Preference-based cone contraction algorithms for interactive evolutionary multiple objective optimization

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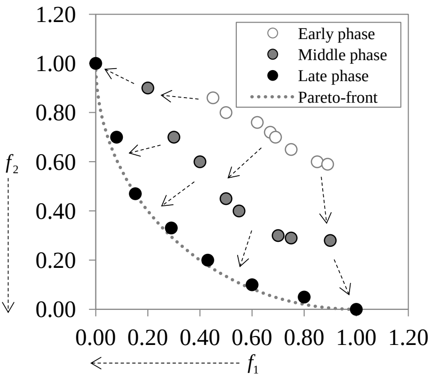
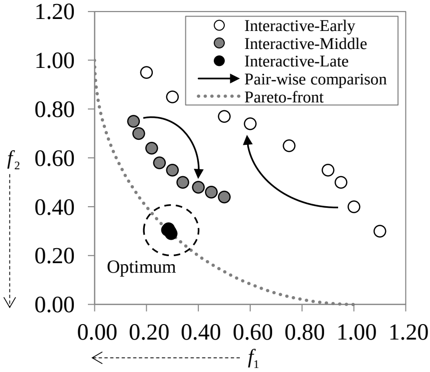
MCDA
MULTIPLE CRITERIA
DECISION AIDING

Evolutionary Multiple-objective Optimization (EMO)



Evolutionary Algorithms for MOO
Mimic the process of naturall evolution to solve optimization problems
Advantages of EMO: – can be applied to problems having complex fitness landscapes – the computational complexity can be reduced since solutions are optimized in an interrelated manner

Preference-based EMOAs: Motivation

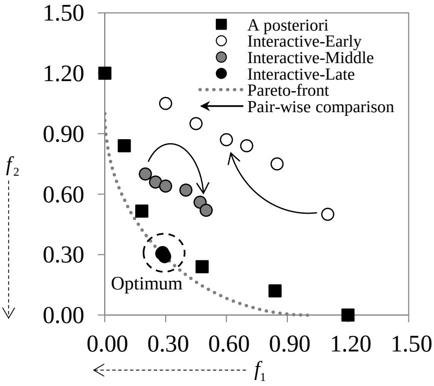


Preference-based EMOAs

Observation: it is not practical to approximate an entire PF since the DM is interested in finding only relevant solutions to him or her

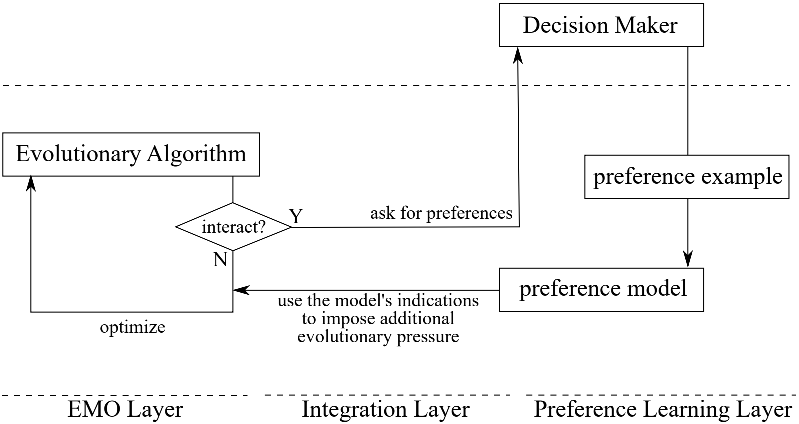
Incorporation of the DM's preferences into EMOA is oriented towards finding a region in the Pareto front, being highly preferred to the DM.

Preference vs. non preference-based EMOAs

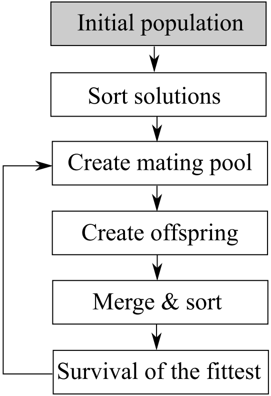


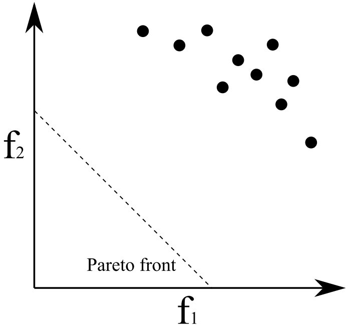
Advantages
The preference information can be used to **constraint** the search space, thereby reducing the complexity of the problem.
The preference information can be used to impose an additional selection pressure, driving population of solutions towards **region in the PF, being highly preferred to the DM**

