AC-trivialization proofs eliminating some potential counterexamples to the Andrews-Curtis conjecture

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The Andrews Curtis conjecture (Andrews and Curtis, 1965) was first posed in 1965, and is of interest to group theorists and low-dimensional topologists. Building on the conclusions of (Swan et al., 2012), the following are the AC-trivializations found by the method of 'Distance Metric Ensemble Learning' (Krawiec and Swan). The identifiers for each case correspond to those given in the list of unsolved instances arising from experiments performed since 2001 (Edjvet et al., 2001; Edjvet, 2003; Cremona and Edjvet, 2010; Edjvet and Swan, 2014) at Martin Edjvet's homepage.

Instance: **T1**. Trivialization sequence length: 6

$$\langle a,b|a^2bAB, b^2aBA\rangle \xrightarrow[(b^2aBA)^A]{} \langle a,b|a^2bAB, ab^2aBA^2\rangle$$
$$\langle a,b|a^2bAB, ab^2aBA^2\rangle \xrightarrow[ab^2aBA^2 * = a^2bAB]{} \langle a,b|ab, a^2bAB\rangle$$
$$\langle a,b|ab, a^2bAB\rangle \xrightarrow[(a^2bAB)^b]{} \langle a,b|ab, Ba^2bA\rangle$$
$$\langle a,b|ab, Ba^2bA\rangle \xrightarrow[(ab)^A]{} \langle a,b|a^2bA, Ba^2bA\rangle$$
$$\langle a,b|a^2bA, Ba^2bA\rangle \xrightarrow[(a^2bA)^{-1}]{} \langle a,b|aBA^2, Ba^2bA\rangle$$
$$\langle a,b|aBA^2, Ba^2bA\rangle \xrightarrow[Ba^2bA * = aBA^2]{} \langle a,b|B, aBA^2\rangle$$

Instance: **T5**. Trivialization sequence length: 10

$$\langle a,b|a^2b^2AB^2, b^2a^2BA^2\rangle \xrightarrow[b^2a^2BA^2*=a^2b^2AB^2]{} \langle a,b|a^2b^2AB^2, b^2a^2bAB^2\rangle$$
$$\langle a,b|a^2b^2AB^2, b^2a^2bAB^2\rangle \xrightarrow[(b^2a^2bAB^2)^b]{} \langle a,b|ba^2bAB, a^2b^2AB^2\rangle$$
$$\langle a,b|ba^2bAB, a^2b^2AB^2\rangle \xrightarrow[(ba^2bAB)^b]{} \langle a,b|a^2bA, a^2b^2AB^2\rangle$$
$$\langle a,b|a^2bA, a^2b^2AB^2\rangle \xrightarrow[(a^2b^2AB^2)^{-1}]{} \langle a,b|a^2bA, b^2aB^2A^2\rangle$$
$$\langle a,b|a^2bA, b^2aB^2A^2\rangle \xrightarrow[b^2aB^2A^2*=a^2bA]{} \langle a,b|a^2bA, b^2aBA\rangle$$
$$\langle a,b|a^2bA, b^2aBA\rangle \xrightarrow[(a^2bA)^{-1}]{} \langle a,b|aBA^2, b^2aBA\rangle$$
$$\langle a,b|aBA^2, b^2aBA\rangle \xrightarrow[(aBA^2)^a]{} \langle a,b|BA, b^2aBA\rangle$$
$$\langle a,b|BA, b^2aBA\rangle \xrightarrow[(BA)^a]{} \langle a,b|AB, b^2aBA\rangle$$
$$\langle a,b|AB, b^2aBA\rangle \xrightarrow[(AB)^B]{} \langle a,b|bAB^2, b^2aBA\rangle$$
$$\langle a,b|bAB^2, b^2aBA\rangle \xrightarrow[bAB^2*=b^2aBA]{} \langle a,b|A, b^2aBA\rangle$$

