Full results and implementation details of layout partitioning for advertisement fit problem

Jakub Marszałkowski, Maciej Drozdowski
Poznań University of Technology,
Piotrowo 2, 60-965 Poznań, Poland

Introduction

This document serves as supplement for the paper [1] with complete computational results and implementation details. For complete model, algorithm, benchmark datasets and results discussion please see the main paper.

In the referred paper website column width optimization problem is formulated. An algorithm selecting column widths for a given set of advertisement units is presented. A computationally improved Wang algorithm [2] for two-dimensional stock cutting problem is used for generating all acceptable combinations of ads that fit different widths of the columns. Possible column widths are valued for several objective functions. Two approaches are proposed. The first constructs a Pareto frontier of column width combinations, provided with their scores for the user choice. The second is a set of column widths optimal in the sense of a weighted linear function of all objectives. To justify the weights expert survey on criteria of web layout usability to fit advertisements is presented. Both solutions are examined on several datasets of internet advertising units.

Implementation details

Wang algorithm: duplicates recognition

Wang two-dimensional cutting stock algorithm [2] is used for finding all feasible ad unit combinations. The main computational disadvantage of this algorithm are combination duplicates. To eliminate creating and checking for unnecessary duplicates few improvements were introduced (see [1], Section 4.1).

Still, checking as fast as possible for duplicates is a key requirement. Duplicates can be recognized by comparing vectors of ad unit multiplicities $K_g = (k_{g1}, \ldots, k_{gn})$ and dimensions $h'_g \times w'_g$. To implement it efficiently, numeric signatures are calculated from K_g where each component k_{gi} is represented as a digit. Number of bits x for such a digit should be selected to represent the largest expected value b_e. The signatures can be stored as `int` variables. Then depending on size of integer in programming environment used, for each $32/x$ or $64/x$ ad units (digits) in the input dataset only one fast `int` comparison is necessary to check them. This plus comparison of the combination envelope sizes suffice for efficient duplicate recognition.

Wang algorithm: constraint checking order

For best performance of the improved Wang algorithm the constraints should be checked in the order that prunes the combinations as fast as possible. We establisehed the following sequence of checking the constraints: 4, 3, 8, 7, 10, 5. The first three are the easiest to check. These constraints can be quickly checked for excluding a combination from further joining (e.g. when its width will not allow to add even the narrowest ad without exceeding page width: $w'_g > W - \min_{1 \le e \le n}\{w_e\}$). Such combination can be omitted without trying to join it with all other combinations from $L^{(\kappa-1)}$. On the other hand, if constraints 8, 7 or 5 are not satisfied on horizontal joining, then there is no need to check for vertical alignment.

For constraint 8 value of t' can be set so hight that $t' = t \max_{1 \le e \le n}\{b_e\}$. In such a case it will not exclude any of the solutions which would not be otherwise eliminated by constraints 7 or 5. Is was set so in our computational experiments. Still, the constraints should be checked in the presented order. It helps to avoid testing whole branches of combinations, that would be finally discarded by constraints 7 or 5. It provided 12.5% gain in the execution time

for the largest dataset.

Finally, constraints 7 and 5 are computationally most demanding and should be checked as late as possible. The first requires enumeration by the ad units, and second comparisons for each ad unit. As a result we obtain a set of combination widths.

Layout browsing index

Generating all possible column width combinations requires exponential time. However, due to the limited sizes of the practical constraints it can be solved in acceptable time. Yet, a method of fast browsing column widths worth checking is needed becasue the most time-consuming part of the proposed algorithm is searching in Y for the next greater or smaller value of column width.

This can be done by ordering Y by column widths, and then building a vector of references from any current width y_r to the next greater/smaller width. This vector can be built with a single pass of Y, and will not be memory-consuming as in practice its cardinality will be much smaller than page width W, where usually $W \leq 1600$ (see [1], Section 6.1).

