Evolutionary algorithms for solving single and multiple-objective political redistricting problems: the case study of Poland

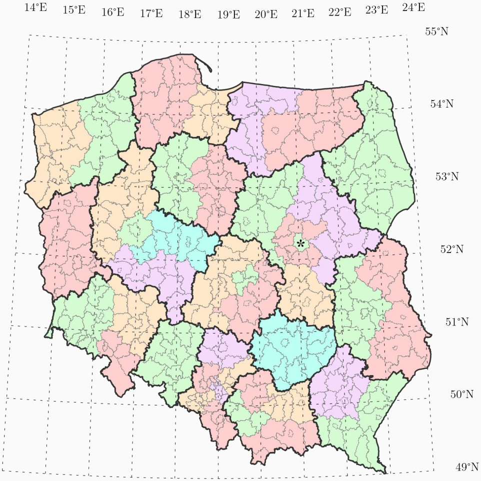
Michał Tomczyk, Miłosz Kadziński
Institute of Computing Science
Poznan University of Technology, Poland
www.cs.put.poznan.pl/mtomczyk/

Political redistricting problem

- **Problem:** Political Redistricting
- **Case study:** Elections to the lower house of Polish parliament (Sejm)
- Based on data from 2019

- The problem concerns determining the borders of electoral districts so that the final plan satisfies assumed criteria.
- There is no one universal set of criteria that should be used.
- We divide the area into districts to ensure each region has representatives in the Sejm.

M. Tomczyk, M. Kadziński, Evolutionary algorithms for solving single- and multiple-objective political redistricting problems: The case study of Poland, Applied Soft Computing 152, 111258, 2024



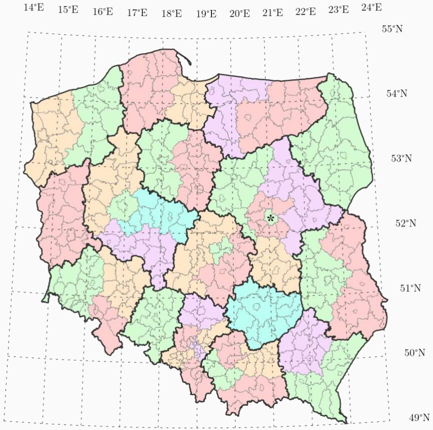
Political redistricting problem

1. There are 460 seats in the Sejm
2. There are 41 electoral districts
3. The number of mandates available in a district is proportional to:

   in-district population / population of Poland

4. Representatives are selected using the **D'Hondt rule in each district. For example,** there are 5 mandates available in a district; 3 parties, X, Y, and Z, attained the following numbers of votes: 10, 6, 1.

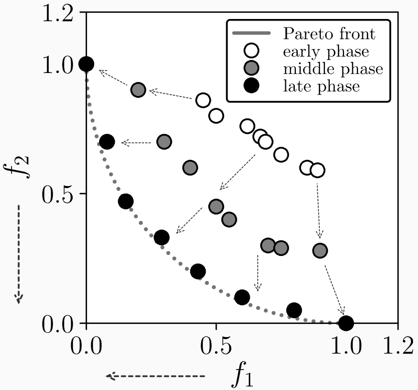
Divbisor	X	Y	Z
1	10	6	1
2	5	3	0.5
3	3.33	2	0.33



Motivation

Most works treat this problem as **single-objective**, with other optimization criteria employed as **hard constraints**. For multi-objective studies, **two-objective linear cases prevail**.

Why is it worth modeling this problem as a **multi-objective** one? Because the result of such optimization is of **much greater value to the decision-maker (broader perspective)**.



Contributions

- Most works treat this problem as **single-objective**, with other optimization criteria employed as **hard constraints**.
- For multi-objective studies, **two-objective linear cases prevail**.
- Typically, developed are optimizers that are **inefficient** in terms of **computational complexity** and the **qualities of constructed recommendations**.

We went far beyond these limitations:

- Advanced evolutionary optimizers were developed.
- The methods were applied to non-linear problem variants.
- The problem involved up to four objective functions.
- It is the first such study devoted to Poland.
- The proposed methods introduce various algorithmic developments that make the optimizers highly efficient.