Scheme of an interactive EMOA



Scheme of a simple EA for MOO





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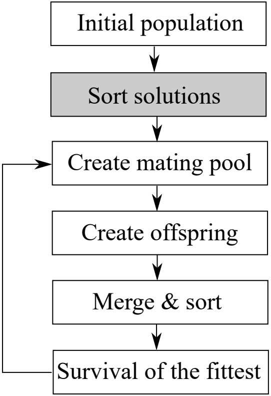
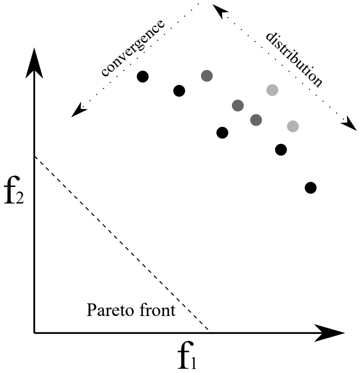
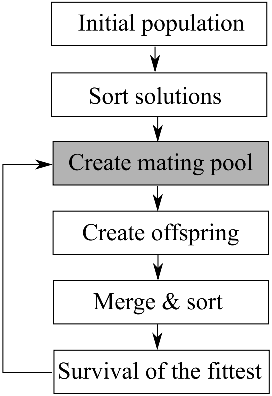
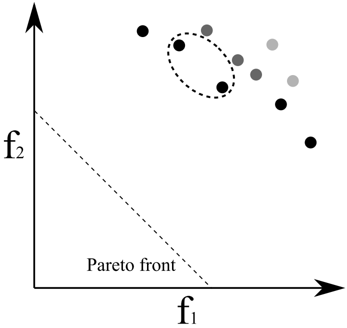




Scheme of a simple EA for MOO





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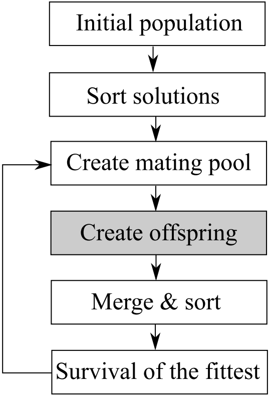
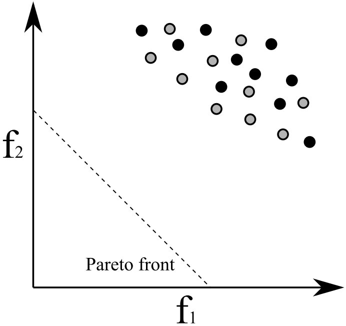
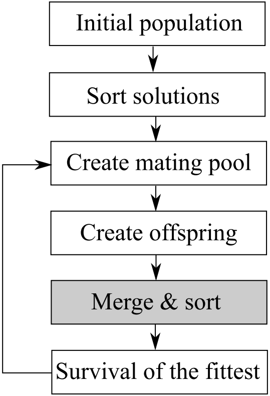
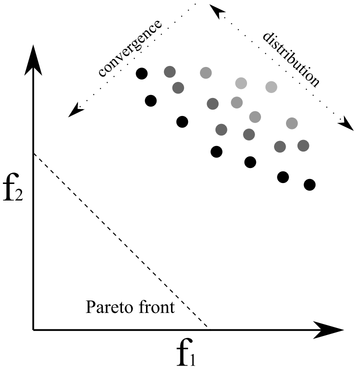


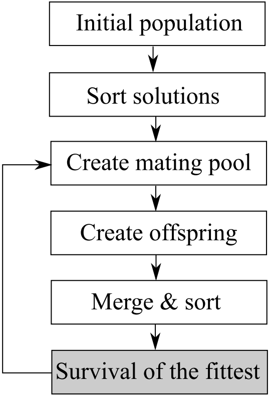


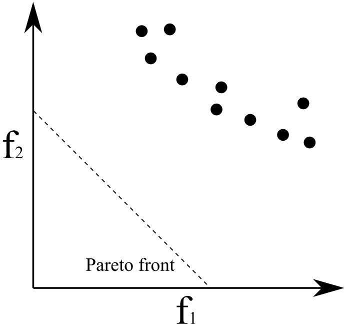
Scheme of a simple EA for MOO



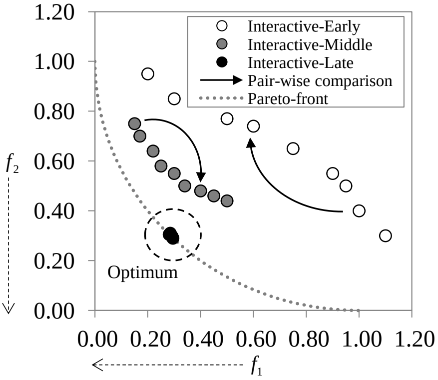


Scheme of a simple EA for MOO





Proposed algorithms: CDEMO and DCEMO



Properties of CDEMO and DCEMO
▷ Interactive
▷ Based on pairwise comparisons
▷ Use an Achievement Scalarizing Function to model DM's preferences
▷ Use preference cones to indicate preferred region in the objective space

Proposed algorithms: CDEMO and DCEMO

Preference model

$$d(s, w, z) = \max_{i=1,\ldots,M} \{w_i |s_i - z_i|\} + \rho \sum_{i=1,\ldots,M} (w_i |s_i - z_i|),$$

where w is an objective weight vector, z is a reference point, and ρ is an augmentation multiplier.

Form of DM's preferences

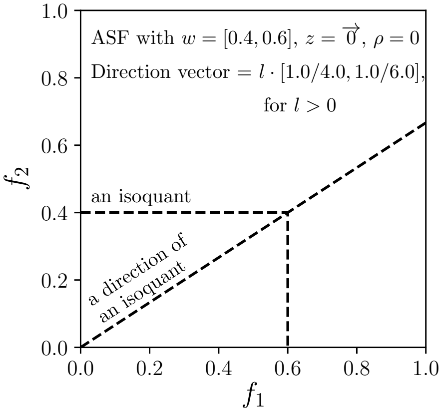
Pairwise comparisons of solutions: $s^a \succ s^b$

Space of compatible model instances

Objective weight vector w is compatible if $\forall_{s^a \succ s^b \in \mathcal{H}} d(s^a, w, z) < d(s^b, w, z)$

The following question arises:
how to exploit the preference model in order to bias the evolutionary search?

