Decomposition-based interactive evolutionary algorithm for multiple objective optimization

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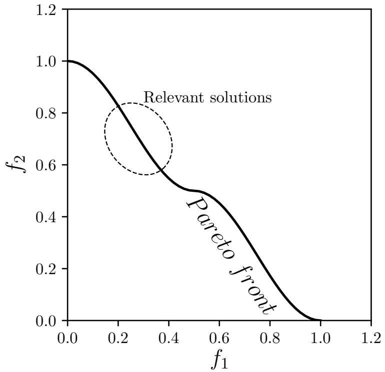
Preference-based EMOAs: Motivation

Incorporation of the DM's preferences into EMOA is oriented toward construction and/or selection of the DM's most preferred solution.



Motivation

The preference information can be used to **constraint** the search space, thereby reducing the complexity of the problem.

The preference information can be used to impose an additional selection pressure, driving population of solutions toward **the most relevant region of PF.**

Desirable characteristics of a preference-based EMOA

Interactiveness

- a priori
- a posteriori
- **interactive**

Evolutionary base

- dominance
- indicator
- **decomposition**

Preference model

- an additive value function
- a Chebyshef function
- ...

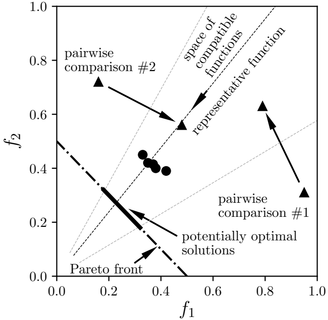
DM's judgements

- direct (e.g., objective weights)
- **indirect (e.g., pairwise comparisons)**

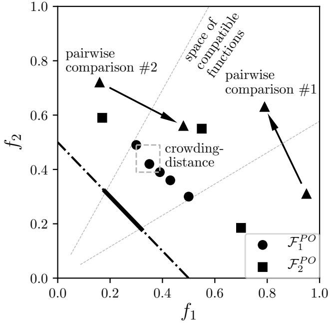
Robustness analysis

- single representative model instance
- control the spread of solutions
- **exploit a whole space of compatible model instances**

Comparison of different EMOAs incorporating preference learning



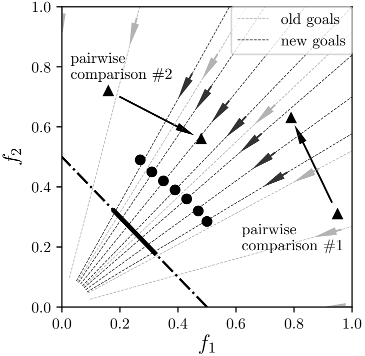
(a) A single representative preference model instance (e.g., NEMO-0)



(b) All compatible preference model instances (e.g., NEMO-II based on NSGA-II)

Figure: The role of a preference model in example preference-based Evolutionary Multiple Objective Optimization algorithms.

Comparison of different EMOAs incorporating preference learning



(c) A subset of all compatible preference model instances (e.g., IEMO/D based on MOEA/D)

Figure: The role of a preference model in example preference-based Evolutionary Multiple Objective Optimization algorithms.

IEMO/D: Preference model and a form of preference judgements

Preference model

$$L_\alpha^w(s^j, z) = \begin{cases} \left(\sum_{i \in \{1,\ldots,M\}} \left| w_i \left(s_i^j - z_i\right)\right|^\alpha\right)^{1/\alpha} & \text{for } \alpha < \infty, \\ max_{i=1,\ldots,M}\{|w_i(s_i^j - z_i)|\} & \text{for } \alpha = \infty. \end{cases}$$

s^j is a solution, w is a normalized weight vector such that $\sum_{i=1,\ldots,M} w_i = 1$, and $z = [z_1, z_2, \ldots, z_M]$ is a reference point. The less the distance of s^j from z ($L_\alpha^w(s^j, z)$), the better.

Form of preference judgements: Pairwise comparisons

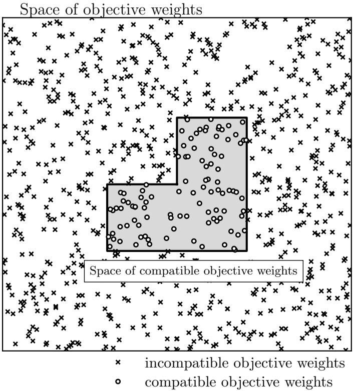
$$\forall_{(s^j \succ s^k) \in \mathcal{H}} \quad L_\alpha^w(s^j, z) < L_\alpha^w(s^k, z).$$

We assume that z and α are fixed and only objective weights w decide whether a particular model instance is consistent with the DM's decision examples.

