in case of confirmation, since the results are also true for the case of disconfirmation.

5.4. Hypothesis-inversion symmetry (HIS)

Eells and Fitelson did not analyze the HIS property. Crupi et al. found the symmetry desirable in case of confirmation and unattractive in case of disconfirmation. In case of confirmation they argue that "*E implies H iff $\neg H$ refutes E*" and therefore the equality $c(H, E) = -c(E, \neg H)$ is attractive. However, our calculations based on the scenario α show a counterexample to HIS. We have that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$, whereas $Pr(E|H) = 100/199$ and $Pr(E|\neg H) = 0$. Obviously, a 1% increase in confirmation is not equal to over 50% drop and $c(H, E) \neq -c(E, \neg H)$. Thus, we regard the HIS as an undesirable property.

5.5. Inversion (commutativity) symmetry (IS)

The inversion symmetry IS, denoted by Eells and Fitelson as commutativity symmetry, is regarded by those authors as unattractive. Crupi et al. found it undesirable, however, only in case of confirmation.

They claim that the fact that $c(face, Jack) > c(Jack, face)$ (i.e., the probability of getting a *face card* having drawn a *Jack* is 1 and is higher than the probability of getting a *Jack* having drawn a *face card*) should be regarded as the counterexample for IS in case of confirmation. On the other hand, they argue that IS is a desirable property in case of disconfirmation since "E *refutes* H *iff* H *refutes* E".

However, if we incorporate the interpretation that confirmation measures should express how much more, in case of confirmation (or less in case of disconfirmation) probable the hypothesis is when the premise is present (E) rather than when it is absent ($\neg E$), we will conclude that inversion symmetry is generally unattractive, both in case of confirmation and disconfirmation. Using the scenario α we have that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$, whereas $Pr(E|\neg H) = 0$ and $Pr(E|H) = 100/199$. Obviously, a 1% increase of confirmation for $E \to H$ rule is smaller than an over 50% increase for a rule $H \to E$. Therefore, the IS is an unattractive symmetry property.

5.6. Evidence-hypothesis (total) symmetry (EHS)

Evidence-hypothesis (also known as Total symmetry) is formed by applying the negation operator to the rule's premise and its conclusion. Eells and Fitelson as well as Crupi et al. regard it as an unattractive property. However, since the EHS is a composition of ES and HS, and both ES and HS are desirable under the (BC') interpretation of the confirmation concept, EHS should also be found desirable. Our result is therefore contrary to the previous analysis in the literature, but it allows to escape from boiling the confirmation down to the presence or absence of positive examples and counterexamples.

5.7. Evidence-hypothesis-inversion symmetry ($EHIS$)

The $EHIS$ property was analyzed only by Crupi et al. who claim that $c(H, E) = c(\neg E, \neg H)$ only in case of confirmation. Our results are contrary. Using the scenario α we have that $Pr(H|\neg E) = 0.99$ and $Pr(H|E) = 1$, whereas $Pr(\neg E|H) = 99/199$ and $Pr(\neg E|\neg H) = 1$. A 1% increase of confirmation for $E \to H$ rule is not equal to an over 50% increase for a rule $\neg H \to \neg E$, thus it is a counterexample proving that $EHIS$ is generally an unattractive symmetry property.

5.8. Summary

The conducted analysis reveals that under the (BC') interpretation it is enough to consider the symmetry properties only in case of confirmation, since the results are valid also for the case of disconfirmation. Moreover, the set of desirable properties contains only the evidence symmetry (ES), the hypothesis symmetry (HS) and their composition, i.e., the evidence-hypothesis symmetry (EHS) (see Table 4). This implies that a valuable Bayesian confirmation measure should satisfy only those symmetry properties. By defining the new set of symmetry properties we gain a tool for assessing the quality of confirmation measures. Only using the truly meaningful measures we can extract useful patterns from data.