Achievement Scalarizing Function – Isoquants

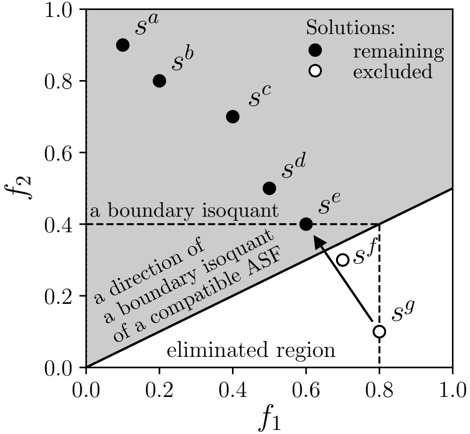


ASF – Isoquant & its direction

Isoquant – a curve representing a set of points being equally evaluated according to some function (e.g., ASF).

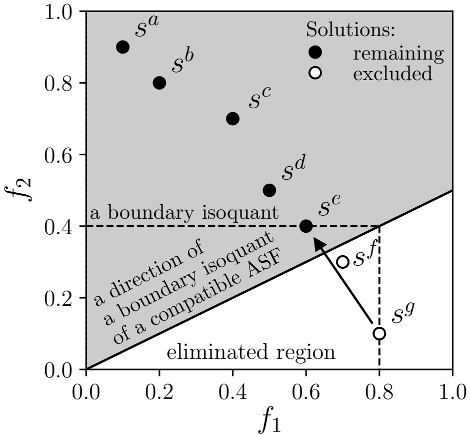
Direciton of isoquant – a line passing through a reference point z and a corner of the isoquant.

Achievement Scalarizing Function – Isoquants & Eliminated Region



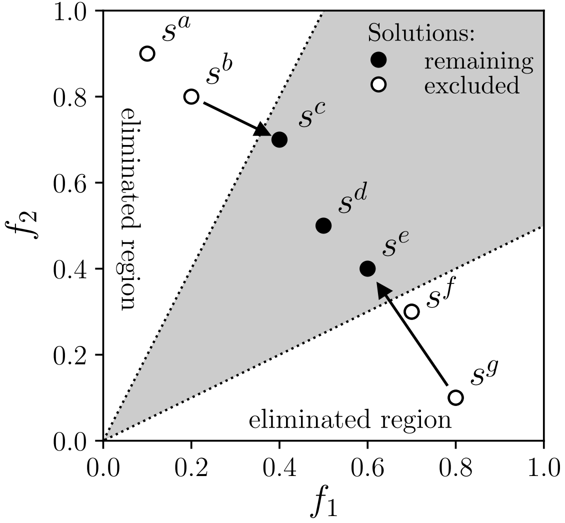
Boundary isoquant
A boundary isoquant evaluates two solutions compared by the DM equally.
Graphically, a direction of a boundary isoquant separates the eliminated region and a region preferred by the DM

Achievement Scalarizing Function – Isoquants & Eliminated Region

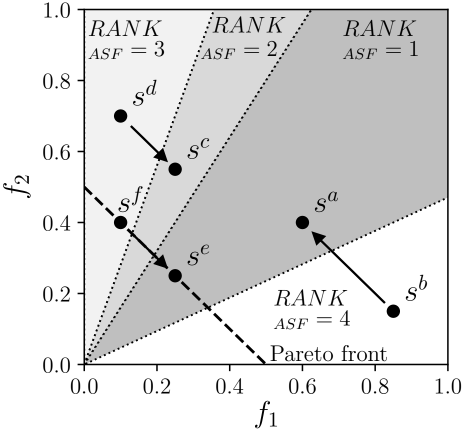


How to check if some solution is in the region of interest? (example for s^c)
1) Find a line passing through s^c
2) Check if an ASF parametrized with w is in agreement with the DM's pair-wise comparison, i.e., $d(s^e, w, z) < d(s^g, w, z)$

Achievement Scalarizing Function – Isoquants & Eliminated Region



The proposed RANK procedure based on preference cones



RANK procedure based on preference cones

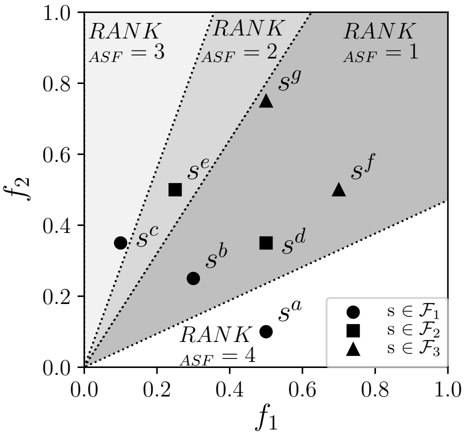
Imagine that the DM provided thee pairwise comparisons in the following order:

1) $s^a \succ s^b$ (the oldest example)
2) $s^c \succ s^d$ (the middle example)
3) $s^e \succ s^f$ (the newest example)

For a given solution s, *RANK* equals:

- ▷ *RANK* $= 1$ if s is in each preference cone;
- ▷ *RANK* $= 2$ if s is in each preference cone, except the newest one;
- ▷ *RANK* $= 3$ if s is in each preference cone, except the two newest ones;
- ▷ *RANK* $= 4$ ...