Complete results

Due to their size complete computational results are presented in this supplement. Instances are defined by a benchmark set of ad units, webpage width W, and the number of columns. Objective functions ranges for all feasible layouts are included. For each instance there the set of results consists of the best weighted solution and the complete Pareto frontier. Each solution is given as column widths with the total layout width, values of the three objective functions, and the value of the weighted linear function.

subset, W=990, 2 columns
$V_1 \in [42, 47], V_2 \in [7, 7], V_3 \in [-2534, -1914]$
248+732=980; 47, 7 , -1914; 100.0

Pareto frontier:
248+732=980; 47, 7 , -1914; 100.0

subset, W=1250, 3 columns
$V_1 \in [39, 55], V_2 \in [5, 7], V_3 \in [-2606, -858]$
164+328+732=1224; 55, 7 , -858; 100.0

Pareto frontier:
164+328+732=1224; 55, 7 , -858; 100.0

subset, W=1250, 4 columns
$V_1 \in [44, 50], V_2 \in [7, 7], V_3 \in [-1986, -858]$
164+164+164+732=1224; 50, 7 , -858; 100.0

Pareto frontier:
164+164+164+732=1224; 50, 7 , -858; 100.0

AdBrite, W=990, 2 columns
$V_1 \in [43, 48], V_2 \in [7, 7], V_3 \in [-2846, -2102]$
248+732=980; 48, 7 , -2102; 100.0

Pareto frontier:
248+732=980; 48, 7 , -2102; 100.0

AdBrite, W=1250, 3 columns
$V_1 \in [40, 56], V_2 \in [5, 7], V_3 \in [-3054, -1062]$
164+328+732=1224; 56, 7 , -1062; 100.0

Pareto frontier:
164+328+732=1224; 56, 7 , -1062; 100.0

AdBrite, W=1250, 4 columns
$V_1 \in [45, 51], V_2 \in [7, 7], V_3 \in [-2310, -1062]$
164+164+164+732=1224; 51, 7 , -1062; 100.0

Pareto frontier:
164+164+164+732=1224; 51, 7 , -1062; 100.0

Clicksor, W=990, 2 columns
$V_1 \in [735, 1123], V_2 \in [16, 20], V_3 \in [-5833, -4493]$
164+816=980; 1012, 20 , -5433; 64.8
Pareto frontier:
124+864=988; 1123, 17 , -5833; 48.2
129+861=990; 1075, 17 , -5783; 44.3
164+816=980; 1012, 20 , -5433; 64.8
184+806=990; 922, 20 , -5233; 60.0
248+742=990; 796, 18 , -4593; 49.6
253+737=990; 779, 17 , -4543; 42.8
258+732=990; 783, 16 , -4493; 38.2

Clicksor, W=1250, 3 columns
$V_1 \in [739, 1386], V_2 \in [15, 25], V_3 \in [-5977, -1655]$
184+254+812=1250; 1044, 24 , -2153; 71.5
Pareto frontier:
129+129+991=1249; 1386, 15 , -4507; 53.2
129+164+948=1241; 1279, 16 , -4286; 50.5
129+184+936=1249; 1246, 16 , -4122; 49.6
129+254+866=1249; 1168, 20 , -3632; 58.3
129+254+864=1247; 1173, 20 , -3638; 58.5
129+258+862=1249; 1152, 20 , -3604; 57.4
129+304+816=1249; 1082, 25 , -3282; 67.8
129+308+812=1249; 1073, 25 , -3254; 67.5
164+184+898=1246; 1194, 18 , -3257; 57.8
164+258+828=1250; 1056, 21 , -2797; 59.9
164+258+816=1238; 1059, 24 , -2845; 67.2
184+184+876=1244; 1147, 18 , -2533; 60.3
184+184+866=1234; 1152, 18 , -2583; 60.2
184+184+864=1232; 1157, 18 , -2593; 60.5
184+254+812=1250; 1044, 24 , -2153; 71.5
184+258+806=1248; 969, 24 , -2143; 66.7
184+293+773=1250; 891, 21 , -1958; 55.6
184+304+761=1249; 893, 20 , -1908; 53.6
184+308+756=1248; 878, 19 , -1893; 50.2
254+254+742=1250; 855, 18 , -1665; 48.0
254+258+737=1249; 836, 17 , -1655; 44.3