6. Meaningful measures

In Table 2 there are gathered popular measures of Bayesian confirmation. It is a set of eight measures also recalled in [5, 10, 11]. In this section, we analyze those measures with respect to their properties. The following desirable properties are concerned:

Table 4. New desirable (YES) and undesirable (NO) symmetry properties

ES	YES: *for any* (H,E) $c(H,E) = -c(H,\neg E)$
HS	YES: *for any* (H,E) $c(H,E) = -c(\neg H,E)$
EIS	NO: *for some* (H,E) $c(H,E) \neq -c(\neg E,H)$
HIS	NO: *for some* (H,E) $c(H,E) \neq -c(E,\neg H)$
IS	NO: *for some* (H,E) $c(H,E) \neq c(E,H)$
EHS	YES: *for any* (H,E) $c(H,E) = c(\neg H,\neg E)$
$EHIS$	NO: *for some* (H,E) $c(H,E) \neq c(\neg E,\neg H)$

- properties of symmetry: ES, HS, EHS. Let us remark that the analysis regarding symmetry properties also verifies whether the unattractive symmetry properties (i.e. IS, HIS, EIS, $EHIS$) are not satisfied by the measures,
- property M of monotonic dependency of the measure on the number of objects satisfying (supporting) or not the premise or the conclusion of the rule [2, 16, 42],
- weak Ex_1 and weak L properties indicating the conditions under which the confirmation measures should obtain their maximal or minimal values.

A thorough analysis allows us to indicate the most valuable and meaningful measures with respect to the above properties.

6.1. Properties of symmetry

Eells and Fitelson in [8] have considered measures $S(H,E)$, $C(H,E)$, $D(H,E)$, $F(H,E)$ and $R(H,E)$ (see definition in Table 2) with respect to the following symmetry properties: ES, HS, IS and EHS. Our analysis concentrated on the measures and symmetry properties not covered in that work. Putting all the results together, two measures emerge as being truly interesting regarding all of the considered symmetry properties. They are measures $S(H,E)$ and $N(H,E)$. These are the only ones from the analyzed set of measures in Table 2 that satisfy the ES, HS and EHS, not possessing the other symmetry properties at the same time.

Theorem 6.1. Measures $S(H,E)$ and $N(H,E)$ are the only measures from Table 2 that satisfy the desirable properties ES, HS and EHS, and do not satisfy the unattractive properties IS, HIS, EIS, $EHIS$.

Proof:
Eells and Fitelson have shown in [8] that measures $D(H,E)$, $R(H,E)$ and $F(H,E)$ violate desirable evidence symmetry ES. Moreover, they have proved that measure $C(H,E)$ satisfies the unattractive inversion property IS. Thus, measures $D(H,E)$, $R(H,E)$, $F(H,E)$ and $C(H,E)$ should not be regarded as the most meaningful interestingness measures.

Let us now present that measures $M(H,E)$ and $G(H,E)$ are also not the most worthy since they violate the desirable HS as generally, $M(H,E) \neq -M(\neg H,E)$ and $G(H,E) \neq -G(\neg H,E)$.

Let us observe that
$M(H,E) = Pr(E|H) - Pr(E) = \frac{ad-bc}{|U|(a+b)}$,
$M(\neg H,E) = Pr(E|\neg H) - Pr(E) = -\frac{ad-bc}{|U|(c+d)}$.
Thus, $M(H,E) \neq -M(\neg H,E)$ unless $a+b = c+d$.
Moreover,
$G(H,E) = 1 - \frac{Pr(\neg H|E)}{Pr(\neg H)} = \frac{ad-bc}{(a+c)(c+d)}$,
$G(\neg H,E) = 1 - \frac{Pr(H|E)}{Pr(H)} = -\frac{ad-bc}{(a+c)(a+b)}$.
Thus, $G(H,E) \neq -G(\neg H,E)$ unless $a+b = c+d$.

Now the let us prove that measures $S(H,E)$ and $N(H,E)$ satisfy just the desirable properties: ES, HS and EHS. Eells and Fitelson [8] have shown that measure $S(H,E)$ satisfies ES, HS and EHS, but not IS. To show that measure $S(H,E)$ does not satisfy EIS, HIS, nor $EHIS$ we need to prove that for at least one (H,E) we have: $S(H,E) \neq -S(\neg E,H)$, $S(H,E) \neq -S(E,\neg H)$ and $S(H,E) \neq S(\neg E,\neg H)$.
Let us observe that:
$S(H,E) = Pr(H|E) - Pr(H|\neg E) = \frac{ad-bc}{(a+c)(b+d)}$,
$S(\neg E,H) = S(E,\neg H) = -S(\neg E,\neg H) = -(Pr(\neg E|\neg H) - Pr(\neg E|H)) = -\frac{ad-bc}{(a+b)(c+d)}$.
This implies that neither $S(H,E) = -S(\neg E,H)$, $S(H,E) = -S(E,\neg H)$ nor $S(H,E) = S(\neg E,\neg H)$ is true unless $b = c$ or $a = d$. Thus, measure $S(H,E)$ does not possess the unattractive EIS, HIS, $EHIS$ properties.