RANK & Non-dominated sorting



Non-dominated sorting

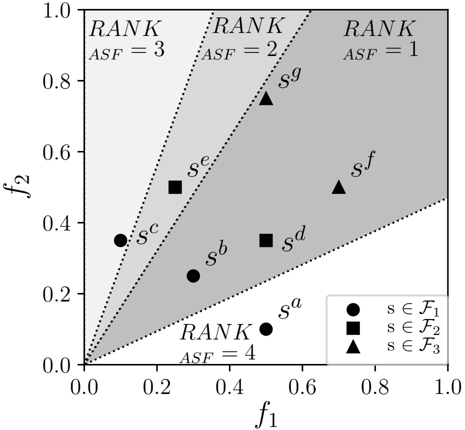
Partitions solutions into non-dominated fronts

2) The first front $\mathcal{F}_1$ contains all non-dominated solutions in population P

2) The second front $\mathcal{F}_2$ contains all non dominated solutions in $P \setminus \mathcal{F}_2$

2) The third front $\mathcal{F}_3$ contains...

CDEMO & DCEMO variants



CDEMO

1) Primary: RANK
2) Secondary: Non-dominated sorting

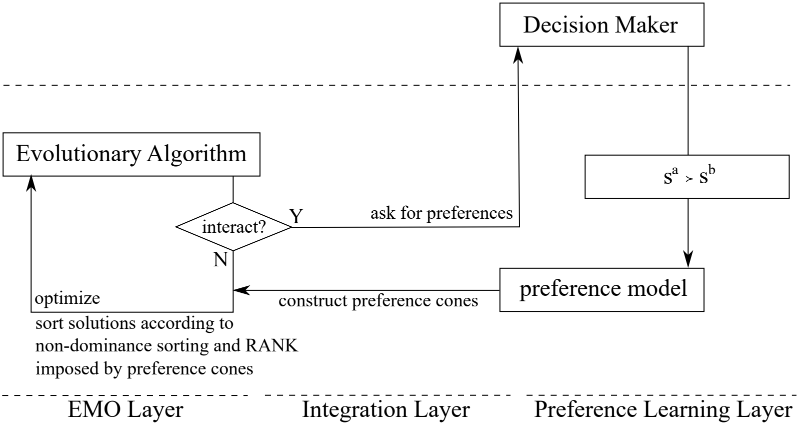
Example order: $(s^b) \succ (s^d) \succ (s^f, s^g) \succ (s^e) \succ (s^c) \succ (s^a)$

DCEMO

1) Primary: Non-dominated sorting
2) Secondary: RANK

Example order: $(s^b) \succ (s^c) \succ (s^a) \succ (s^d) \succ (s^e) \succ (s^f, s^g)$

Scheme of CDEMO & DCEMO



Novel visualization method for EMO: Trace for Evolutionary Multiple-objective Optimization (TEMO)

Drawbacks of traditional visualization techniques

1) Illustrate solutions constructed in the final generation, or after few pre-selected numbers of generation

2) Concern only single run of the method

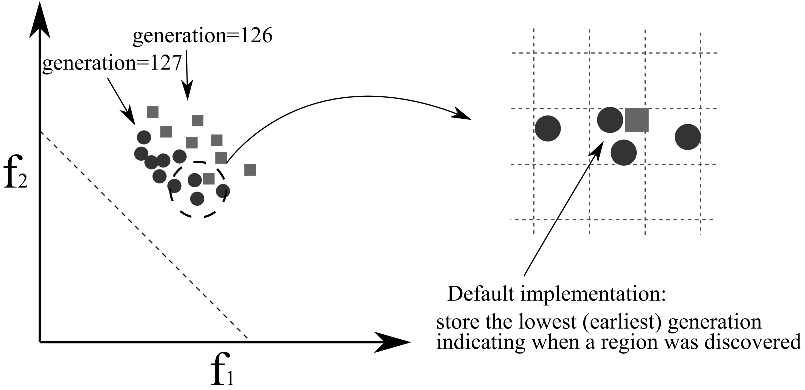
Features of TEMO

1) Illustrates solutions constructed throughout whole evolutionary run

2) Aggregates populations constructed for many independent method's runs

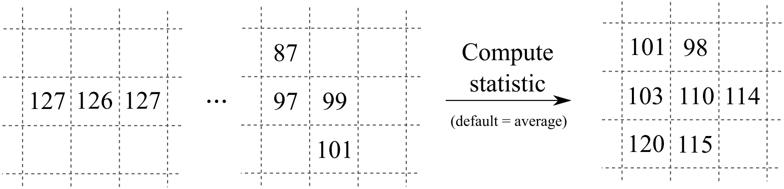
TEMO – Basic concept

Statistics derived from a single run



TEMO – Basic concept

Averaging many independent runs



Having derived statistics, TEMO uses a pre-defined color pattern to illustrate the average performance of the analyzed method.