Clicksor, W=1250, 4 columns
$V_1 \in [743, 1147], V_2 \in [16, 22], V_3 \in [-4737, -1280]$
124+124+184+816=1248; 1026, 22 , -2073; 79.9
Pareto frontier:
124+124+184+816=1248; 1026, 22 , -2073; 79.9
124+129+129+866=1248; 1142, 19 , -3632; 64.5
124+129+129+864=1246; 1147, 19 , -3638; 65.0
124+129+184+812=1249; 1018, 22 , -1988; 79.8
124+129+254+742=1249; 829, 18 , -1290; 50.2
129+129+129+862=1249; 1129, 19 , -3604; 63.4
129+129+164+828=1250; 1033, 19 , -2692; 62.2
129+129+164+816=1238; 1036, 22 , -2740; 74.5
129+129+184+806=1248; 946, 22 , -1978; 72.4
129+129+254+737=1249; 813, 17 , -1280; 44.4
129+164+184+773=1250; 860, 21 , -1773; 61.3

Google Ads, W=990, 2 columns
$V_1 \in [1659, 2181], V_2 \in [18, 25], V_3 \in [-7123, -5515]$
248+742=990; 1769, 25 , -5635; 64.4
Pareto frontier:
124+864=988; 2181, 18 , -7123; 42.0
129+861=990; 2123, 18 , -7063; 38.6
129+848=977; 2077, 19 , -7063; 38.4
164+816=980; 2032, 22 , -6643; 54.1
184+806=990; 1908, 22 , -6403; 49.1
204+762=966; 1812, 22 , -6163; 46.3
248+742=990; 1769, 25 , -5635; 64.4
253+737=990; 1756, 24 , -5575; 61.0
258+732=990; 1807, 23 , -5515; 62.8

Google Ads, W=1250, 3 columns
$V_1 \in [1663, 2571], V_2 \in [16, 28], V_3 \in [-7539, -1723]$
184+254+812=1250; 2150, 26 , -3335; 67.2
Pareto frontier:
124+258+864=1246; 2313, 22 , -5667; 53.2
124+313+812=1249; 2214, 27 , -5111; 62.2
129+129+992=1250; 2567, 16 , -6046; 50.3
129+129+991=1249; 2571, 16 , -6049; 50.5
129+164+952=1245; 2351, 16 , -5746; 42.0
129+164+948=1241; 2377, 16 , -5758; 43.1
129+184+936=1249; 2307, 16 , -5554; 41.1
129+254+864=1247; 2301, 22 , -4930; 56.8
129+258+862=1249; 2295, 22 , -4888; 56.8
129+308+812=1249; 2209, 27 , -4438; 65.8
129+313+806=1248; 2092, 27 , -4396; 60.6
129+340+778=1247; 2042, 28 , -4156; 61.7
129+340+773=1242; 2043, 28 , -4171; 61.7
129+340+762=1231; 2049, 28 , -4204; 61.8
164+184+898=1246; 2271, 19 , -4619; 50.9
164+204+864=1232; 2223, 19 , -4515; 49.3
164+258+828=1250; 2161, 23 , -4011; 57.6
164+258+816=1238; 2164, 26 , -4059; 63.8
164+313+773=1250; 1977, 27 , -3571; 60.0
164+313+762=1239; 1983, 27 , -3615; 60.0
184+184+864=1232; 2215, 19 , -3915; 52.3
184+204+862=1250; 2203, 19 , -3685; 53.1
184+254+812=1250; 2150, 26 , -3335; 67.2
184+258+806=1248; 2040, 26 , -3317; 62.2
184+304+762=1250; 1969, 27 , -2985; 62.9
184+308+756=1248; 1910, 27 , -2967; 60.3
204+204+842=1250; 2113, 20 , -3007; 54.9
204+204+816=1224; 2085, 23 , -3163; 58.9
204+238+806=1248; 1961, 23 , -2815; 55.2
204+254+792=1250; 1936, 26 , -2707; 60.9
204+258+778=1240; 1937, 26 , -2743; 60.7
204+258+773=1235; 1938, 26 , -2773; 60.6
204+258+762=1224; 1944, 26 , -2839; 60.5
204+288+756=1248; 1883, 27 , -2515; 61.6
238+254+756=1248; 1864, 26 , -2201; 60.4
254+254+742=1250; 1930, 25 , -1731; 64.1
254+258+737=1249; 1926, 24 , -1723; 61.8
258+258+734=1250; 1932, 23 , -1731; 60.0
258+258+732=1248; 1939, 23 , -1747; 60.2

