Scheduling with precedence constraints:

Mixed graph coloring in series-parallel graphs

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Abstract. We consider the mixed graph coloring problem. A *mixed graph* $G_M = (V, E, A)$ is a graph with vertex set V and containing edges (set E) and arcs (set A). An edge joining vertices i and j is denoted by $\{i, j\}$, while an arc with tail p and head q is denoted by (p, q). A k-coloring of G_M is a function $\varphi : V \rightarrow \{1, 2, \ldots, k\}$ such that $\varphi(i) \neq \varphi(j)$ for $\{i, j\} \in E$ and $\varphi(p) < \varphi(q)$ for $(p, q) \in A$. Observe that the mixed graph G_M must be acyclic, i.e., must not contain any directed circuit, otherwise no proper k-coloring exists.

The mixed graph coloring model can be used for formulating scheduling problems where both incompatibility and precedence constraints are present. Formally, let T be a collection of jobs (with unit processing times). These jobs have to be processed taking into account the following constraints:

1. *Precedence constraints.* There is a set of ordered jobs (i, j) such that i must be processed before j.
2. *Disjunctive constraints.* For a family $\mathcal{I} = \{I_1, \ldots, I_l\}$ of subsets of T, no two jobs in I_α can be processed simultaneously, $\alpha = 1, \ldots, l$.

Consider now a mixed graph $G_M = (V, E, A)$ obtained as follows:

1. With each job j in T we associate a vertex j in V. G_M currently has no other vertices, and no arcs and edges.
2. For each ordered pair (i, j) of jobs we introduce an arc (i, j) in G_M.
3. For each subset I_α, $\alpha = 1, \ldots, l$, we introduce a clique associated with the jobs in I_α. (If an edge is needed between vertices i and j, we introduce it only if there was no previous arc or edge joining i and j.)

It easy to see that there is one-to-one correspondence between feasible schedules in k time units and k-colorings of the mixed graph G_M.

Herein, we present an $O(n^{3.376} \log n)$ time algorithm for finding the optimal scheduling in the case when incompatibility and precedence constraints form a series-parallel mixed graph.