# Solving the Poisson equations by an interval difference method of the second order

Andrzej Marciniak[a, b, *]

[a]Poznan University of Technology, Institute of Computing Science, ul. Piotrowo 2, 60-965 Poznan, Poland
[b]Adam Mickiewicz University, Faculty of Mathematics and Computer Science, ul. Umultowska 87, 61-614 Poznan, Poland

**Abstract**

The paper deals with an interval difference method for solving the Poisson equation based on the conventional central-difference method. We present the interval method in full details and prove that the exact solution is included in the interval solution obtained. Some numerical results obtained in floating-point interval arithmetic are also presented.

*MSC:* 65N06, 65N15, 65G20, 65G99

*Keywords:* Poisson's equation, central-difference method, interval difference method for solving the Poisson equation

## 1. Introduction

In a number of our previous paper we have presented interval methods for solving the initial value problem in ordinary differential equations based on classical explicit and implicit Runge-Kutta methods and linear multistep methods (see e.g. [4], [5], [6]). All these works have been collected in [7]. It seems that similar techniques for constructing interval methods can be applied to various problems in partial differential equations.

In this paper we present an interval difference method of the second order for solving the Poisson equation with the Dirichlet boundary conditions. The interval method is based on the conventional central-difference method. It appears that at the mesh points the exact solution is within the interval solution obtained (see Section 4). Numerical experiments in floating-point interval arithmetic, presented in Section 5, confirm the theoretical justifications.

## 2. The Poisson equation

An elliptic partial-differential equation, known as the Poisson equation, is of the form (see e.g. [1] or [3])

$$\frac{\partial^2 u}{\partial x^2}(x, y) + \frac{\partial^2 u}{\partial y^2}(x, y) = f(x, y).$$

We assume that the function $f$ describes the input to the problem on a plane region $R$ whose boundary will be denoted by $\Gamma$.

Equations of this type arise naturally in the study of various time-independent problems such as:

- two-dimensional steady-state problems involving incompressible fluids,
- the potential energy of a point in a plane acted on by gravitational forces in the plane,
- the steady-state distribution of heat in a plane region.

* Corresponding address: Poznan University of Technology, Institute of Computing Science, ul. Piotrowo 2, 60-965 Poznan, Poland.
*E-mail address:* Andrzej.Marciniak@put.poznan.pl