IEMO/D: Robustness analysis

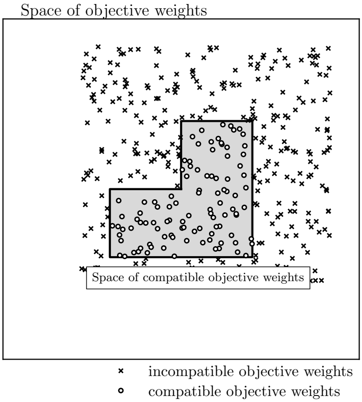
Compatible model instances

- IEMO/D exploits a set of all compatible preference functions by deriving from the uniform distribution a subset of compatible instances of the L_α-norm;
- IEMO/D employs such compatible functions to define the search directions in the evolutionary process.



IEMO/D: Robustness analysis

In order to maximize the success rate of the sampling method, IEMO/D reduces the search space.



IEMO/D: Iteratively Constrained Rejection Sampling With Upsampling

Reduction of the search space

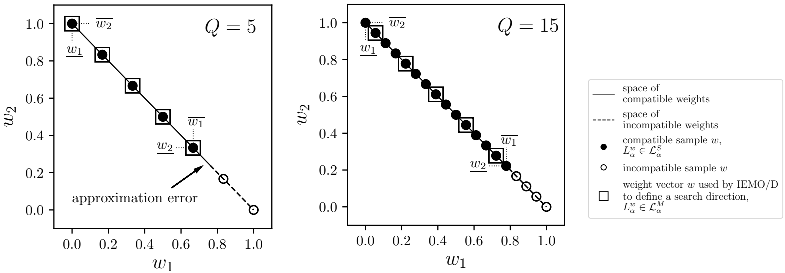
IEMO/D uses Hit-and-Run [1] algorithm to generate a set of candidate weight vectors from a uniform distribution over the following convex space:

$$\mathcal{C}^{\underline{w}\overline{w}} = \begin{cases} \sum_{i=1,..,M} & w_i = 1, \\ \forall_{i=1,\ldots,M} & \underline{w_i} \leq w_i \leq \overline{w_i}. \end{cases}$$

IEMO/D systematically approximates the bounds of candidate weights, hence reducing the space of weight vectors to be exploited and decreasing the rejection rate.

T. Tervonen, G. van Valkenhoef, N. Basturk, and D. Postmus, "Hit-and-Run enables efficient weight generation for simulation-based multiple criteria decision analysis," European Journal of Operational Research, vol. 224, no. 3, pp. 552 - 559, 2013.

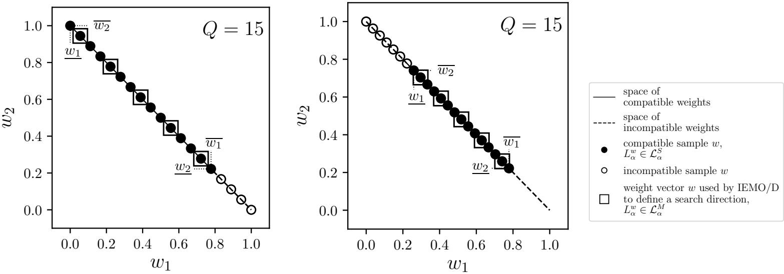
IEMO/D: Iteratively Constrained Rejection Sampling With Upsampling



(a) first iteration (ICRS) (b) first iteration (ICRSU)

Figure: The Iteratively Constrained Rejection Sampling method with (*ICRSU*, $Q = 15$) or without (*ICRS*, $Q = 5$) upsampling (the number of samples used by IEMO/D to define the search directions is equal to 5).

IEMO/D: Iteratively Constrained Rejection Sampling With Upsampling



(a) first iteration (ICRSU) (b) second iteration (ICRSU)

Figure: The Iteratively Constrained Rejection Sampling method with ($ICRSU, Q = 15$) or without ($ICRS$, $Q = 5$) upsampling (the number of samples used by IEMO/D to define the search directions is equal to 5).

IEMO/D: The replacement of optimization goals.

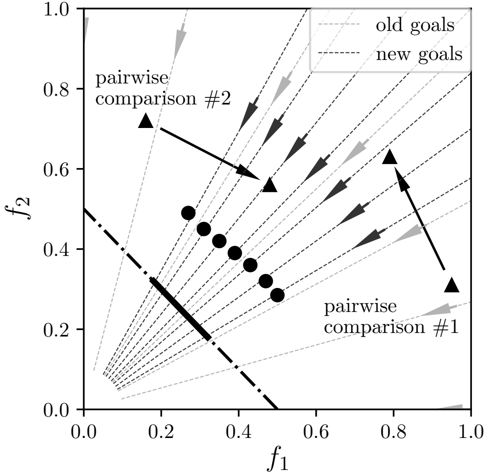


Figure: The replacement of optimization goals.