Considered objective functions

Optimization criteria assumed in the work:

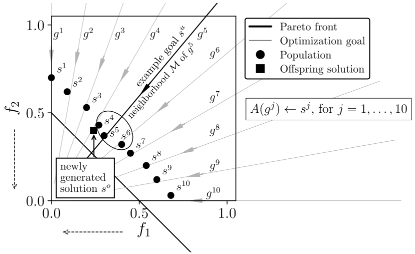
1. Population equality in districts (deviation from ideal value; min)
2. Compactness of districts (deviation from ideal value; min)
3. Dissimilarity with the current plan (min)
4. The number of mandates attainable by a party (max)

+ constrains that are unique to the country/electoral system

Developed algorithms

Top-level (conducting evolutionary search)

- MOEA/D-based (decomposition-based) evolutionary optimizer coupled with a dedicated clustering-based local search procedure
- Performs similarly to the baseline method, except that the evaluation phase is fed with MOEA/D's scalarizing function.



Bottom level (solution evaluation): A three-level solution representation was developed

Genotype

Centroids: Each represents one district *(here: counties)*

→ **Decision vector**

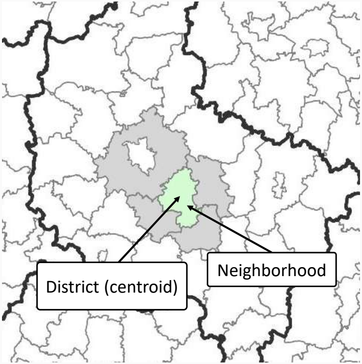
Redistricting plan

→ **Objective vector**

Clustering driven by a scalarizing function

Evaluation

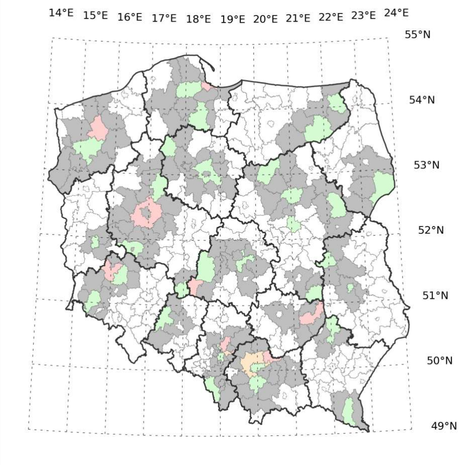
Developed algorithms



Clustering:

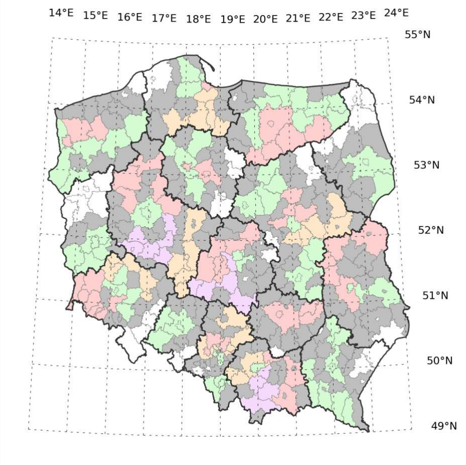
- Initial conditions: centroids (one per each district).
- The clusters are iteratively expanded.
- In each iteration, one neighboring country is added to a district represented by a centroid.
- The selection of a country to be included is driven via an appointed scalarizing function
- The implemented procedure imposes low computational burden.