Measure $N(H,E)$ satisfies ES, HS and EHS since unconditionally $N(H,E) = -N(H,\neg E)$, $N(H,E) = -N(\neg H,E)$ and $N(H,E) = N(\neg H,\neg E)$. Simple calculations show that:
$N(H,E) = -N(H,\neg E) = -N(\neg H,E) = N(\neg H,\neg E) = \frac{ad-bc}{(a+b)(c+d)}$.

Moreover, measure $N(H,E)$ does not satisfy the unattractive properties of IS, EIS, HIS, $EHIS$, since there is always at least one (H,E) for which the following equations are not satisfied:
$N(H,E) = N(E,H)$, $N(H,E) = -N(\neg E,H)$, $N(H,E) = -N(E,\neg H)$ and
$N(H,E) = N(\neg E,\neg H)$.
Let us observe that $N(H,E) = Pr(E|H) - Pr(E|\neg H) = \frac{ad-bc}{(a+b)(c+d)}$.
Moreover $N(E,H) = -N(\neg E,H) = -N(E,\neg H) = N(\neg E,\neg H) = \frac{ad-bc}{(a+c)(b+d)}$.

Thus, clearly unless $b = c$ or $a = d$ the above equations defining the properties IS, EIS, HIS, $EHIS$ are not satisfied. This implies that measure $N(H,E)$ does not possess these symmetry properties. □

6.2. Property of monotonicity M

Greco, Pawlak and Słowiński have considered in [16] Bayesian confirmation measures from the viewpoint of their usefulness for measuring interestingness of decision rules. According to [16], a confirmation measure should enjoy a property, called property of monotonicity M. It requires that a confirmation measure $c(H,E)$ is a function non-decreasing with respect to a and d, and non-increasing with respect to b and c. The property of M with respect to a (or, analogously, with respect to d) means that any evidence in which the premise E and the conclusion H (or, analogously, $\neg E$ and $\neg H$) hold together increases (or at least does not decrease) the confirmation of the rule $E \to H$. On the other hand, the property M with respect to b (or, analogously, with respect to c) means that any evidence in which $\neg E$ and H holds (or, analogously, E and $\neg H$ hold) decreases (or at least does not increase) the confirmation of the rule $E \to H$.

The property M has also proved to be very useful regarding multicriteria evaluation of patterns in form of rules [2, 42]. Since measures $S(H,E)$ and $N(H,E)$ are truly valuable with respect to symmetry properties, let us now analyze them regarding property M.

Theorem 6.2. Measures $S(H,E)$ and $N(H,E)$ satisfy the property of monotonicity M.

Proof:
The possession of property M by measure $S(H,E)$ has been proved in [16].

In the following we suppose that $N(H,E)$ can be calculated, i.e., remembering that $N(H,E) = \frac{a}{a+b} - \frac{c}{c+d}$, we suppose that at least one from a and b, and at least one from c and d are greater than zero. Measure $N(H,E)$ satisfies property M because it is

- non-decreasing with respect to a,
- non-increasing with respect to b,
- non-increasing with respect to c,
- non-decreasing with respect to d.

Let us define measures $N(H,E)$ as follows: $N(H,E) = Pr(E|H) - Pr(E|\neg H) = \frac{a}{a+b} - \frac{c}{c+d}$.
Increasing a by $\Delta > 0$ and remembering that a and b are always non negative, we get
$\frac{a}{a+b} - \frac{c}{c+d} \leq \frac{a+\Delta}{a+\Delta+b} - \frac{c}{c+d} \iff a(a+\Delta+b) \leq (a+\Delta)(a+b) \iff \Delta \geq 0.$

Thus, measure $N(H,E)$ is non-decreasing with respect to a.
The proof regarding b, c and d is analogous. □

6.3. Weak Ex_1 and weak L properties

The weak Ex_1 property [19, 43] is a generalization of Ex_1 property introduced by Crupi et al. in [5]. It allows to escape from potential paradoxes that can be caused by using Ex_1.

On the basis of classical deductive logic let us construct a function v:

$$v(H,E) = \begin{cases} \text{the same positive value, denoted as} \quad V \quad \text{if} \quad E \models H, \\ \text{the same negative value, denoted as} \quad -V \quad \text{if} \quad E \models \neg H, \\ 0 \quad \text{otherwise.} \end{cases} \tag{3}$$

For any argument (H,E) function v assigns it the same positive value V (e.g., $+1$) if and only if the premise E of the rule entails the conclusion H (i.e. $E \models H$). The same value but of opposite sign $-V$ (e.g., -1) is assigned if and only if the premise E refutes the conclusion H (i.e. $E \models \neg H$). In all other cases (i.e. when the premise is not conclusively confirmatory nor conclusively disconfirmatory) function v obtains value 0.