Instance: **T11**. Trivialization sequence length: 14

$$\langle a,b|a^3b^2A^2B^2,b^3a^2B^2A^2\rangle \xrightarrow[(b^3a^2B^2A^2)^A]{} \langle a,b|a^3b^2A^2B^2,ab^3a^2B^2A^3\rangle$$
$$\langle a,b|a^3b^2A^2B^2,ab^3a^2B^2A^3\rangle \xrightarrow[ab^3a^2B^2A^3*=a^3b^2A^2B^2]{} \langle a,b|ab,a^3b^2A^2B^2\rangle$$
$$\langle a,b|ab,a^3b^2A^2B^2\rangle \xrightarrow[(a^3b^2A^2B^2)^A]{} \langle a,b|ab,a^4b^2A^2B^2A\rangle$$
$$\langle a,b|ab,a^4b^2A^2B^2A\rangle \xrightarrow[a^4b^2A^2B^2A*=ab]{} \langle a,b|ab,a^4b^2A^2B\rangle$$
$$\langle a,b|ab,a^4b^2A^2B\rangle \xrightarrow[(ab)^B]{} \langle a,b|ba,a^4b^2A^2B\rangle$$
$$\langle a,b|ba,a^4b^2A^2B\rangle \xrightarrow[a^4b^2A^2B*=ba]{} \langle a,b|ba,a^4b^2A\rangle$$
$$\langle a,b|ba,a^4b^2A\rangle \xrightarrow[(a^4b^2A)^a]{} \langle a,b|ba,a^3b^2\rangle$$
$$\langle a,b|ba,a^3b^2\rangle \xrightarrow[(ba)^B]{} \langle a,b|b^2aB,a^3b^2\rangle$$
$$\langle a,b|b^2aB,a^3b^2\rangle \xrightarrow[(b^2aB)^{-1}]{} \langle a,b|bAB^2,a^3b^2\rangle$$
$$\langle a,b|bAB^2,a^3b^2\rangle \xrightarrow[(a^3b^2)^a]{} \langle a,b|bAB^2,a^2b^2a\rangle$$
$$\langle a,b|bAB^2,a^2b^2a\rangle \xrightarrow[(bAB^2)^b]{} \langle a,b|AB,a^2b^2a\rangle$$
$$\langle a,b|AB,a^2b^2a\rangle \xrightarrow[a^2b^2a*=AB]{} \langle a,b|AB,a^2b\rangle$$
$$\langle a,b|AB,a^2b\rangle \xrightarrow[(AB)^A]{} \langle a,b|BA,a^2b\rangle$$
$$\langle a,b|BA,a^2b\rangle \xrightarrow[a^2b*=BA]{} \langle a,b|a,BA\rangle$$

Instance: **T13**. Trivialization sequence length: 7

$$\langle a,b|a^2bAbAB, b^2aBaBA\rangle \xrightarrow[(b^2aBaBA)^A]{} \langle a,b|a^2bAbAB, ab^2aBaBA^2\rangle$$
$$\langle a,b|a^2bAbAB, ab^2aBaBA^2\rangle \xrightarrow[ab^2aBaBA^2*=a^2bAbAB]{} \langle a,b|ab, a^2bAbAB\rangle$$
$$\langle a,b|ab, a^2bAbAB\rangle \xrightarrow[(ab)^A]{} \langle a,b|a^2bA, a^2bAbAB\rangle$$
$$\langle a,b|a^2bA, a^2bAbAB\rangle \xrightarrow[(a^2bA)^{-1}]{} \langle a,b|aBA^2, a^2bAbAB\rangle$$
$$\langle a,b|aBA^2, a^2bAbAB\rangle \xrightarrow[(aBA^2)^b]{} \langle a,b|BaBA^2b, a^2bAbAB\rangle$$
$$\langle a,b|BaBA^2b, a^2bAbAB\rangle \xrightarrow[(a^2bAbAB)^b]{} \langle a,b|BaBA^2b, Ba^2bAbA\rangle$$
$$\langle a,b|BaBA^2b, Ba^2bAbA\rangle \xrightarrow[BaBA^2b*=Ba^2bAbA]{} \langle a,b|A, Ba^2bAbA\rangle$$

Instance: **T29**. Trivialization sequence length: 21

$$\langle a,b|a^3b^3A^2B^3,b^3a^3B^2A^3\rangle \xrightarrow[a^3b^3A^2B^3*=b^3a^3B^2A^3]{} \langle a,b|b^3a^3B^2A^3,a^3b^3aB^2A^3\rangle$$
$$\langle a,b|b^3a^3B^2A^3,a^3b^3aB^2A^3\rangle \xrightarrow[(a^3b^3aB^2A^3)^a]{} \langle a,b|a^2b^3aB^2A^2,b^3a^3B^2A^3\rangle$$
$$\langle a,b|a^2b^3aB^2A^2,b^3a^3B^2A^3\rangle \xrightarrow[(a^2b^3aB^2A^2)^{-1}]{} \langle a,b|a^2b^2AB^3A^2,b^3a^3B^2A^3\rangle$$
$$\langle a,b|a^2b^2AB^3A^2,b^3a^3B^2A^3\rangle \xrightarrow[(b^3a^3B^2A^3)^A]{} \langle a,b|a^2b^2AB^3A^2,ab^3a^3B^2A^4\rangle$$
$$\langle a,b|a^2b^2AB^3A^2,ab^3a^3B^2A^4\rangle \xrightarrow[(ab^3a^3B^2A^4)^a]{} \langle a,b|a^2b^2AB^3A^2,b^3a^3B^2A^3\rangle$$
$$\langle a,b|a^2b^2AB^3A^2,b^3a^3B^2A^3\rangle \xrightarrow[(a^2b^2AB^3A^2)^{-1}]{} \langle a,b|a^2b^3aB^2A^2,b^3a^3B^2A^3\rangle$$
$$\langle a,b|a^2b^3aB^2A^2,b^3a^3B^2A^3\rangle \xrightarrow[(a^2b^3aB^2A^2)^a]{} \langle a,b|ab^3aB^2A,b^3a^3B^2A^3\rangle$$
$$\langle a,b|ab^3aB^2A,b^3a^3B^2A^3\rangle \xrightarrow[(ab^3aB^2A)^a]{} \langle a,b|b^3aB^2,b^3a^3B^2A^3\rangle$$
$$\langle a,b|b^3aB^2,b^3a^3B^2A^3\rangle \xrightarrow[(b^3a^3B^2A^3)^{-1}]{} \langle a,b|b^3aB^2,a^3b^2A^3B^3\rangle$$
$$\langle a,b|b^3aB^2,a^3b^2A^3B^3\rangle \xrightarrow[a^3b^2A^3B^3*=b^3aB^2]{} \langle a,b|b^3aB^2,a^3b^2A^2B^2\rangle$$
$$\langle a,b|b^3aB^2,a^3b^2A^2B^2\rangle \xrightarrow[(b^3aB^2)^b]{} \langle a,b|b^2aB,a^3b^2A^2B^2\rangle$$
$$\langle a,b|b^2aB,a^3b^2A^2B^2\rangle \xrightarrow[a^3b^2A^2B^2*=b^2aB]{} \langle a,b|b^2aB,a^3b^2AB\rangle$$
$$\langle a,b|b^2aB,a^3b^2AB\rangle \xrightarrow[(b^2aB)^b]{} \langle a,b|ba,a^3b^2AB\rangle$$
$$\langle a,b|ba,a^3b^2AB\rangle \xrightarrow[(ba)^A]{} \langle a,b|ab,a^3b^2AB\rangle$$
$$\langle a,b|ab,a^3b^2AB\rangle \xrightarrow[(ab)^a]{} \langle a,b|ba,a^3b^2AB\rangle$$
$$\langle a,b|ba,a^3b^2AB\rangle \xrightarrow[a^3b^2AB*=ba]{} \langle a,b|ba,a^3b^2\rangle$$
$$\langle a,b|ba,a^3b^2\rangle \xrightarrow[(ba)^{-1}]{} \langle a,b|AB,a^3b^2\rangle$$
$$\langle a,b|AB,a^3b^2\rangle \xrightarrow[(a^3b^2)^a]{} \langle a,b|AB,a^2b^2a\rangle$$
$$\langle a,b|AB,a^2b^2a\rangle \xrightarrow[a^2b^2a*=AB]{} \langle a,b|AB,a^2b\rangle$$
$$\langle a,b|AB,a^2b\rangle \xrightarrow[(a^2b)^a]{} \langle a,b|AB,aba\rangle$$
$$\langle a,b|AB,aba\rangle \xrightarrow[aba*=AB]{} \langle a,b|a,AB\rangle$$