Experimental Setting

1) To simulate the answers of a real-world DM, we constructed an artificial DM modeled with ASF incorporating some pre-defined objective weight vector w^{DM}.

2) During the preference elicitation phase, two distinct non-dominated solutions were selected form the population and compared as imposed by the preference function of the artificial DM.

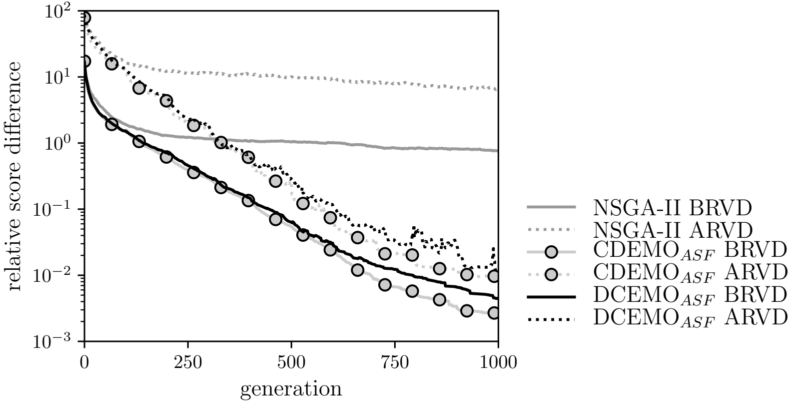
3) The interactions were performed 10 times, at regular intervals.

4) The methods were run 100 times. For each run, the artificial DM incorporated different objective weight vector w^{DM}.

5) For each artificial DM and each benchmark problem, before running the experiments, we found an optimal solution. This solution helped us to assess the performance of the EMOA. Specifically, we computed a relative score difference between the optimum and (BRSD) the best solution in the population and (ARSD), on average, each solution in the population.

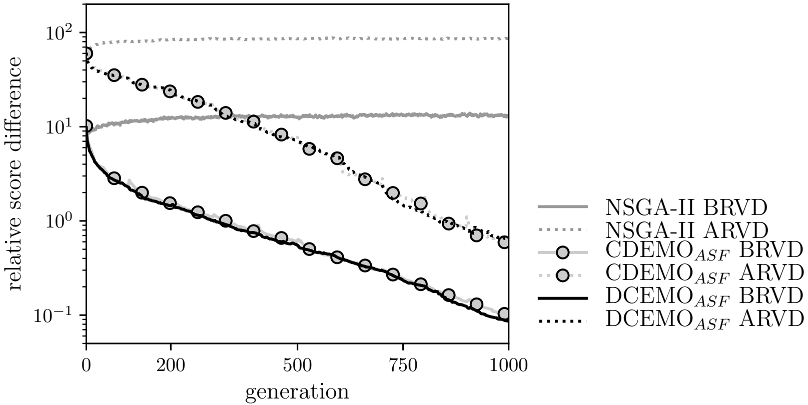
Convergence plots for CDEMO and DCEMO

DTLZ1 with 3 objectives



Convergence plots for CDEMO and DCEMO

DTLZ1 with 5 objectives



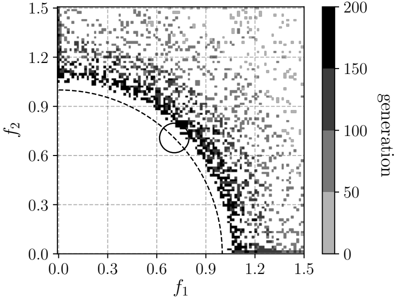
Example numerical results

Average BRSD (first row) and ARSD (second row) for the populations generated in the last iteration by five algorithms for the DTLZ2 with $M = 2, \ldots 5$ objectives. Average ranks $\overline{R}$ attained by the algorithms according to either BRSD or ARSD.

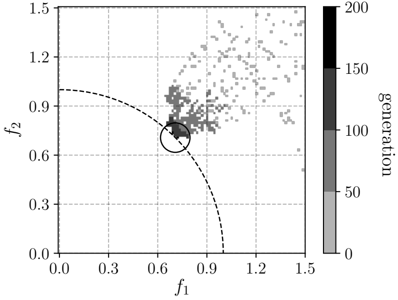
		NSGA-II			CDEMO_{ASF}			DCEMO_{ASF}		
	M	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$
DTLZ2	2	0.0065	0.0066	2.98	0.0002	0.0003	1.35	0.0002	0.0008	1.67
		0.3588	0.1520	3.00	0.0044	0.0103	1.41	0.0061	0.0123	1.59
	3	0.0154	0.0127	2.66	0.0108	0.0237	1.72	0.0105	0.0234	1.62
		0.3261	0.0879	3.00	0.0282	0.0334	1.51	0.0312	0.0371	1.49
	4	0.0515	0.0349	2.62	0.0118	0.0231	1.72	0.0132	0.0238	1.66
		0.5788	0.1361	3.00	0.0427	0.0322	1.52	0.0431	0.0348	1.48
	5	0.0722	0.0482	2.86	0.0112	0.0171	1.58	0.0116	0.0175	1.56
		0.6987	0.1451	3.00	0.0503	0.0371	1.54	0.0424	0.0298	1.46