Google Ads, W=1250, 4 columns
$V_1 \in [1667, 2205], V_2 \in [20, 25], V_3 \in [-6051, -1344]$
124+129+254+742=1249; 1834, 25 , -1358; 70.9
Pareto frontier:
124+124+258+744=1250; 1842, 25 , -1503; 70.5
124+124+258+742=1248; 1846, 25 , -1519; 70.7
124+129+129+864=1246; 2205, 20 , -4932; 49.8
124+129+184+812=1249; 2054, 24 , -3172; 70.4
124+129+254+742=1249; 1834, 25 , -1358; 70.9
129+129+129+862=1249; 2187, 20 , -4888; 48.7
129+129+129+848=1235; 2101, 21 , -4930; 46.7
129+129+164+816=1238; 2056, 24 , -3954; 65.1
129+129+184+806=1248; 1932, 24 , -3152; 61.0
129+129+254+737=1249; 1818, 24 , -1348; 64.8
129+129+258+734=1250; 1824, 23 , -1344; 60.3

IAB, W=990, 2 columns
$V_1 \in [7033, 10453], V_2 \in [37, 51],$
$V_3 \in [-11183, -8195]$
258+732=990; 10441, 37 , -8195; 74.9
Pareto frontier:
129+852=981; 8512, 51 , -10517; 50.5
164+824=988; 8413, 48 , -9887; 50.9
164+820=984; 8339, 51 , -9887; 55.4
184+806=990; 8146, 47 , -9527; 49.8
216+773=989; 7872, 42 , -8951; 43.9
216+761=977; 8220, 41 , -8951; 46.4
248+742=990; 9195, 39 , -8375; 61.1
253+737=990; 9990, 38 , -8285; 70.1
254+732=986; 10453, 37 , -8267; 74.2
258+732=990; 10441, 37 , -8195; 74.9

IAB, W=1250, 3 columns
$V_1 \in [6495, 13601], V_2 \in [37, 72],$
$V_3 \in [-13567, -2990]$
164+258+828=1250; 10301, 69 , -5275; 71.2
Pareto frontier:
92+313+844=1249; 10707, 72 , -9477; 62.7
92+313+842=1247; 10714, 72 , -9479; 62.7
92+313+834=1239; 10716, 69 , -9487; 60.5
92+330+828=1250; 10704, 69 , -9187; 61.4
92+340+816=1248; 10768, 52 , -9019; 50.2
92+382+776=1250; 11459, 43 , -8303; 50.0
92+383+773=1248; 11842, 42 , -8288; 51.6
92+396+761=1249; 12193, 41 , -8066; 53.7
124+288+836=1248; 10037, 71 , -6543; 67.1
124+313+812=1249; 10794, 50 , -6213; 57.6
124+368+756=1248; 11160, 40 , -5503; 54.9
124+383+742=1249; 13005, 39 , -5303; 65.7
129+293+828=1250; 10410, 70 , -5729; 71.2
129+304+816=1249; 10551, 52 , -5603; 59.5
129+308+812=1249; 10783, 50 , -5555; 59.6
129+313+808=1250; 10889, 47 , -5489; 58.3
129+328+792=1249; 10675, 44 , -5315; 55.5
129+340+780=1249; 10786, 43 , -5171; 55.8
129+340+764=1233; 11089, 41 , -5267; 55.9
129+340+761=1230; 11119, 41 , -5285; 56.0
129+362+756=1247; 11136, 40 , -4919; 56.6
129+378+742=1249; 12901, 39 , -4715; 66.9
129+383+737=1249; 13285, 38 , -4655; 68.7
129+383+734=1246; 13590, 37 , -4673; 69.7
129+383+732=1244; 13601, 37 , -4685; 69.7