Instance: **T31**. Trivialization sequence length: 10

$$\langle a,b|a^3bAbAB^2, b^3aBaBA^2\rangle \xrightarrow[(a^3bAbAB^2)^B]{} \langle a,b|b^3aBaBA^2, ba^3bAbAB^3\rangle$$

$$\langle a,b|b^3aBaBA^2, ba^3bAbAB^3\rangle \xrightarrow[ba^3bAbAB^3*=b^3aBaBA^2]{} \langle a,b|ba, b^3aBaBA^2\rangle$$

$$\langle a,b|ba, b^3aBaBA^2\rangle \xrightarrow[(b^3aBaBA^2)^a]{} \langle a,b|ba, Ab^3aBaBA\rangle$$

$$\langle a,b|ba, Ab^3aBaBA\rangle \xrightarrow[(ba)^A]{} \langle a,b|ab, Ab^3aBaBA\rangle$$

$$\langle a,b|ab, Ab^3aBaBA\rangle \xrightarrow[Ab^3aBaBA*=ab]{} \langle a,b|ab, Ab^3aBa\rangle$$

$$\langle a,b|ab, Ab^3aBa\rangle \xrightarrow[(ab)^{-1}]{} \langle a,b|BA, Ab^3aBa\rangle$$

$$\langle a,b|BA, Ab^3aBa\rangle \xrightarrow[(Ab^3aBa)^A]{} \langle a,b|BA, b^3aB\rangle$$

$$\langle a,b|BA, b^3aB\rangle \xrightarrow[(b^3aB)^b]{} \langle a,b|BA, b^2a\rangle$$

$$\langle a,b|BA, b^2a\rangle \xrightarrow[(BA)^a]{} \langle a,b|AB, b^2a\rangle$$

$$\langle a,b|AB, b^2a\rangle \xrightarrow[b^2a*=AB]{} \langle a,b|b, AB\rangle$$

Instance: **T34**. Trivialization sequence length: 10

$$\langle a,b|a^2b^2aBA^2B, b^2a^2bAB^2A\rangle \xrightarrow[(a^2b^2aBA^2B)^B]{} \langle a,b|b^2a^2bAB^2A, ba^2b^2aBA^2B^2\rangle$$

$$\langle a,b|b^2a^2bAB^2A, ba^2b^2aBA^2B^2\rangle \xrightarrow[ba^2b^2aBA^2B^2*=b^2a^2bAB^2A]{} \langle a,b|ba, b^2a^2bAB^2A\rangle$$

$$\langle a,b|ba, b^2a^2bAB^2A\rangle \xrightarrow[(ba)^B]{} \langle a,b|b^2aB, b^2a^2bAB^2A\rangle$$

$$\langle a,b|b^2aB, b^2a^2bAB^2A\rangle \xrightarrow[(b^2aB)^A]{} \langle a,b|ab^2aBA, b^2a^2bAB^2A\rangle$$

