

Forum (Robustness Analysis)

Robustness in OR-DA: a generic framework and its application to the configuration of power distribution networks

A. Aubry¹, A. Rossi², M. Jacomino³

¹CRAN, Nancy-Université, CNRS, Boulevard des
Aiguillettes, F-54506 Vandoeuvre lès Nancy
alexis.aubry@cran.uhp-nancy.fr

²Lab-STICC, Université de Bretagne-Sud, Centre de
recherche, F-56321 Lorient
andre.rossi@univ-ubs.fr

³G-SCOP, Grenoble-INP, UJF, CNRS, 46 avenue Félix
Viallet, F-38031 Grenoble
mireille.jacomino@grenoble-inp.fr

1. Introduction

Usually works made in optimisation field assumed that the problem is entirely exactly known. Then a solution S for one given forecast instance I^{ref} is computed with regard to a criterion z that will be considered as a maximisation criterion without loss of generality. An optimal solution for an instance I is denoted S_I^* and its associated performance is denoted z_I^* . The performance of a solution S applied to an instance I relatively to an optimisation criterion z is denoted $z_I(S)$. The classic way to solve an optimisation problem without uncertainty is the predictive approach. An off-line algorithm builds an optimal solution $S_{I^{ref}}^*$ for the forecast instance I^{ref} , and guarantees an

optimal performance for this instance only, valued by $z_{I^{ref}}^*$. In practice, the real system is subject to perturbations such that the solution $S_{I^{ref}}^*$ is applied to the actual instance I that may be different from the forecast instance I^{ref} , and $S_{I^{ref}}^*$ may not be optimal and even admissible for I . In the most optimistic case (when S remains admissible for I), the actual performance $z_I(S_{I^{ref}}^*)$ can be "far" from the forecast performance $z_{I^{ref}}^*$, leading a costly resolution step to return a poor performance solution. Developing robustness features has appeared to be an efficient way to cope with uncertainties and inaccuracy even though researchers do not use the same definition depending on the application. Roughly speaking, robustness measures the solution ability to remain "good" despite variability of the data. What is exactly a so called good solution and the considered class of uncertainties is strongly application-dependent, and has led researchers to develop a large variety of approaches (See [2] for a commented survey of some approaches).

The goal of this paper is to present a generic robustness framework to deal with uncertainty in optimisation. In the next section, the framework is proposed. A robustness definition is given and five robustness issues, which are highlighted by the robustness definition, are detailed, discussed and compared to existing literature. Finally this framework is illustrated through an example in section 3.

1. A generic robustness framework for Operational Research and Decision-Aid

Many robustness definitions can be found in the literature as shown in [1]. The robustness definition given in [7] is used in this paper: the robustness is defined as the solution ability to guarantee a performance level L_λ , according to a robustness criterion λ , on a risk to be covered P (a set of instances or versions in this paper). The usual robustness criteria have been defined in [5] as follows:

1. Absolute robustness:

$$\lambda_1(S, z, P) = \min_{I \in P} z_I(S)$$

2. Robust deviation:

$$\lambda_2(S, z, P) = \max_{I \in P} \{z_I^* - z_I(S)\}$$

3. Relative robustness:

$$\lambda_3(S, z, P) = \max_{I \in P} \frac{z_I^* - z_I(S)}{z_I^*}$$

Definition 1. In [7], a solution S is said to be L_λ -robust on the set of instances P relatively to the robustness criterion λ if it satisfies the following inequality:

$$\lambda(S, z, P) \geq L_\lambda \quad (1)$$

Note that this definition generalises the p -robustness [8] and the β -robustness [4]. The L_λ -robustness highlights five robustness issues. These issues

are identified by their input data or decisions variables in the definition of L_λ -robustness. In the following of the paper, λ and z are assumed to be given.

2.a. *First robustness issue: stability analysis*

It is assumed that only a forecast instance I^{ref} , a performance level L_λ and a solution $S_{I^{ref}}$ are given. Then, the first robustness issue can be stated as follows: "knowing a solution $S_{I^{ref}}$ and given an expected performance level L_λ , what is the neighbourhood P of I^{ref} such that $\lambda(S_{I^{ref}}, z, P) \geq L_\lambda$?" This question can be seen as a *stability analysis problem*. Addressing this problem means finding the neighbourhood P in which the solution $S_{I^{ref}}$ remains stable in the sense of L_λ . This problem includes the first and second questions of *sensitivity analysis* defined in [6]:

1) In what neighbourhood P of I^{ref} does $S_{I^{ref}}^*$ remain optimal?