Developed algorithms



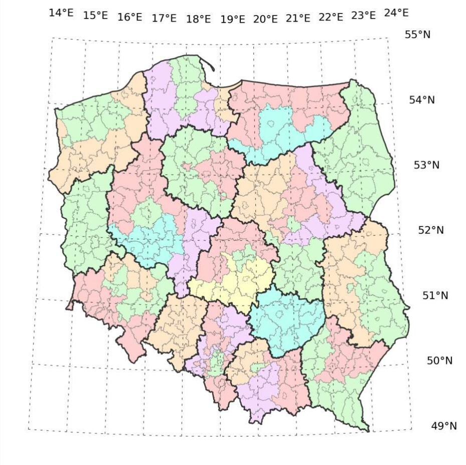
Iteration = 0

Developed algorithms



Iteration = 130

Developed algorithms



Iteration = 339

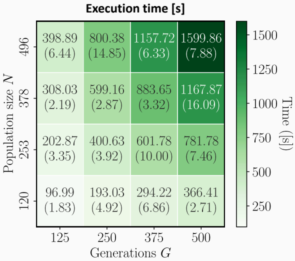
Experimental verification

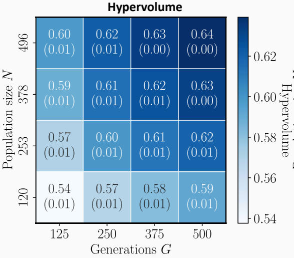
1. Performance verification in single-objective scenarios.
2. Performance verification in multi-objective scenarios:
   I. Experiments involving apolitical criteria (population equality, district compactness, dissimilarity).
   II. Experiments involving the political criterium (the number of attainable mandates):
      a) Analysis for the default number of districts: 41
      b) Analysis for different numbers of districts to construct.

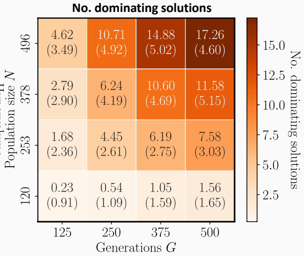
Apolitical experiments

Sensitivity analysis

- An extensive sensitivity analysis was conducted: 16 combinations of population sizes and generation limits were used.
- Execution time, hypervolume, and a number of solutions that dominate the current plan, given population quality and district compactness, were measured.
- The results are averaged over 100 runs.