Experimental evaluation

Experimental setting

- DTLZ1-7 and WFG1-2 benchmarks with $2-5$ objectives.
- The number of generations G was set to 300 except for DTLZ3 ($G = 900$), DTLZ6 ($G = 900$) and WFG1 ($G = 1500$)
- The DM was asked to compare pairwise solutions from the current population.
- The number of preference elicitation iterations was limited to a realistic level of 12, and hence $EI = G/12$.
- α in the preference model used by IEMO/D as well as α^{DM} employed for simulating a decision model $L^{w^{DM}}_{\alpha^{DM}}$ of an artificial DM were set to 5.
- z was set to a utopian point.
- Tournament selection of size 5 for NSGA-II and NEMO methods. We used a random selection of a pair of solutions for MOEA/D and IEMO/D.
- Simulated binary crossover (probability of 1.0) with a distribution index of 10.0 and a polynomial mutation with a distribution index of 10.0 and probability of $1/dv$, where dv is a number of decision variables.

Experimental results

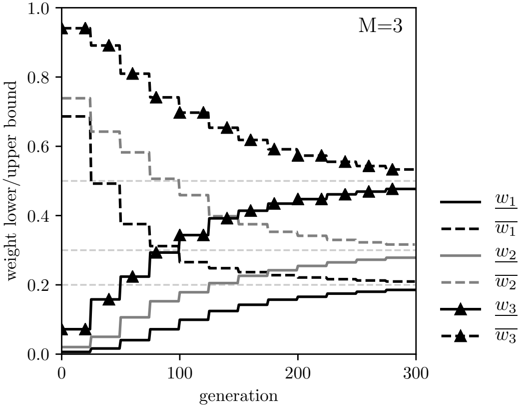
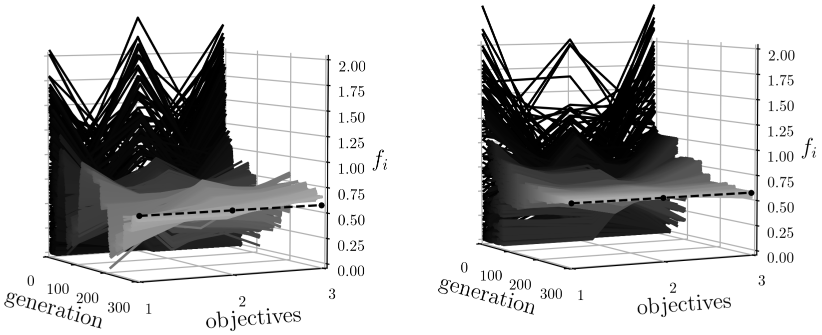


Figure: Bounds of objectives weights ($\underline{w}$ and $\overline{w}$) approximated with ICRSU ($Q = 1000$ and $T = 100000$) throughout 300 generations for DTLZ2 with $M = 3$ objectives, averaged over 100 runs.

Experimental results

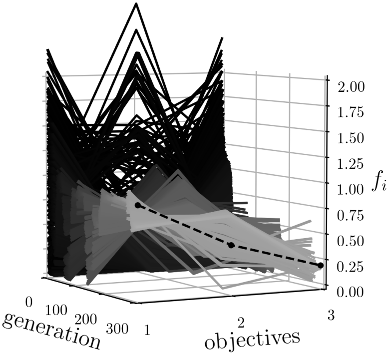


(a) NEMO-0-L_α

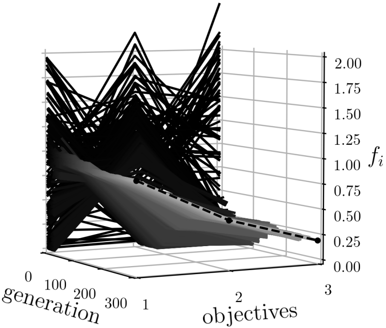
(b) IEMO/D

Figure: Solutions constructed by the algorithms throughout the evolutionary search for DTLZ2 with $M = 3$ ($w^{DM} = [1/3, 1/3, 1/3]$).