Using the L_λ -robustness formalism, $S = S_{I^{ref}}^*$, $\lambda = \lambda_3$ and $L_\lambda = 0\%$.

2) In what neighbourhood P of I^{ref} does $S_{I^{ref}}$ remains feasible, with acceptable performance?

Now S is supposed to be given, $\lambda = \lambda_3$ and the value of L_λ defines what is a so-called acceptable performance.

2.b. *Second robustness issue: sensitivity analysis*

It is assumed that only a forecast instance I^{ref} , a neighbourhood P of I^{ref} and a solution $S_{I^{ref}}$ are given. Then, the second robustness issue can be stated as follows: "knowing a solution $S_{I^{ref}}$ and assuming a neighbourhood P of I^{ref} , what is the performance level L_λ that is guaranteed by $S_{I^{ref}}$ such that $\lambda(S_{I^{ref}}, z, P) \geq L_\lambda$?" This question can be seen as a *sensitivity analysis problem* where the sensitivity is measured by L_λ on the set of instances P . This problem generalises the third question of *sensitivity analysis* defined in [5]:

3) Considering I a neighbour of I^{ref} , is the solution $S_{I^{ref}}^*$ feasible for I and then, what is its performance degradation?

Using the L_λ -robustness formalism, $S = S_{I^{ref}}^*$, $\lambda = \lambda_3$ and $P = \{I\}$.

In our framework, the instance I is generalised by the neighbourhood P , and the performance deviation is assessed in the worst case on the neighbourhood.

2.c. *Third robustness issue: finding a robust solution*

It is assumed that only a forecast instance I^{ref} , a neighbourhood P of I^{ref} and a performance level L_λ are given. Then, the third robustness issue can be stated as follows: "knowing a performance level L_λ that must be guaranteed on a given neighbourhood P of I^{ref} , what is a robust solution S such that $\lambda(S, z, P) \geq L_\lambda$?"

2.d. *Fourth robustness issue: maximising stability*

It is assumed that only a forecast instance I^{ref} and a performance level L_λ are given. Then, the fourth robustness issue can be stated as follows: "knowing a performance level L_λ that must be guaranteed, find a solution S that maximises the neighbourhood P of I^{ref} such that $\lambda(S, z, P) \geq L_\lambda$ ". To answer this question, the neighbourhood P covered by the solution S must be measurable. That means that the first issue must be addressed beforehand.

2.e. *Fifth robustness issue: minimising sensitivity*

It is assumed that only a forecast instance I^{ref} and a neighbourhood P of I^{ref} are given. Then, the fifth robustness issue can be stated as follows: "assuming a neighbourhood P of I^{ref} that must be covered, find a solution S that maximises the performance level L_λ such that $\lambda(S, z, P) \geq L_\lambda$ ". To answer this question, the performance level L_λ must be measurable. That means that the second robustness issue must be addressed beforehand.

2. Configuration under uncertainty of a power distribution network

The aim of this section is to illustrate the previously presented robustness framework towards a power distribution network.

In the context of electrical energy, the market deregulation is deeply modifying the conditions of control of the operational safety of the networks. This trend results in exploiting the networks closer to their physical limits. If the present operating system remains flexible insofar as the sources capacities are much higher than the customers load, this situation cannot last in a context of quick increase of the loads and of stabilisation of production capacities. The challenge of the next years will be to exploit the networks with an available power which tends to balance with the loads. In this context, taking into account uncertainties on the sources capacity and on the loads will induce challenging problems. These uncertainties are mainly due to new technologies such as renewable energies whose production remains very fluctuating. The aerogenerators can be disconnected from the network for safety reasons and can induce voltage drops. Moreover their production is very related to weather conditions. In the same way, the production of

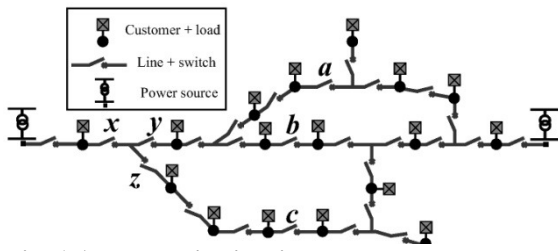


Fig. 1 A power distribution network

photovoltaic cells is dependant on the sunning. When voltage drop occurs, the only present solution to face these problems is to resort to load-shedding.