Application of TEMO

NSGA-II and CDEMO run for DTLZ2 and $w^{DM} = [0.5, 0.5]$



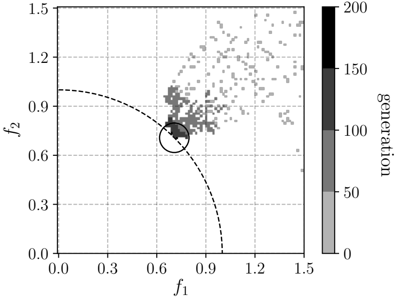
NSGA-II



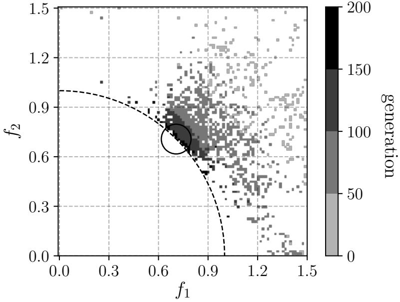
CDEMO

Application of TEMO

CDEMO and DCEMO run for DTLZ2 and $w^{DM} = [0.5, 0.5]$



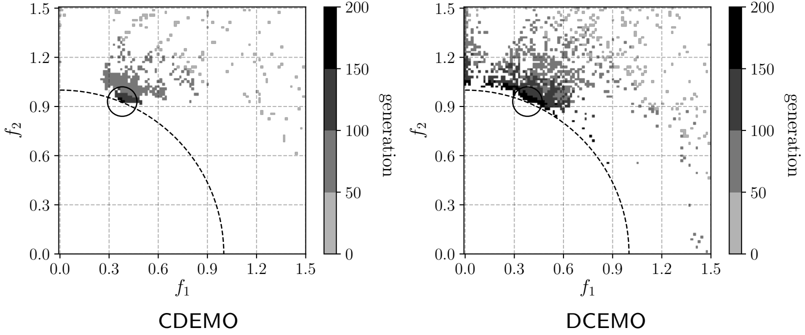
CDEMO



DCEMO

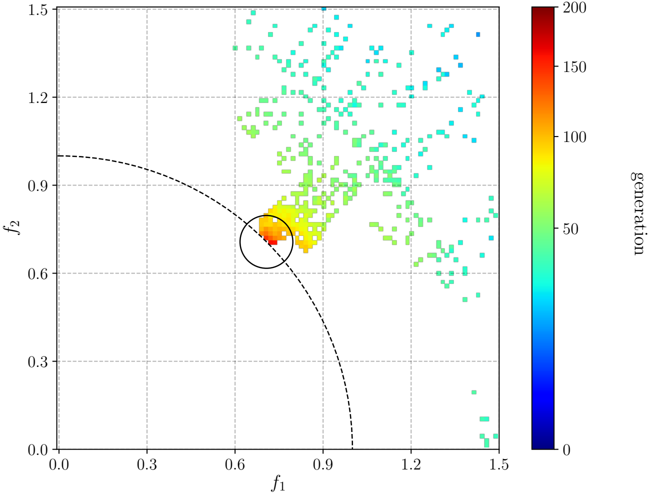
Application of TEMO

CDEMO and DCEMO run for DTLZ2 and $w^{DM} = [0.7, 0.3]$



Application of TEMO

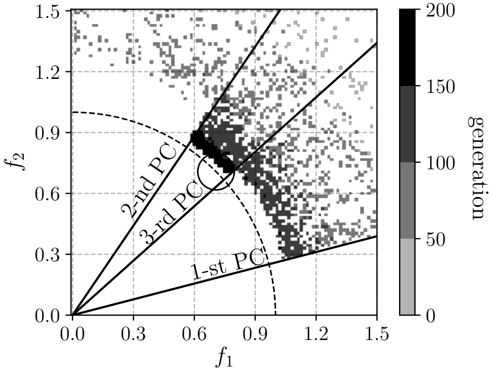
TEMO with different coloring pattern



CDEMO for DTLZ2 with $w^{DM} = [0.5, 0.5]$

Application of TEMO

Illustrating the preference cones



The simulation used 3 pre-defined pairwise comparisons for the preference elicitation phase:

50^{th} generation: $s^{1a} = [1.20; 0.35] \succ s^{1b} = [1.35, 0.20]$.

100^{th} generation: $s^{2a} = [0.85; 1.10] \succ s^{2b} = [0.70, 1.25]$.

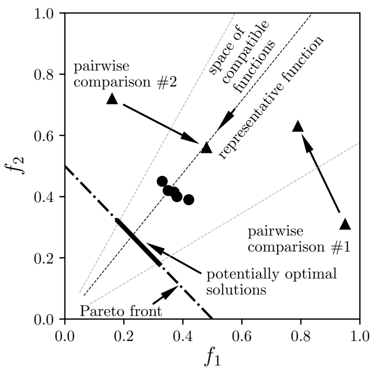
150^{th} generation: $s^{3a} = [0.85; 0.85] \succ s^{3b} = [1.35, 0.20]$.