Experimental results



(a) NEMO-0-L_α



(b) IEMO/D

Figure: Solutions constructed by the algorithms throughout the evolutionary search for DTLZ2 with $M = 3$ ($w^{DM} = [0.2, 0.3, 0.5]$).

Evaluation strategy

Artificial DMs and optimal solutions

For each test problem, we simulated 100 artificial DMs with the randomly selected weight vectors w^{DM} incorporated into $L_{\alpha^{DM}}^{w^{DM}}$ and found the optimal solutions s^{opt} for such DMs:

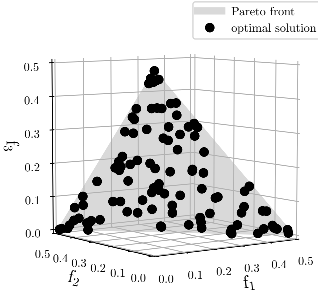


Figure: Optimal solutions s^{opt} for DTLZ1 with 3 objectives.

Evaluation strategy

Reported measures

For each run with a unique artificial DM, we reported the following measures for the compared algorithms:

• BRSD: a relative score difference of the best constructed solution $s^j \in P$ to the optimal solution $s^{opt}_{w^{DM}}$ according to the DM's model: $BRSD(P, L^{w^{DM}}_{\alpha^{DM}}) =$

$$min_{s^j \in P}\{\left(L^{w^{DM}}_{\alpha^{DM}}(s^{opt}_{w^{DM}}, z) - L^{w^{DM}}_{\alpha^{DM}}(s^j, z)\right) / L^{w^{DM}}_{\alpha^{DM}}(s^{opt}_{w^{DM}}, z)\};$$

• ARSD: an average relative score difference of all constructed solutions to the optimal solution $s^{opt}_{w^{DM}}$ according to the DM's model: $ARSD(P, L^{w^{DM}}_{\alpha^{DM}}) =$

$$\left(\sum_{s^j \in P} \left(L^{w^{DM}}_{\alpha^{DM}}(s^{opt}_{w^{DM}}, z) - L^{w^{DM}}_{\alpha^{DM}}(s^j, z)\right) / L^{w^{DM}}_{\alpha^{DM}}(s^{opt}_{w^{DM}}, z)\right) \Big/ |P|.$$

To aggregate the results from the experimental runs involving different artificial DMs, we computed the mean values of BRSD and ARSD along with the standard deviations (StD) as well as with the **averaged ranks $\overline{R}$ attained by different algorithms.**

Experimental results

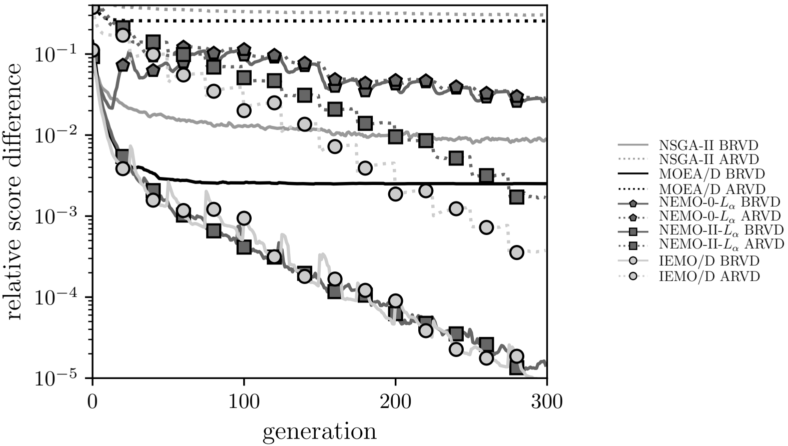


Figure: Convergence plot for average BRSD and ARSD for different algorithms applied to DTLZ2 with $M = 3$.

Experimental results

Table: Average BRSD (first row) and ARSD (second row) for the populations generated in the last iteration by six algorithms for the DTLZ and WFG test problems with $M = 2, 3, 4$, and 5 objectives. Average ranks $\overline{R}$ attained by the algorithms according to either BRSD or ARSD.