A power distribution network is composed of several power sources, electrical lines with their switch and customers with their load (see figure 1). The set of power sources must serve the set of customers with feeding their load and satisfying some electrotechnical constraints (like no connection between two sources). Configuring the network is choosing which power source serves which customer by setting the appropriate switches. Thus, the switches positions are the network configuration.

3.a. Problem modelling

The power distribution network is composed of m power sources that have to feed n loads (residential, commercial or industrial customers).

The power distribution network is modelled as a directed graph like in figure 1. However, the nodes have not all the same role in the network, and the set of nodes N is decomposed into three subsets: the customer nodes N_c , the junction nodes N_t and the source nodes N_s such that $N = N_c \cup N_t \cup N_s$. Each node i of N_c represents a customer with its load whereas N_s represents the set of sources with their capacity. N_t represents the set of junction nodes. The network of the figure 1 is modelled as the graph of the figure 2. This network is composed of $n = 17$ customers (1,...,17), of $t = 6$ junction nodes (18,...,23) and of $m = 2$ sources (24 and 25). The junction node 18 represents the junction point between switches which are denoted $x, y,$ and z on the network of figure 1. The junction nodes behave as customers without load, which are not to be necessarily served. The load of the customers and the junction points is represented by an integer $(n+t)$ -sized vector L such that $\forall k \in N_t, L_k = 0$. Each source j from N_s is power-limited by a capacity C_j . C is an integer m -sized vector. The network structure is

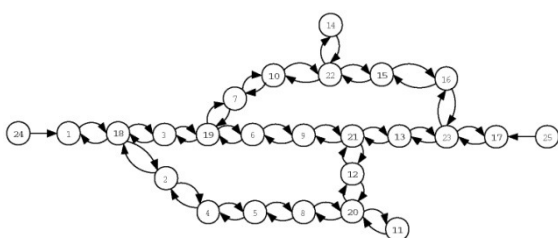


Fig. 2 Model of the network of the figure 1

represented by arcs between nodes of N . Each arc represents an electrical line with a switch. As flow direction is not pre-defined, each electrical line is represented by two arcs modelling the two possible orientations. However, a power flow cannot arrive to a source, thus there is no incoming arc for source nodes.

The distribution network configuration is modelled by a $(n+t+m)^2$ -binary matrix denoted S . $S_{i,j} = 1$ if the switch represented by the arc (i, j) is closed: the electrical current flows from i to j . $S_{i,j} = 0$ if the same switch is opened or if it does not exist: the electrical current does not flow from i to j .

We consider the power distribution network as a service production system. The most important constraint to be satisfied by the configuration is thus a service constraint: *each customer must be served and the service must cover the total load*.

The other constraints to be satisfied are electrotechnical ones which express the operation and safety conditions of the network. They can be defined as follows:

1. *Network radiality*: there cannot be cycle in the configuration and each node must have at most one predecessor.
2. *Disconnected sources*: an admissible configuration cannot contain any path connecting two sources.
3. *Power limitation of sources*: each source can provide only a limited quantity of power characterised by the capacity C .
4. *Power limitation of electrical lines*: each arc (i, j) is constrained by a capacity of maximum flow denoted $f_{i,j}^{\max}$.
5. *Constraints on voltage drops*: the depth of the solution forest is limited by an integer D_{\max} .

An admissible solution is a directed forest whose roots are the source nodes. Moreover, this forest must span all the customers nodes and satisfy the previous constraints. We can illustrate these requirements on the example of the figure 1 completed by the following data:

$$\left\{ \begin{array}{l} \forall i \in N_c, L_i = 1 \\ \forall k \in N_t, L_k = 0 \\ \forall j \in N_s, C_j = 10 \\ D_{\max} = 10 \\ \forall (i, j) \in N^2, f_{i,j}^{\max} = 10 \end{array} \right. \quad (2)$$

An admissible configuration for this example is the forest of the figure 3. This solution consists in opening the switches represented by the arcs $\{(5, 8); (8, 5)\}, \{(6, 9); (9, 6)\}$, and $\{(10, 22); (22, 10)\}$ (resp. the switches $a, b,$ and c of figure 1) while keeping closed the other switches: i.e., on the two arcs representing each other switch, one arc is used following the sense of the current.