$$\langle a,b|ab^2aBA, b^2a^2bAB^2A\rangle \xrightarrow[(b^2a^2bAB^2A)^A]{} \langle a,b|ab^2aBA, ab^2a^2bAB^2A^2\rangle$$

$$\langle a,b|ab^2aBA, ab^2a^2bAB^2A^2\rangle \xrightarrow[(ab^2a^2bAB^2A^2)^a]{} \langle a,b|ab^2aBA, b^2a^2bAB^2A\rangle$$

$$\langle a,b|ab^2aBA, b^2a^2bAB^2A\rangle \xrightarrow[b^2a^2bAB^2A*=ab^2aBA]{} \langle a,b|b^2a, ab^2aBA\rangle$$

$$\langle a,b|b^2a, ab^2aBA\rangle \xrightarrow[(ab^2aBA)^a]{} \langle a,b|b^2a, b^2aB\rangle$$

$$\langle a,b|b^2a, b^2aB\rangle \xrightarrow[(b^2a)^{-1}]{} \langle a,b|AB^2, b^2aB\rangle$$

$$\langle a,b|AB^2, b^2aB\rangle \xrightarrow[AB^2*=b^2aB]{} \langle a,b|B, b^2aB\rangle$$

Instance: **T35**. Trivialization sequence length: 24

$$\langle a,b|a^2b^2AbAB^2, b^2a^2BaBA^2\rangle \xrightarrow[b^2a^2BaBA^2*=a^2b^2AbAB^2]{} \langle a,b|a^2b^2AbAB^2, b^2a^2BabAbAB^2\rangle$$
$$\langle a,b|a^2b^2AbAB^2, b^2a^2BabAbAB^2\rangle \xrightarrow[(b^2a^2BabAbAB^2)^b]{} \langle a,b|a^2b^2AbAB^2, ba^2BabAbAB\rangle$$
$$\langle a,b|a^2b^2AbAB^2, ba^2BabAbAB\rangle \xrightarrow[(ba^2BabAbAB)^b]{} \langle a,b|a^2BabAbA, a^2b^2AbAB^2\rangle$$
$$\langle a,b|a^2BabAbA, a^2b^2AbAB^2\rangle \xrightarrow[(a^2b^2AbAB^2)^b]{} \langle a,b|a^2BabAbA, Ba^2b^2AbAB\rangle$$
$$\langle a,b|a^2BabAbA, Ba^2b^2AbAB\rangle \xrightarrow[(a^2BabAbA)^a]{} \langle a,b|aBabAb, Ba^2b^2AbAB\rangle$$
$$\langle a,b|aBabAb, Ba^2b^2AbAB\rangle \xrightarrow[(aBabAb)^B]{} \langle a,b|baBabA, Ba^2b^2AbAB\rangle$$
$$\langle a,b|baBabA, Ba^2b^2AbAB\rangle \xrightarrow[Ba^2b^2AbAB*=baBabA]{} \langle a,b|baBabA, Ba^2b^3A\rangle$$
$$\langle a,b|baBabA, Ba^2b^3A\rangle \xrightarrow[(Ba^2b^3A)^B]{} \langle a,b|baBabA, a^2b^3AB\rangle$$
$$\langle a,b|baBabA, a^2b^3AB\rangle \xrightarrow[a^2b^3AB*=baBabA]{} \langle a,b|baBabA, a^2b^2abA\rangle$$
$$\langle a,b|baBabA, a^2b^2abA\rangle \xrightarrow[(a^2b^2abA)^a]{} \langle a,b|ab^2ab, baBabA\rangle$$
$$\langle a,b|ab^2ab, baBabA\rangle \xrightarrow[baBabA*=ab^2ab]{} \langle a,b|ab^2ab, baBab^3ab\rangle$$
$$\langle a,b|ab^2ab, baBab^3ab\rangle \xrightarrow[(baBab^3ab)^b]{} \langle a,b|ab^2ab, aBab^3ab^2\rangle$$
$$\langle a,b|ab^2ab, aBab^3ab^2\rangle \xrightarrow[(aBab^3ab^2)^{-1}]{} \langle a,b|ab^2ab, B^2AB^3AbA\rangle$$
$$\langle a,b|ab^2ab, B^2AB^3AbA\rangle \xrightarrow[(B^2AB^3AbA)^a]{} \langle a,b|ab^2ab, AB^2AB^3Ab\rangle$$
$$\langle a,b|ab^2ab, AB^2AB^3Ab\rangle \xrightarrow[(ab^2ab)^B]{} \langle a,b|bab^2a, AB^2AB^3Ab\rangle$$
$$\langle a,b|bab^2a, AB^2AB^3Ab\rangle \xrightarrow[(bab^2a)^{-1}]{} \langle a,b|AB^2AB, AB^2AB^3Ab\rangle$$
$$\langle a,b|AB^2AB, AB^2AB^3Ab\rangle \xrightarrow[(AB^2AB^3Ab)^{-1}]{} \langle a,b|AB^2AB, Bab^3ab^2a\rangle$$
$$\langle a,b|AB^2AB, Bab^3ab^2a\rangle \xrightarrow[Bab^3ab^2a*=AB^2AB]{} \langle a,b|Bab^2, AB^2AB\rangle$$
$$\langle a,b|Bab^2, AB^2AB\rangle \xrightarrow[(AB^2AB)^b]{} \langle a,b|Bab^2, BAB^2A\rangle$$
$$\langle a,b|Bab^2, BAB^2A\rangle \xrightarrow[(Bab^2)^{-1}]{} \langle a,b|B^2Ab, BAB^2A\rangle$$
$$\langle a,b|B^2Ab, BAB^2A\rangle \xrightarrow[(BAB^2A)^{-1}]{} \langle a,b|B^2Ab, ab^2ab\rangle$$
$$\langle a,b|B^2Ab, ab^2ab\rangle \xrightarrow[(B^2Ab)^B]{} \langle a,b|BA, ab^2ab\rangle$$
$$\langle a,b|BA, ab^2ab\rangle \xrightarrow[ab^2ab*=BA]{} \langle a,b|BA, ab^2\rangle$$
$$\langle a,b|BA, ab^2\rangle \xrightarrow[BA*=ab^2]{} \langle a,b|b, ab^2\rangle$$