From definition, any confirmation measure $c(H,E)$ agrees with function $v(H,E)$ in the way that if $v(H,E)$ is positive, then the same is true of $c(H,E)$, and when $v(H,E)$ is negative, so is $c(H,E)$. However, the relationship between the logical entailment or refutation of H by E, and the conditional probability of H subject to E should go further and fulfill the following weak Ex_1 property:

$$\text{(weak Ex}_1\text{)} \qquad \begin{array}{l} if \quad v(H_1, E_1) \geq v(H_2, E_2) \quad and \quad v(H_1, \neg E_1) \leq v(H_2, \neg E_2) \\ \quad \text{with at least one inequality strictly satisfied} \\ \quad then \quad c(H_1, E_1) > c(H_2, E_2). \end{array} \qquad (4)$$

The above weak Ex_1, formulated in a slightly different way than in [19], guarantees that a confirmation measure $c(H, E)$ cannot attain its maximal value unless the two following conditions are satisfied:

- $Pr(H|E) = 1$, or equivalently, $c = 0$, i.e. there are no counterexamples to the rule, and
- $Pr(H|\neg E) = 0$, or equivalently, $b = 0$, i.e. there are no objects in U satisfying the rule's conclusion but not its premise.

Analogously, weak Ex_1 property guarantees that the confirmation measure $c(H, E)$ cannot attain its minimal value unless the two following conditions are satisfied:

- $Pr(H|E) = 0$, or equivalently, $a = 0$, i.e. there are no positive examples to the rule, and
- $Pr(H|\neg E) = 1$, or equivalently, $d = 0$, i.e. there are no objects in U not satisfying neither the rule's premise nor its conclusion.

Theorem 6.3. Measures $S(H, E)$ and $N(H, E)$ satisfy the weak Ex_1 property. See proof in [19].

Besides possessing the valuable weak Ex_1 property, measures $S(H, E)$ and $N(H, E)$ are also interesting due to not satisfying the original Ex_1 property [5]. As it is explained in [19, 43], property Ex_1 can lead to some undesirable ranking on rules induced from a dataset U. Thus, the fact that measures $S(H, E)$ and $N(H, E)$ do not satisfy Ex_1, but only weak Ex_1 is their advantage.

Closely related to weak Ex_1 property is weak L property. It is a generalization of the logicality L property [3, 12]. Weak L is a property assuring that:

- $c(H, E)$ is maximal when $E \models H$ and $\neg E \models \neg H$ (i.e. when $b = c = 0$),
- and $c(H, E)$ is minimal when $E \models \neg H$ and $\neg E \models H$ (i.e. when $a = d = 0$).

Theorem 6.4. Measures $S(H, E)$ and $N(H, E)$ satisfy the weak L property. See proof in [19].

Again, measures $S(H, E)$ and $N(H, E)$ are also meaningful due to the fact that they do not satisfy the original logicality L, that could lead to paradoxical ranking of rules [19].

7. Conclusions

The article concerns the properties of confirmation measures used to evaluate patterns in form of rules induced from datasets. Bayesian confirmation measures constitute an important group of interestingness measures as they have, from definition, the ability to discard rules whose premises do not sufficiently confirm their conclusions.

To distinguish between many confirmation measures, their properties have been widely discussed and analyzed in the literature. This article has focused on the group of symmetry properties. Adopting an incremental rather than absolute perspective, we proposed a new set of desirable symmetry properties. Since confirmation measures should reflect how much more it is probable to have the conclusion H when the premise (E) is present rather than when the negation of the premise ($\neg E$) is present, only symmetries formed by applying the negation operator to the rule's premise, conclusion or both (i.e. ES, HS and EHS) are desirable. Properties IS, EIS, HIS, $EHIS$ are unattractive. Thus, valuable confirmation measures should only satisfy ES, HS and EHS.

Having determined a set of desirable symmetry properties, we have analyzed a set of popular confirmation measures with respect to symmetries, and other valuable properties being property M, weak Ex_1 and weak L. The results we have obtained point out measure $S(H, E)$ and measure $N(H, E)$ as the best regarding those useful properties. This way we have gained a meaningful tool for assessing the information value carried by rules induced from datasets.

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