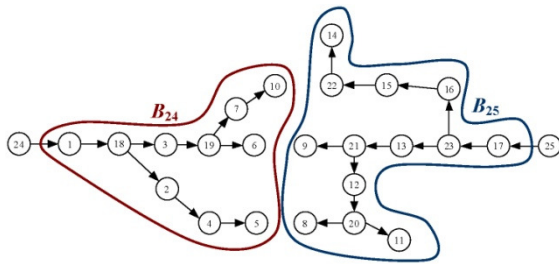


Fig. 3 An admissible configuration

3.b. *Perturbations: highlighting and robustness definition*

Naturally a lot of perturbations can occur in power distribution networks:

- risks: loss of an electrical line, loss of the functioning of a switch ...

Buried lines may be inopportunately cut by mechanical diggers on a construction site. For the case of overhead lines, an ordinary road crash can cause the fall of an electrical pylon and so the cable disconnection.

- uncertainties: load variations, source actual capacities ...

Actually loads vary during the days (cooking, television, washing machines), the weeks (evenings, week-ends) and the seasons (heaters, lights). Moreover, the profile of the load varies with the type of customer (residential, commercial, and industrial). Concerning the uncertainties on source capacities, they are mainly due to the introduction of the new technologies of energy production like renewable energies that remains very fluctuating as their efficiency strongly depends on weather conditions.

The challenge of the next years will be to exploit the networks with an available power which tends to balance with the loads. That is the reason why, to our point of view, taking into account uncertainties on the sources capacity (vector C) and on the loads (vector L) induce challenging problems. Thus in this paper, the network is regarded as being free from failures: i.e. lines and switches are always well functioning.

So, in the following of the paper, we will take into accounts the uncertainties on loads and on source capacities to propose robust configurations.

Uncertainties on load and on source capacities can lead to load-shedding. Meeting energy demand was a constraint in certain context; it becomes a performance to guarantee by taking into account the uncertainties on load and source capacities. Let an instance I be defined as follows:

$$I = \begin{cases} L_i & \forall i \in N_c \cup N_l \\ C_i & \forall i \in N_s \end{cases}$$

For an admissible configuration S , a service level can now be defined by:

$$z^{SL}(S, I) = \frac{\sum_{i \in N_c} \sum_{j \in N} S_{j,i}}{n} \quad (3)$$

where $\sum_{i \in N_c} \sum_{j \in N} S_{j,i}$ is the number of served customers.

To measure the global performance of a configuration, the service level in the worst case appears to be a relevant criterion. Thus, we can use the absolute robustness as a robustness measure. That means that:

$$\lambda_1(S, z^{SL}(S, I), P) = \min_{I \in P} \{z^{SL}(S, I)\}.$$

It measures the minimal rate of served customers when the source capacities C and the customer loads L vary in P . A configuration S is then said to be robust (in reference to the definition 1) if the value $\lambda_1(S, z^{SL}(S, I), P)$ is higher than a performance level L_λ (a minimal waited rate of served customers).

Now the five robustness problems defined in section 2 can be instantiated to our problem.

3.c. *Stability analysis*

In the addressed problem, stability analysis consists in finding the set P of instances I that can be covered by the configuration S without load-shedding ($L_\lambda = 100\%$).

A configuration S defines m sets B_j that partition $N_c \cup N_l$ where B_j is the set of loads served by the source j in the configuration S . In the example of figure 3, we have $B_{24} = \{1; 2; 3; 4; 5; 6; 7; 10; 18; 19\}$ and $B_{25} = \{8; 9; 11; 12; 13; 14; 15; 16; 17; 20; 21; 22; 23\}$.