Instance: **T39**. Trivialization sequence length: 10

$$\langle a,b|a^2bAb^2AB^2,b^2aBa^2BA^2\rangle \xrightarrow[a^2bAb^2AB^2*=b^2aBa^2BA^2]{} \langle a,b|b^2aBa^2BA^2,a^2bAba^2BA^2\rangle$$
$$\langle a,b|b^2aBa^2BA^2,a^2bAba^2BA^2\rangle \xrightarrow[(a^2bAba^2BA^2)^a]{} \langle a,b|abAba^2BA,b^2aBa^2BA^2\rangle$$
$$\langle a,b|abAba^2BA,b^2aBa^2BA^2\rangle \xrightarrow[(abAba^2BA)^{-1}]{} \langle a,b|abA^2BaBA,b^2aBa^2BA^2\rangle$$
$$\langle a,b|abA^2BaBA,b^2aBa^2BA^2\rangle \xrightarrow[(abA^2BaBA)^a]{} \langle a,b|bA^2BaB,b^2aBa^2BA^2\rangle$$
$$\langle a,b|bA^2BaB,b^2aBa^2BA^2\rangle \xrightarrow[(bA^2BaB)^b]{} \langle a,b|A^2Ba,b^2aBa^2BA^2\rangle$$
$$\langle a,b|A^2Ba,b^2aBa^2BA^2\rangle \xrightarrow[(A^2Ba)^A]{} \langle a,b|AB,b^2aBa^2BA^2\rangle$$
$$\langle a,b|AB,b^2aBa^2BA^2\rangle \xrightarrow[(AB)^B]{} \langle a,b|bAB^2,b^2aBa^2BA^2\rangle$$
$$\langle a,b|bAB^2,b^2aBa^2BA^2\rangle \xrightarrow[bAB^2*=b^2aBa^2BA^2]{} \langle a,b|a^2BA^2,b^2aBa^2BA^2\rangle$$
$$\langle a,b|a^2BA^2,b^2aBa^2BA^2\rangle \xrightarrow[(a^2BA^2)^a]{} \langle a,b|aBA,b^2aBa^2BA^2\rangle$$
$$\langle a,b|aBA,b^2aBa^2BA^2\rangle \xrightarrow[(aBA)^a]{} \langle a,b|B,b^2aBa^2BA^2\rangle$$