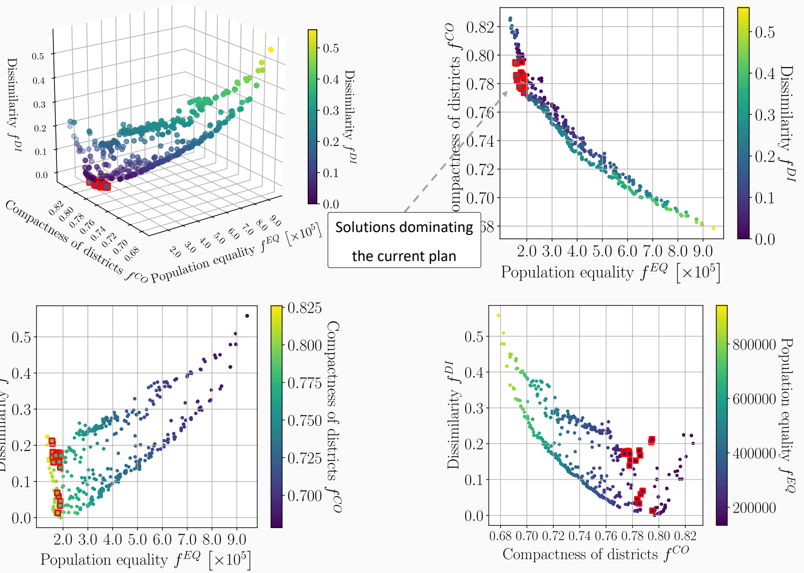






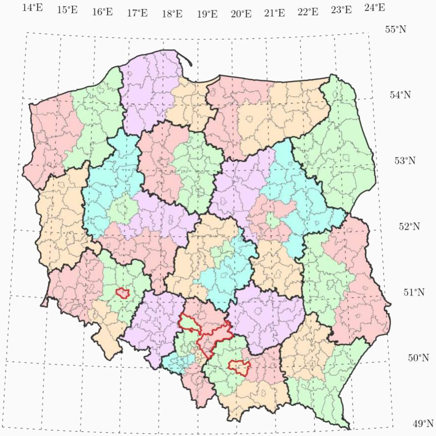
Apolitical experiments

Example final population

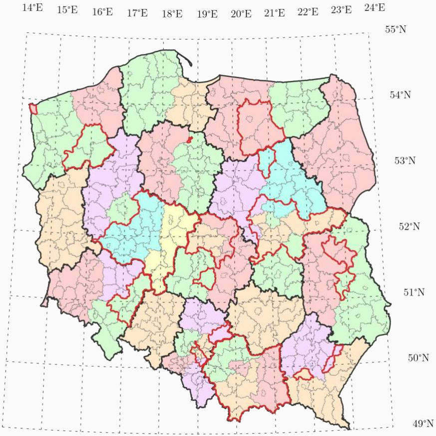


Apolitical experiments

Example redistricting plans

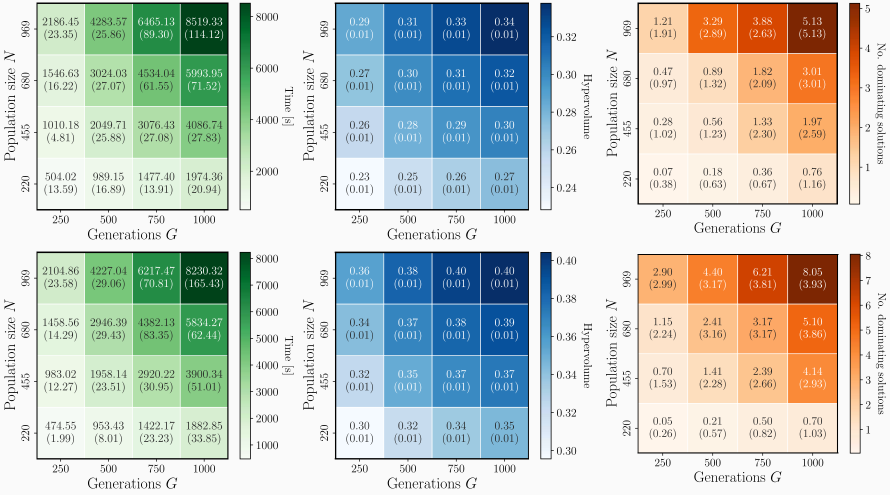


Dissimilarity:	1%
Improvement on "population equality":	5.9%
Improvement on "district compactness":	1.3%



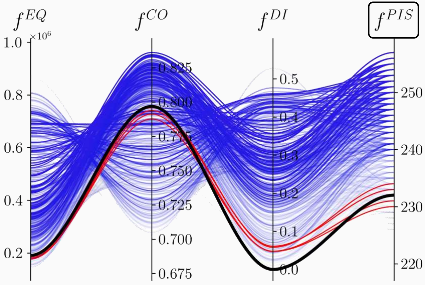
Dissimilarity:	18%
Improvement on "population equality":	7.4%
Improvement on "district compactness":	2.5%

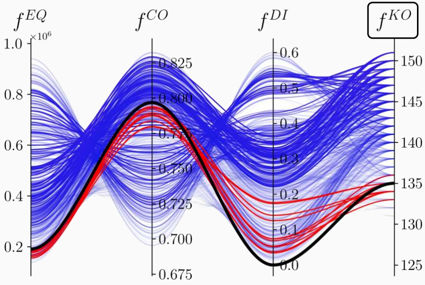
Two additional scenarios regarding the fourth objective were considered: maximization of mandates attainable by PIS and KO.



Political experiments

Example final populations

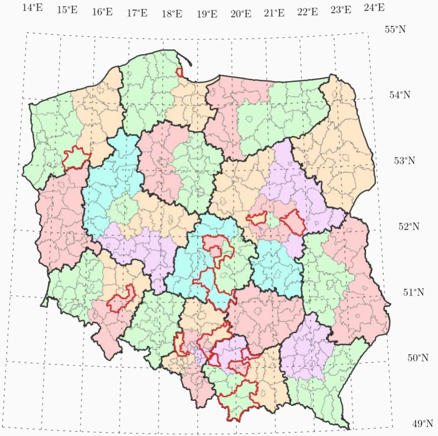




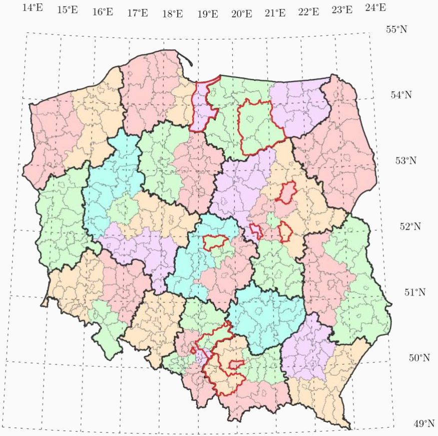
- Example final populations generated for the two scenarios.
- Black line: evaluation of the current plan
- Red lines: solutions dominating the current plan according to population equality and district compactness.
- The more mandates a party aims to get, the more significant changes in the current plan are required.
- **Observation**: There exists dominating solutions that additionally improve the number of attainable mandates.