164+254+828=1246; 10313, 69 , -5347; 71.1
164+258+828=1250; 10301, 69 , -5275; 71.2
164+308+778=1250; 10289, 43 , -4725; 54.3
164+313+773=1250; 10605, 42 , -4670; 55.6
164+313+764=1241; 10923, 41 , -4733; 56.6
164+313+761=1238; 10953, 41 , -4754; 56.7
164+330+756=1250; 10943, 40 , -4483; 56.8
184+248+816=1248; 9285, 52 , -4687; 54.9
184+254+812=1250; 10263, 50 , -4611; 59.5
184+258+808=1250; 10030, 47 , -4571; 56.1
184+288+778=1250; 9805, 43 , -4271; 52.9
184+293+773=1250; 9855, 42 , -4221; 52.6
184+304+761=1249; 10435, 41 , -4119; 55.6
184+308+756=1248; 10645, 40 , -4087; 56.2
238+244+768=1250; 8496, 42 , -4073; 45.0
238+248+764=1250; 9114, 41 , -4037; 48.1
238+248+761=1247; 9144, 41 , -4064; 48.2
238+254+756=1248; 10100, 40 , -4001; 53.3
244+244+761=1249; 8937, 41 , -3565; 48.5
244+248+756=1248; 9188, 40 , -3543; 49.3
254+254+742=1250; 12061, 39 , -3007; 67.3
254+254+734=1242; 12330, 37 , -3095; 67.2
254+254+732=1240; 12341, 37 , -3117; 67.2
254+258+737=1249; 12013, 38 , -2990; 66.3
254+258+734=1246; 12318, 37 , -3023; 67.3
254+258+732=1244; 12329, 37 , -3045; 67.3
258+258+734=1250; 12306, 37 , -2995; 67.3
258+258+732=1248; 12317, 37 , -3017; 67.3

IAB, W=1250, 4 columns
$V_1 \in [6600, 11477], V_2 \in [37, 53]$,
$V_3 \in [-11911, -2125]$
92+92+254+812=1250; 9715, 50 , -3335; 76.1
Pareto frontier:
92+92+244+820=1248; 8469, 51 , -3903; 65.0
92+92+248+816=1248; 8737, 52 , -3907; 68.8
92+92+254+812=1250; 9715, 50 , -3335; 76.1
92+92+304+761=1249; 9887, 41 , -2125; 67.6
92+92+308+756=1248; 10097, 40 , -2175; 67.6
92+92+308+754=1246; 10444, 39 , -2203; 69.0
92+92+313+744=1241; 11056, 39 , -2318; 73.8
92+92+313+742=1239; 11068, 39 , -2346; 73.9
92+124+254+780=1250; 9417, 43 , -2591; 65.1
92+124+258+776=1250; 9397, 43 , -2587; 64.9
92+129+184+844=1249; 8996, 52 , -4582; 68.8
92+129+184+842=1247; 9003, 52 , -4598; 68.8
92+129+253+776=1250; 9578, 43 , -2976; 65.2
92+129+254+773=1248; 9718, 42 , -2473; 66.5
92+129+254+761=1236; 10066, 41 , -2605; 67.5
92+129+258+764=1243; 10024, 41 , -2520; 67.4
92+129+258+761=1240; 10054, 41 , -2553; 67.6
124+124+184+816=1248; 8767, 51 , -4387; 65.9
124+129+254+742=1249; 10881, 39 , -2250; 72.6
129+129+129+852=1239; 9536, 53 , -6215; 69.5
129+129+164+820=1242; 9363, 52 , -5121; 70.1
129+129+254+737=1249; 11161, 38 , -2240; 73.5
129+129+254+734=1246; 11466, 37 , -2273; 74.4
129+129+254+732=1244; 11477, 37 , -2295; 74.4
129+129+258+734=1250; 11454, 37 , -2221; 74.5
129+129+258+732=1248; 11465, 37 , -2243; 74.5