Instance: **T56**. Trivialization sequence length: 25

$$\langle a,b|a^4b^3A^3B^3, b^4a^3B^3A^3\rangle \xrightarrow[(a^4b^3A^3B^3)^B]{} \langle a,b|b^4a^3B^3A^3, ba^4b^3A^3B^4\rangle$$
$$\langle a,b|b^4a^3B^3A^3, ba^4b^3A^3B^4\rangle \xrightarrow[ba^4b^3A^3B^4*=b^4a^3B^3A^3]{} \langle a,b|ba, b^4a^3B^3A^3\rangle$$
$$\langle a,b|ba, b^4a^3B^3A^3\rangle \xrightarrow[(b^4a^3B^3A^3)^{-1}]{} \langle a,b|ba, a^3b^3A^3B^4\rangle$$
$$\langle a,b|ba, a^3b^3A^3B^4\rangle \xrightarrow[(a^3b^3A^3B^4)^b]{} \langle a,b|ba, Ba^3b^3A^3B^3\rangle$$
$$\langle a,b|ba, Ba^3b^3A^3B^3\rangle \xrightarrow[(Ba^3b^3A^3B^3)^b]{} \langle a,b|ba, B^2a^3b^3A^3B^2\rangle$$
$$\langle a,b|ba, B^2a^3b^3A^3B^2\rangle \xrightarrow[(B^2a^3b^3A^3B^2)^b]{} \langle a,b|ba, B^3a^3b^3A^3B\rangle$$
$$\langle a,b|ba, B^3a^3b^3A^3B\rangle \xrightarrow[B^3a^3b^3A^3B*=ba]{} \langle a,b|ba, B^3a^3b^3A^2\rangle$$
$$\langle a,b|ba, B^3a^3b^3A^2\rangle \xrightarrow[(ba)^{-1}]{} \langle a,b|AB, B^3a^3b^3A^2\rangle$$
$$\langle a,b|AB, B^3a^3b^3A^2\rangle \xrightarrow[(B^3a^3b^3A^2)^B]{} \langle a,b|AB, B^2a^3b^3A^2B\rangle$$
$$\langle a,b|AB, B^2a^3b^3A^2B\rangle \xrightarrow[(AB)^{-1}]{} \langle a,b|ba, B^2a^3b^3A^2B\rangle$$
$$\langle a,b|ba, B^2a^3b^3A^2B\rangle \xrightarrow[(B^2a^3b^3A^2B)^B]{} \langle a,b|ba, Ba^3b^3A^2B^2\rangle$$
$$\langle a,b|ba, Ba^3b^3A^2B^2\rangle \xrightarrow[(Ba^3b^3A^2B^2)^b]{} \langle a,b|ba, B^2a^3b^3A^2B\rangle$$
$$\langle a,b|ba, B^2a^3b^3A^2B\rangle \xrightarrow[B^2a^3b^3A^2B*=ba]{} \langle a,b|ba, B^2a^3b^3A\rangle$$
$$\langle a,b|ba, B^2a^3b^3A\rangle \xrightarrow[(ba)^b]{} \langle a,b|ab, B^2a^3b^3A\rangle$$
$$\langle a,b|ab, B^2a^3b^3A\rangle \xrightarrow[(B^2a^3b^3A)^B]{} \langle a,b|ab, Ba^3b^3AB\rangle$$
$$\langle a,b|ab, Ba^3b^3AB\rangle \xrightarrow[(ab)^B]{} \langle a,b|ba, Ba^3b^3AB\rangle$$
$$\langle a,b|ba, Ba^3b^3AB\rangle \xrightarrow[Ba^3b^3AB*=ba]{} \langle a,b|ba, Ba^3b^3\rangle$$
$$\langle a,b|ba, Ba^3b^3\rangle \xrightarrow[(Ba^3b^3)^{-1}]{} \langle a,b|ba, B^3A^3b\rangle$$
$$\langle a,b|ba, B^3A^3b\rangle \xrightarrow[(B^3A^3b)^B]{} \langle a,b|ba, B^2A^3\rangle$$
$$\langle a,b|ba, B^2A^3\rangle \xrightarrow[(B^2A^3)^B]{} \langle a,b|ba, BA^3B\rangle$$
$$\langle a,b|ba, BA^3B\rangle \xrightarrow[BA^3B*=ba]{} \langle a,b|ba, BA^2\rangle$$
$$\langle a,b|ba, BA^2\rangle \xrightarrow[(ba)^{-1}]{} \langle a,b|AB, BA^2\rangle$$
$$\langle a,b|AB, BA^2\rangle \xrightarrow[(BA^2)^B]{} \langle a,b|AB, A^2B\rangle$$
$$\langle a,b|AB, A^2B\rangle \xrightarrow[(AB)^{-1}]{} \langle a,b|ba, A^2B\rangle$$
$$\langle a,b|ba, A^2B\rangle \xrightarrow[A^2B*=ba]{} \langle a,b|A, ba\rangle$$

Instance: **T61**. Trivialization sequence length: 14

$$\langle a,b|a^3b^2AbA^2B^2, b^3a^2BaB^2A^2\rangle \xrightarrow[(b^3a^2BaB^2A^2)^A]{} \langle a,b|a^3b^2AbA^2B^2, ab^3a^2BaB^2A^3\rangle$$
$$\langle a,b|a^3b^2AbA^2B^2, ab^3a^2BaB^2A^3\rangle \xrightarrow[ab^3a^2BaB^2A^3*=a^3b^2AbA^2B^2]{} \langle a,b|ab, a^3b^2AbA^2B^2\rangle$$
$$\langle a,b|ab, a^3b^2AbA^2B^2\rangle \xrightarrow[(a^3b^2AbA^2B^2)^A]{} \langle a,b|ab, a^4b^2AbA^2B^2A\rangle$$
$$\langle a,b|ab, a^4b^2AbA^2B^2A\rangle \xrightarrow[a^4b^2AbA^2B^2A*=ab]{} \langle a,b|ab, a^4b^2AbA^2B\rangle$$
$$\langle a,b|ab, a^4b^2AbA^2B\rangle \xrightarrow[(ab)^B]{} \langle a,b|ba, a^4b^2AbA^2B\rangle$$
$$\langle a,b|ba, a^4b^2AbA^2B\rangle \xrightarrow[a^4b^2AbA^2B*=ba]{} \langle a,b|ba, a^4b^2AbA\rangle$$
$$\langle a,b|ba, a^4b^2AbA\rangle \xrightarrow[(ba)^b]{} \langle a,b|ab, a^4b^2AbA\rangle$$
$$\langle a,b|ab, a^4b^2AbA\rangle \xrightarrow[(a^4b^2AbA)^a]{} \langle a,b|ab, a^3b^2Ab\rangle$$
$$\langle a,b|ab, a^3b^2Ab\rangle \xrightarrow[(a^3b^2Ab)^a]{} \langle a,b|ab, a^2b^2Aba\rangle$$
$$\langle a,b|ab, a^2b^2Aba\rangle \xrightarrow[(a^2b^2Aba)^{-1}]{} \langle a,b|ab, ABaB^2A^2\rangle$$
$$\langle a,b|ab, ABaB^2A^2\rangle \xrightarrow[(ab)^A]{} \langle a,b|a^2bA, ABaB^2A^2\rangle$$
$$\langle a,b|a^2bA, ABaB^2A^2\rangle \xrightarrow[ABaB^2A^2*=a^2bA]{} \langle a,b|a^2bA, ABaBA\rangle$$
$$\langle a,b|a^2bA, ABaBA\rangle \xrightarrow[(ABaBA)^A]{} \langle a,b|a^2bA, BaBA^2\rangle$$
$$\langle a,b|a^2bA, BaBA^2\rangle \xrightarrow[BaBA^2*=a^2bA]{} \langle a,b|B, a^2bA\rangle$$