Comparison with different methods

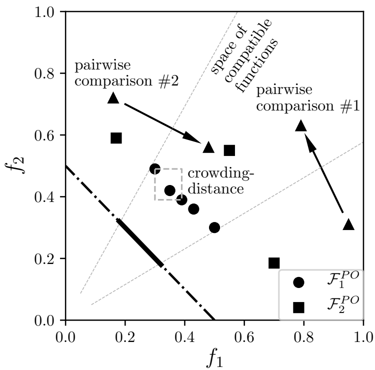
NEMO-0	NEMO-II
▷ Interactive	▷ Interactive
▷ Based of pairwise comparisons	▷ Based of pairwise comparisons
▷ Generational evolutionary base	▷ Generational evolutionary base
▷ Use an Additive Value Function (AVF) to model DM's preferences	▷ Use an Additive Value Function (AVF) to model DM's preferences
▷ Sort solutions according to (1) non-dominated sorting (2) representative additive value function	▷ Sort solutions according to (1) fronts of potential optimality (2) crowding-distance

Comparison with different methods

NEMO-0 method



NEMO-II method



Comparison with different methods

Average BRSD (first row) and ARSD (second row) for the populations generated in the last iteration by different methods for the DTLZ2 with $M = 2, \dots 5$ objectives. Average ranks $\overline{R}$ attained by the algorithms according to either BRSD or ARSD.

Artificial DM modeled with a Weighted Sum

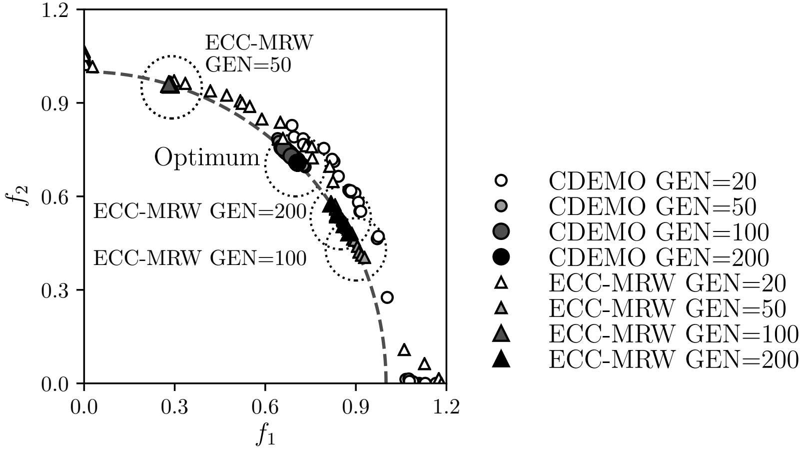
		CDEMO_{ASF}			$\mathrm{ECC\text{-}MRW}_{ASF}$			NEMO-II $CP = 2$			NEMO-0 $CP = 2$		
	M	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$
DTLZ2C	2	0.0016	0.0039	2.70	0.0100	0.0297	3.46	0.0000	0.0001	1.11	0.0040	0.0110	2.73
	3	0.0136	0.0209	2.47	0.0737	0.0985	3.39	0.0032	0.0064	1.40	0.0392	0.0627	2.74
	4	0.0487	0.0504	2.36	0.1483	0.1523	3.37	0.0215	0.0192	1.71	0.1019	0.1308	2.56
	5	0.0960	0.0913	2.46	0.1915	0.1616	3.14	0.0569	0.0379	1.84	0.1446	0.1656	2.56

Artificial DM modeled with a Chebyshev Function

		CDEMO_{ASF}			$\mathrm{ECC\text{-}MRW}_{ASF}$			NEMO-II $CP = 5$			NEMO-0 $CP = 5$		
	M	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$
DTLZ2	2	0.0044	0.0103	1.28	0.0641	0.0870	2.32	0.3207	0.1404	3.91	0.0856	0.1136	2.49
	3	0.0282	0.0334	1.60	0.0690	0.0893	2.17	0.2988	0.0917	3.89	0.0899	0.1043	2.34
	4	0.0427	0.0322	1.75	0.0824	0.0929	2.17	0.5647	0.1538	4.00	0.0785	0.0828	2.08
	5	0.0503	0.0371	1.92	0.0810	0.0981	2.15	0.6965	0.1422	4.00	0.0636	0.0705	1.93

CDEMO vs. ECC-MRW

DTLZ2; DM modeled with a Chebyshef function incorporating $w^{DM} = [0.5, 0.5]$.

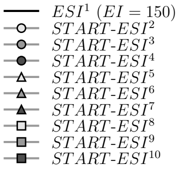


Interaction patterns

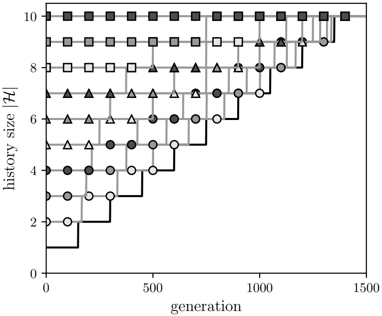
$START\text{-}ESI^{STR}$ pattern

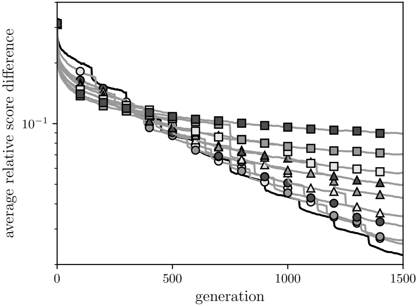
1) STR interactions are performed in the first generation;

2) the remaining $10 - STR$ interactions are evenly distributed throughout optimization.