superset, W=990, 2 columns
$V_1 \in [8979, 12475], V_2 \in [39, 51]$,
$V_3 \in [-12221, -8901]$
258+732=990; 12467, 39 , -8901; 74.9
Pareto frontier:
129+852=981; 10487, 51 , -11481; 50.5
164+824=988; 10356, 48 , -10781; 49.6
164+820=984; 10277, 51 , -10781; 54.9
164+812=976; 10320, 49 , -10781; 51.3
184+806=990; 10043, 49 , -10381; 51.9
204+780=984; 9832, 47 , -9981; 49.2
216+773=989; 9891, 46 , -9741; 50.2
216+762=978; 10245, 45 , -9741; 52.4
248+742=990; 11162, 41 , -9101; 61.4
253+737=990; 11957, 40 , -9001; 69.9
254+732=986; 12475, 39 , -8981; 74.2
258+732=990; 12467, 39 , -8901; 74.9

superset, W=1250, 3 columns
$V_1 \in [7904, 16708], V_2 \in [39, 72]$,
$V_3 \in [-14941, -3225]$
129+388+732=1249; 16708, 39 , -5303; 69.1
Pareto frontier:
92+333+824=1249; 13636, 72 , -10029; 66.2
92+340+816=1248; 13735, 57 , -9897; 55.7
92+340+812=1244; 13783, 55 , -9901; 54.4
92+362+792=1246; 13299, 48 , -9481; 47.9
92+382+776=1250; 14450, 47 , -9097; 53.7
92+388+768=1248; 14946, 46 , -8985; 55.7
92+396+762=1250; 15330, 45 , -8831; 57.2
124+288+836=1248; 12112, 71 , -7461; 65.4
124+313+812=1249; 12887, 55 , -7081; 58.0
124+333+792=1249; 13247, 48 , -6781; 55.3
124+368+756=1248; 14148, 43 , -6261; 57.3
124+383+742=1249; 16007, 41 , -6031; 65.3
129+254+852=1235; 12478, 64 , -7263; 62.4
129+258+852=1239; 12470, 64 , -7183; 62.6
129+293+828=1250; 12460, 70 , -6627; 68.6
129+304+816=1249; 12596, 57 , -6479; 59.9
129+308+812=1249; 12876, 55 , -6423; 59.8
129+313+808=1250; 12893, 51 , -6347; 57.1
129+328+792=1249; 13157, 48 , -6143; 56.7
129+340+780=1249; 13691, 47 , -5975; 58.9
129+340+773=1242; 13743, 46 , -6017; 58.3