Instance: **T63**. Trivialization sequence length: 24

$$\langle a,b|a^3b^2AB^3Ab, b^3a^2BA^3Ba\rangle \xrightarrow[(a^3b^2AB^3Ab)^B]{} \langle a,b|ba^3b^2AB^3A, b^3a^2BA^3Ba\rangle$$
$$\langle a,b|ba^3b^2AB^3A, b^3a^2BA^3Ba\rangle \xrightarrow[(b^3a^2BA^3Ba)^A]{} \langle a,b|ab^3a^2BA^3B, ba^3b^2AB^3A\rangle$$
$$\langle a,b|ab^3a^2BA^3B, ba^3b^2AB^3A\rangle \xrightarrow[ba^3b^2AB^3A*=ab^3a^2BA^3B]{} \langle a,b|ab^3a^2BA^3B, ba^3b^2aBA^3B\rangle$$
$$\langle a,b|ab^3a^2BA^3B, ba^3b^2aBA^3B\rangle \xrightarrow[(ab^3a^2BA^3B)^{-1}]{} \langle a,b|ba^3bA^2B^3A, ba^3b^2aBA^3B\rangle$$
$$\langle a,b|ba^3bA^2B^3A, ba^3b^2aBA^3B\rangle \xrightarrow[(ba^3b^2aBA^3B)^{-1}]{} \langle a,b|ba^3bA^2B^3A, ba^3bAB^2A^3B\rangle$$
$$\langle a,b|ba^3bA^2B^3A, ba^3bAB^2A^3B\rangle \xrightarrow[(ba^3bAB^2A^3B)^b]{} \langle a,b|a^3bAB^2A^3, ba^3bA^2B^3A\rangle$$
$$\langle a,b|a^3bAB^2A^3, ba^3bA^2B^3A\rangle \xrightarrow[(a^3bAB^2A^3)^a]{} \langle a,b|a^2bAB^2A^2, ba^3bA^2B^3A\rangle$$
$$\langle a,b|a^2bAB^2A^2, ba^3bA^2B^3A\rangle \xrightarrow[(a^2bAB^2A^2)^a]{} \langle a,b|abAB^2A, ba^3bA^2B^3A\rangle$$
$$\langle a,b|abAB^2A, ba^3bA^2B^3A\rangle \xrightarrow[(abAB^2A)^a]{} \langle a,b|bAB^2, ba^3bA^2B^3A\rangle$$
$$\langle a,b|bAB^2, ba^3bA^2B^3A\rangle \xrightarrow[(bAB^2)^b]{} \langle a,b|AB, ba^3bA^2B^3A\rangle$$
$$\langle a,b|AB, ba^3bA^2B^3A\rangle \xrightarrow[(ba^3bA^2B^3A)^a]{} \langle a,b|AB, Aba^3bA^2B^3\rangle$$
$$\langle a,b|AB, Aba^3bA^2B^3\rangle \xrightarrow[(Aba^3bA^2B^3)^{-1}]{} \langle a,b|AB, b^3a^2BA^3Ba\rangle$$
$$\langle a,b|AB, b^3a^2BA^3Ba\rangle \xrightarrow[(b^3a^2BA^3Ba)^b]{} \langle a,b|AB, b^2a^2BA^3Bab\rangle$$
$$\langle a,b|AB, b^2a^2BA^3Bab\rangle \xrightarrow[(AB)^A]{} \langle a,b|BA, b^2a^2BA^3Bab\rangle$$
$$\langle a,b|BA, b^2a^2BA^3Bab\rangle \xrightarrow[b^2a^2BA^3Bab*=BA]{} \langle a,b|BA, b^2a^2BA^3B\rangle$$
$$\langle a,b|BA, b^2a^2BA^3B\rangle \xrightarrow[(b^2a^2BA^3B)^{-1}]{} \langle a,b|BA, ba^3bA^2B^2\rangle$$
$$\langle a,b|BA, ba^3bA^2B^2\rangle \xrightarrow[(BA)^B]{} \langle a,b|AB, ba^3bA^2B^2\rangle$$
$$\langle a,b|AB, ba^3bA^2B^2\rangle \xrightarrow[(AB)^{-1}]{} \langle a,b|ba, ba^3bA^2B^2\rangle$$
$$\langle a,b|ba, ba^3bA^2B^2\rangle \xrightarrow[(ba^3bA^2B^2)^b]{} \langle a,b|ba, a^3bA^2B\rangle$$
$$\langle a,b|ba, a^3bA^2B\rangle \xrightarrow[a^3bA^2B*=ba]{} \langle a,b|ba, a^3bA\rangle$$
$$\langle a,b|ba, a^3bA\rangle \xrightarrow[(ba)^A]{} \langle a,b|ab, a^3bA\rangle$$
$$\langle a,b|ab, a^3bA\rangle \xrightarrow[(ab)^A]{} \langle a,b|a^2bA, a^3bA\rangle$$
$$\langle a,b|a^2bA, a^3bA\rangle \xrightarrow[(a^3bA)^{-1}]{} \langle a,b|a^2bA, aBA^3\rangle$$
$$\langle a,b|a^2bA, aBA^3\rangle \xrightarrow[a^2bA*=aBA^3]{} \langle a,b|A, aBA^3\rangle$$