WFG1 with $M = 3$ objectives





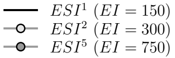
Interaction patterns

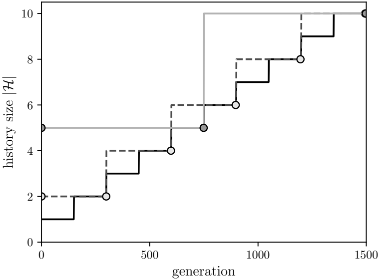
ESI^{PCS} pattern

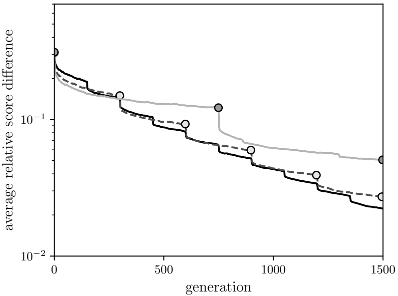
1) interactions are evenly distributed throughout optimization.

2) during each interaction, *PCS* pairwise comparisons are provided by the *DM*.

WFG1 with $M = 3$ objectives



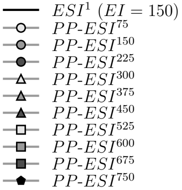




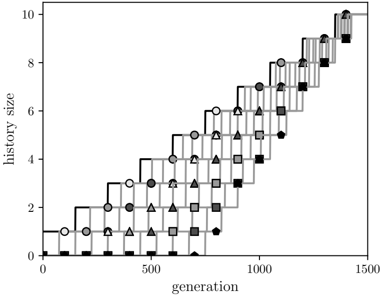
Interaction patterns

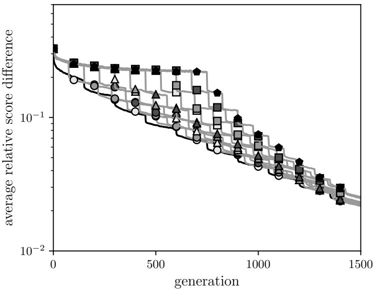
$PP\text{-}ESI^{GEN}$ pattern

1) the preference elicitation is postponed until *GEN* generation
2) after *GEN* generation, interactions are evenly distributed throughout evolutionary search.



WFG1 with $M = 3$ objectives





Interaction patterns

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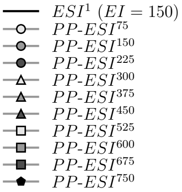
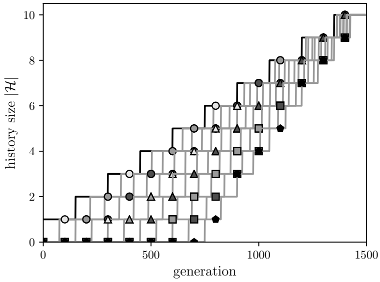
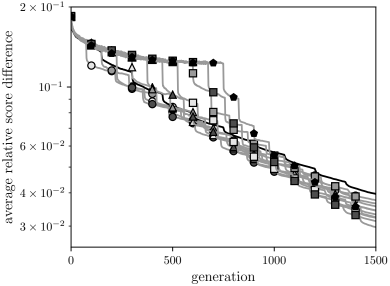


WFG1 with $M = 5$ objectives

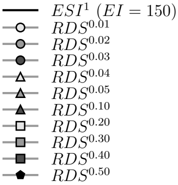




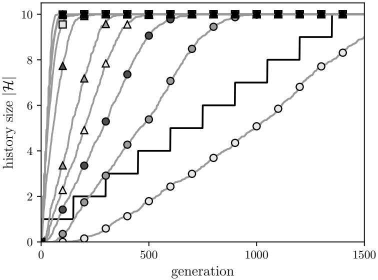
Interaction patterns

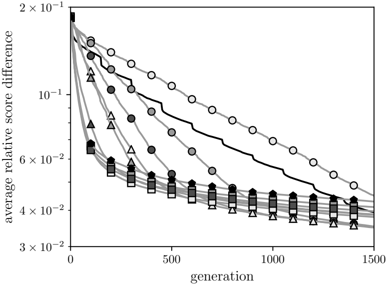
RDS^{th} pattern

Preference elicitation is triggered when the ratio of parent solutions being dominated by at least one offspring solution is lesser than a threshold *th*.



WFG1 with $M = 5$ objectives





Conclusions

▷ We proposed a novel family of interactive evolutionary algorithms – CDEMO and DCEMO methods – for multiple-objective optimization.

▷ CDEMO and DCEMO are based on pairwise comparisons and, throughout the evolutionary search, use the preference information to construct preference cones indicating preferred region in the objective space.

▷ CDEMO and DCEMO proved to perform well on a large set of benchmark problems involving from 2 to 5 objectives.

▷ CDEMO and DCEMO proved to perform better than selected state-of-the-art interactive algorithms based on pairwise comparisons, when the DM's decision policy is compatible with the preference model incorporated by the proposed methods.

▷ We showed that the consistency between the preference model used to model the DM's preferences and the model incorporated by the method is essential for finding the best possible recommendation.

▷ We proposed a novel visualization technique, called TEMO, which illustrates average, i.e., expected, evolutionary run performed by an EMOA.

▷ We proposed several interaction patterns – static and dynamic – for interactive EMOAs. The results indicate that, usually, the interactions should be evenly distributed throughout optimization. However, due to problem's characteristics, sometimes the interactions should be distributed in another way in order to improve the results.