It has been shown in [2] that P can be evaluated as follows:

$$P = \left\{ I \mid I_j - \sum_{i \in B_j} I_i \geq 0, \forall j \in N_s \right\} \quad (4)$$

This result is trivial and can be evaluated in polynomial time. This result only gives an implicit measure of P . If two configurations have to be compared by the stability analysis, an explicit value has to be proposed. If a forecast load L^{ref} and a nominal capacity C^{nom} can be given, then a forecast instance denoted I^{ref} can be defined as:

$$I^{ref} = \begin{cases} L_i^{ref} & \forall i \in N_c \\ 0 & \forall i \in N_l \\ C_i^{nom} & \forall i \in N_s \end{cases}$$

Now a load-shedding margin for each source j can be deduced from formula (4) and valued by:

$$M_j(S, I^{ref}) = \frac{C_j^{nom} - \sum_{i \in B_j} L_i^{ref}}{C_j^{nom}} \quad (5)$$

And finally P can be measured by the minimal load-shedding margin valued by:

$$M = \min_{j \in N_c} M_j(S, I^{ref}) \quad (6)$$

This value is proposed as an explicit measure of P .

In the example of figure 3, with the same values as in equation (2), $M_{24} = 20\%$ and $M_{25} = 10\%$.

3.d. Sensitivity analysis

For the considered example, sensitivity analysis means finding the rate of served customers L_λ by the configuration S in the worst case on a given set P of instances I .

It is assumed that the set P is defined as:

$$P = \{I \mid I_i \in [\alpha_i \ \beta_i]\} \quad (7)$$

where α_i and β_i are exactly known.

Considering the fact that:

1) the worst case is defined by I^{wc} :

$$I^{wc} = \begin{cases} \beta_i & i \in N_c \\ 0 & i \in N_t \\ \alpha_i & i \in N_s \end{cases}$$

2) the configuration S defines m sets B_j that partition

$$N_c \cup N_t,$$

it has been shown in [2] that solving this problem is equivalent as solving m knapsack-problems with precedence constraints (one knapsack for each source). As the precedence constraints are defined by a tree, this problem is only weakly NP-hard [3] and a lot of efficient approaches exist.

3.e. Finding a robust configuration

In the addressed problem, finding a robust configuration means finding a configuration that guarantees the service to customers ($L_\lambda = 100\%$) on a set P of instances I . This problem remains open.

3.f. Maximising stability

In the context of power distribution networks, maximising stability consists in finding the configuration S that maximises the set P of instances I without resorting to load-shedding ($L_\lambda = 100\%$).

Using the minimal load-shedding margin proposed at section 3.c. to measure the stability of a given configuration, the problem consists in finding the configuration that maximises the minimal load-shedding margin.

After having proved that this problem is strongly NP-hard, we have proposed in [2] a MILP formulation and a tabu search to solve this problem.

3.g. Minimising sensitivity

In the addressed problem, minimising sensitivity means finding the configuration that maximises the rate of served customers L_λ in the worst case on a given set P of instances I . This problem remains open.

3. Conclusion

In this paper, a generic robust framework has been proposed and illustrated through an example in which uncertainty is a major issue. This example shows that the five robustness problems that are highlighted by our

framework are relevant for real-life applications. This framework appears to be an interesting decision-aid scheme for managers having to take decisions under uncertainties.

- [1] Aloulou M.A., Kalaï R. and Vanderpooten D. Une nouvelle approche de robustesse : α -robustesse lexicographique. In *Newsletters of the European Working Group "Multiple Criteria Aid for Decisions"*, Series 3, number 11, pp 5-8, 2005.
- [2] Aubry A. *Optimisation pour la configuration robuste de systèmes de production de biens et de services*. Ph.D. Thesis, Institut Polytechnique de Grenoble, France, 2007.
- [3] Garey M. and Johnson D. *Computers and Intractability: a Guide to the Theory of NP-Completeness*. W.H. Freeman and Company, 1979.
- [4] Kalaï R. and Lamboray C. L' α -robustesse lexicographique : une relaxation de la β -robustesse. In *Annales du LAMSADE*, n°7, France, 2007.
- [5] Kouvelis P. and Yu G. *Robust Discrete Optimization and its Applications*. Kluwer Academic Publishers, 1997.
- [6] Mahjoub A., Moukrim A. and Sanlaville E. In *Flexibility and Robustness in Scheduling*, chapter 4. ISTE Ltd, 2008.
- [7] Rossi A. *Ordonnancement en milieu incertain, mise en œuvre d'une démarche robuste*. Ph.D. Thesis, Institut National Polytechnique de Grenoble, France, 2003.
- [8] Snyder L. Facility location under uncertainty: a review. *IIE Transactions*, 38(7): 547-564, 2006.