Political experiments

Example redistricting plans



Dissimilarity:	1%
Improvement on “population equality”:	1.7%
Improvement on “district compactness”:	1.3%
Additional mandates for PIS	2



Dissimilarity:	1%
Improvement on “population equality”:	5.5%
Improvement on “district compactness”:	1.3%
Additional mandates for KO	1

Motivation: The D'Hondt's method favors parties that attained many votes. The number of attainable mandates should increase relatively fast with the number of districts.

Experiments : The following numbers of districts were considered: 41 (current), 60, 80, 100.

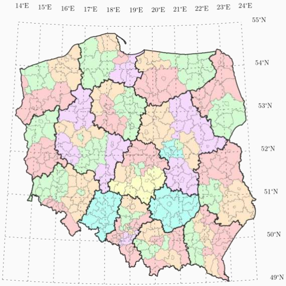
Simple summary: average and standard deviation of the attainable number of mandates yielded by all solutions in example final populations.

No. districts	PIS	KO
41	238.67 ± 0.24	138.83 ± 0.17
60	254.47 ± 0.26	144.89 ± 0.22
80	267.15 ± 0.28	147.79 ± 0.24
100	280.06 ± 0.34	151.06 ± 0.28

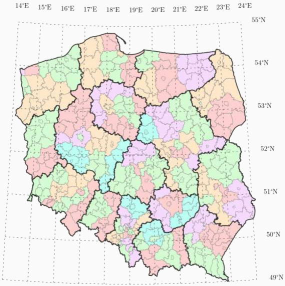
Political experiments

Analysis concerning different numbers of districts to construct

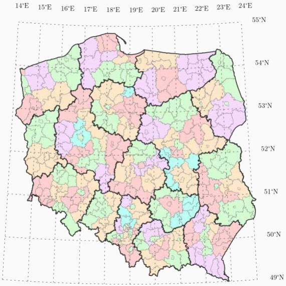
Case for PIS



No. districts:	60
No. mandates (PIS):	251



No. districts:	80
No. mandates (PIS):	261

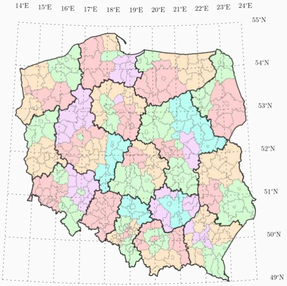


No. districts:	100
No. mandates (PIS):	272

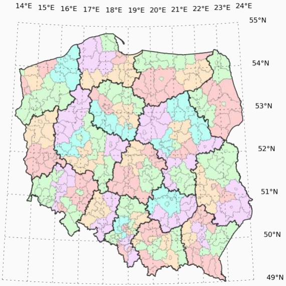
Political experiments

Analysis concerning different numbers of districts to construct

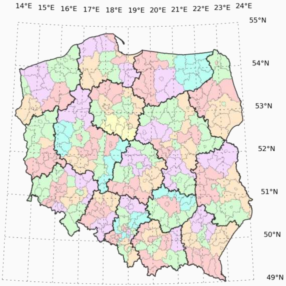
Case for KO



No. districts:	60
No. mandates (KO):	139



No. districts:	80
No. mandates (KO):	141



No. districts:	100
No. mandates (KO):	143

Summary

Summary:

- Advanced evolutionary optimizers for solving the political redistricting problem were proposed (at the time of project realization, only 4 such works were available !!!).
- The introduced methods are founded on dedicated, novel algorithmic advances.
- The work concerned a simultaneous optimization of even four linear/non-linear objective functions.
- The proposed methods have excellent computational complexity.
- The methods can construct satisfactory approximations of the Pareto fronts, which is significant to the decision maker (post-analysis).

Future development plans:

- Further improvement of computational complexity.
- Performing analysis based on future election outcomes (not on past data).
- Adapting the methods to other countries and election systems.
- Determining how different counties contribute to final plans.
- Analysis that involves a simultaneous optimization of the number of mandates attainable by more than one party.