		NSGA-II			MOEA/D			NEMO-0-L_α			NEMO-II-L_α			IEMO/D			ρ
	M	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	
DTLZ2	2	17.59	18.33	4.72	17.33	38.49	3.86	17.82	80.70	3.27	0.01	0.01	1.58	0.02	0.02	1.57	4
		3409.85	1707.38	4.47	3320.65	1548.83	4.53	39.63	133.33	2.82	3.82	20.93	1.78	0.02	0.03	1.40	
	3	88.24	62.50	4.66	25.13	48.19	3.54	208.46	552.48	3.62	0.17	0.42	1.82	0.11	0.28	1.36	4
		3051.18	992.21	4.88	2561.79	851.06	4.10	213.45	554.56	2.48	18.07	54.69	2.04	3.85	8.39	1.50	
	4	498.53	364.08	4.75	24.51	39.13	3.23	394.81	649.79	3.85	0.74	1.26	1.82	0.48	0.93	1.35	4
		5892.20	1459.48	5.00	2133.01	526.03	3.97	399.37	651.41	2.41	99.37	148.60	2.05	31.18	52.96	1.57	
	5	703.14	463.65	4.70	11.42	18.83	3.03	455.72	561.80	3.95	1.55	3.51	2.05	0.78	1.51	1.27	4
		7055.04	1317.63	5.00	1825.73	339.62	3.95	470.68	564.63	2.43	208.68	178.94	2.15	63.95	99.67	1.47	

Experimental results

Table: Average ranks $\overline{R}$ attained by the algorithms according to either BRSD or ARSD for all test problems.

		NSGA-II		MOEA/D		NEMO-0-L_α		NEMO-II-L_α		IEMO/D	
		Mean	StD	Mean	StD	Mean	StD	Mean	StD	Mean	StD
Average ranks ($\overline{R}$)	2	4.59	0.31	3.07	0.69	3.13	0.38	1.85	0.44	2.36	0.69
		4.69	0.23	4.24	0.26	2.60	0.25	1.68	0.27	1.79	0.36
	3	4.69	0.38	3.25	0.44	3.53	0.13	1.81	0.29	1.72	0.33
		4.83	0.21	4.12	0.21	2.63	0.11	1.87	0.19	1.55	0.13
	4	4.66	0.53	3.18	0.29	3.63	0.24	1.94	0.17	1.60	0.38
		4.95	0.09	4.00	0.08	2.47	0.10	2.05	0.08	1.53	0.10
	5	4.67	0.56	3.12	0.33	3.68	0.21	2.01	0.17	1.52	0.41
		4.97	0.06	3.96	0.09	2.32	0.17	2.19	0.14	1.55	0.19

Experimental results

Table: Average ARSD for the populations constructed in the final generation by IEMO/D using different preference models L_α with various artificial DM's models $L_{\alpha^{DM}}$ ($\rho = 3$), for DTLZ2 with $M = 3$. Average ranks $\overline{R}$ attained by different variants of IEMO/D according to ARSD.

	$\alpha^{DM}=3$			$\alpha^{DM}=5$			$\alpha^{DM}=7$			$\alpha^{DM}=9$			$\alpha^{DM}=11$			$\alpha^{DM}=\infty$		
	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$	Mean	StD	$\overline{R}$
α	DTLZ2, $M=3$, $\rho=5$																	
3	0.07	0.19	1.74	0.55	1.90	3.07	4.67	31.76	3.30	7.51	48.47	3.72	2.70	5.47	3.77	10.03	19.85	3.71
5	0.33	0.88	2.93	0.38	0.84	2.79	0.75	2.88	3.10	0.74	1.45	3.22	1.80	4.10	3.44	6.26	9.44	3.59
7	0.48	1.60	3.21	0.38	0.91	2.96	0.63	1.26	3.14	0.81	1.69	2.86	0.63	0.93	3.15	4.75	6.40	3.25
9	0.54	1.21	4.00	0.57	1.10	3.53	0.79	2.30	3.07	0.97	2.37	3.18	1.14	2.56	2.92	4.18	6.16	3.08
11	0.94	2.59	3.89	0.98	2.14	3.72	0.92	2.00	3.45	1.24	4.35	3.28	0.93	1.64	3.03	4.23	5.99	3.20
∞	3.78	12.10	5.23	2.26	4.41	4.93	3.07	5.10	4.94	3.29	5.31	4.74	3.43	5.66	4.69	7.76	10.25	4.17

Summary

We proposed an **interactive** evolutionary multiple objective optimization algorithm IEMO/D implementing the paradigm of **decomposition**.

IEMO/D generates a set uniformly distributed instances of L_α-norms that are compatible with the DM's **indirect** pieces of preferences. This process involves the Monte Carlo simulation based on a **suitably adapted rejection sampling method.**

Our experimental results proved that both an **evolutionary mechanism** and a **robustness preoccupation** had a strong impact on the results of the interactive optimization.

We demonstrated that the results are vastly improved when IEMO/D employs L_α-norm **was highly consistent** with the DM's judgement policy.

Avenues for future research: dynamic model adjustment, dynamic interaction patterns (when to interact, selection of solutions to be compared).