129+340+764=1233; 14067, 45 , -6071; 58.9
129+340+762=1231; 14097, 45 , -6083; 59.0
129+362+756=1247; 14080, 43 , -5679; 58.6
129+378+742=1249; 15897, 41 , -5443; 66.4
129+383+737=1249; 16287, 40 , -5373; 67.7
129+388+732=1249; 16708, 39 , -5303; 69.1
164+254+828=1246; 12359, 69 , -6253; 68.5
164+258+828=1250; 12351, 69 , -6173; 68.6
164+313+773=1250; 12655, 46 , -5458; 54.7
164+313+764=1241; 12979, 45 , -5521; 55.3
164+313+762=1239; 13009, 45 , -5535; 55.4
164+330+756=1250; 13441, 43 , -5237; 56.8
164+340+746=1250; 14379, 41 , -5107; 60.1
164+340+744=1248; 14640, 41 , -5121; 61.3
164+340+742=1246; 14652, 41 , -5135; 61.3
164+340+734=1238; 14934, 39 , -5191; 61.0
164+340+732=1236; 14970, 39 , -5205; 61.1
184+248+816=1248; 11309, 57 , -5565; 56.3
184+254+812=1250; 12352, 55 , -5477; 60.0
184+258+808=1250; 12034, 51 , -5429; 55.6
184+293+773=1250; 11905, 46 , -5009; 52.4
184+304+762=1250; 12491, 45 , -4877; 54.8
184+308+756=1248; 12644, 43 , -4845; 54.1
184+333+732=1249; 14828, 39 , -4537; 62.3
204+204+842=1250; 11154, 52 , -5359; 52.3
204+216+828=1248; 11114, 49 , -5245; 50.2
204+216+820=1240; 11023, 52 , -5317; 51.8
204+238+808=1250; 10773, 50 , -4985; 50.1
204+244+802=1250; 10639, 49 , -4919; 48.9
204+254+792=1250; 11878, 48 , -4809; 54.3
204+288+756=1248; 12191, 43 , -4453; 53.0
216+216+816=1248; 11025, 50 , -5221; 50.6
216+216+812=1244; 11073, 50 , -5257; 50.7
216+254+780=1250; 11830, 47 , -4785; 53.4
216+254+773=1243; 11882, 46 , -4848; 52.7
216+258+776=1250; 11814, 47 , -4741; 53.4
216+258+773=1247; 11874, 46 , -4768; 52.9
216+258+764=1238; 12198, 45 , -4849; 53.5
216+258+762=1236; 12228, 45 , -4867; 53.5
238+244+768=1250; 10597, 46 , -4321; 48.1
238+248+764=1250; 11228, 45 , -4281; 50.4
238+248+762=1248; 11258, 45 , -4301; 50.5
238+254+756=1248; 12174, 43 , -4241; 53.5
244+244+762=1250; 11044, 45 , -3803; 50.9
244+248+756=1248; 11245, 43 , -3789; 50.4
254+254+742=1250; 14148, 41 , -3245; 64.2
254+254+732=1240; 14466, 39 , -3365; 63.9
254+258+737=1249; 14104, 40 , -3225; 63.3
254+258+732=1244; 14458, 39 , -3285; 64.1
258+258+734=1250; 14414, 39 , -3229; 64.0
258+258+732=1248; 14450, 39 , -3253; 64.1

superset, W=1250, 4 columns
$V_1 \in [8308, 14280], V_2 \in [39, 57]$,
$V_3 \in [-13101, -2419]$
92+92+254+812=1250; 11804, 55 , -3643; 76.0
Pareto frontier:
92+92+248+816=1248; 10761, 57 , -4217; 69.7
92+92+254+812=1250; 11804, 55 , -3643; 76.0
92+92+304+762=1250; 11943, 45 , -2419; 66.9
92+92+308+756=1248; 12096, 43 , -2485; 65.0
92+92+313+744=1241; 13044, 41 , -2635; 68.4
92+92+313+742=1239; 13056, 41 , -2665; 68.4
92+92+328+737=1249; 13520, 40 , -2650; 70.3
92+92+330+734=1248; 13849, 39 , -2683; 71.2
92+92+333+732=1249; 14280, 39 , -2695; 74.1
92+124+254+780=1250; 11396, 47 , -2867; 64.4
92+124+258+776=1250; 11380, 47 , -2863; 64.3
92+129+253+776=1250; 11540, 47 , -3247; 64.3
92+129+254+773=1248; 11764, 46 , -2746; 66.0
92+129+254+762=1237; 12118, 45 , -2878; 66.7
92+129+258+764=1243; 12080, 45 , -2798; 66.7
92+129+258+762=1241; 12110, 45 , -2822; 66.8
124+129+254+742=1249; 12865, 41 , -2490; 67.6
129+129+254+737=1249; 13145, 40 , -2475; 68.2
129+129+254+732=1244; 13499, 39 , -2535; 69.1
129+129+258+734=1250; 13455, 39 , -2455; 69.1
129+129+258+732=1248; 13491, 39 , -2479; 69.3

References

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[2] P. Y. Wang. Two algorithms for constrained two-dimensional cutting stock problems. *Operations Research*, 31(3):573–586, 1983.