Instance: **T66**. Trivialization sequence length: 14

$$\langle a,b|a^3bA^2b^2AB^2, b^3aB^2a^2BA^2\rangle \xrightarrow[(a^3bA^2b^2AB^2)^a]{} \langle a,b|a^2bA^2b^2AB^2a, b^3aB^2a^2BA^2\rangle$$

$$\langle a,b|a^2bA^2b^2AB^2a, b^3aB^2a^2BA^2\rangle \xrightarrow[b^3aB^2a^2BA^2*=a^2bA^2b^2AB^2a]{} \langle a,b|ba, a^2bA^2b^2AB^2a\rangle$$

$$\langle a,b|ba, a^2bA^2b^2AB^2a\rangle \xrightarrow[(ba)^b]{} \langle a,b|ab, a^2bA^2b^2AB^2a\rangle$$

$$\langle a,b|ab, a^2bA^2b^2AB^2a\rangle \xrightarrow[(a^2bA^2b^2AB^2a)^{-1}]{} \langle a,b|ab, Ab^2aB^2a^2BA^2\rangle$$

$$\langle a,b|ab, Ab^2aB^2a^2BA^2\rangle \xrightarrow[(Ab^2aB^2a^2BA^2)^a]{} \langle a,b|ab, A^2b^2aB^2a^2BA\rangle$$

$$\langle a,b|ab, A^2b^2aB^2a^2BA\rangle \xrightarrow[A^2b^2aB^2a^2BA*=ab]{} \langle a,b|ab, A^2b^2aB^2a^2\rangle$$

$$\langle a,b|ab, A^2b^2aB^2a^2\rangle \xrightarrow[(A^2b^2aB^2a^2)^A]{} \langle a,b|ab, Ab^2aB^2a\rangle$$

$$\langle a,b|ab, Ab^2aB^2a\rangle \xrightarrow[(Ab^2aB^2a)^A]{} \langle a,b|ab, b^2aB^2\rangle$$

$$\langle a,b|ab, b^2aB^2\rangle \xrightarrow[(ab)^{-1}]{} \langle a,b|BA, b^2aB^2\rangle$$

$$\langle a,b|BA, b^2aB^2\rangle \xrightarrow[b^2aB^2*=BA]{} \langle a,b|BA, b^2aB^3A\rangle$$

$$\langle a,b|BA, b^2aB^3A\rangle \xrightarrow[(BA)^{-1}]{} \langle a,b|ab, b^2aB^3A\rangle$$

$$\langle a,b|ab, b^2aB^3A\rangle \xrightarrow[b^2aB^3A*=ab]{} \langle a,b|ab, b^2aB^2\rangle$$

$$\langle a,b|ab, b^2aB^2\rangle \xrightarrow[(b^2aB^2)^b]{} \langle a,b|ab, baB\rangle$$

$$\langle a,b|ab, baB\rangle \xrightarrow[(baB)^b]{} \langle a,b|a, ab\rangle$$

Instance: **T67**. Trivialization sequence length: 22

$$\langle a,b|a^3bAb^2AB^3, b^3aBa^2BA^3\rangle \xrightarrow[b^3aBa^2BA^3*=a^3bAb^2AB^3]{} \langle a,b|a^3bAb^2AB^3, b^3aBab^2AB^3\rangle$$
$$\langle a,b|a^3bAb^2AB^3, b^3aBab^2AB^3\rangle \xrightarrow[(b^3aBab^2AB^3)^b]{} \langle a,b|b^2aBab^2AB^2, a^3bAb^2AB^3\rangle$$
$$\langle a,b|b^2aBab^2AB^2, a^3bAb^2AB^3\rangle \xrightarrow[(a^3bAb^2AB^3)^a]{} \langle a,b|b^2aBab^2AB^2, a^2bAb^2AB^3a\rangle$$
$$\langle a,b|b^2aBab^2AB^2, a^2bAb^2AB^3a\rangle \xrightarrow[(a^2bAb^2AB^3a)^A]{} \langle a,b|b^2aBab^2AB^2, a^3bAb^2AB^3\rangle$$
$$\langle a,b|b^2aBab^2AB^2, a^3bAb^2AB^3\rangle \xrightarrow[(b^2aBab^2AB^2)^b]{} \langle a,b|baBab^2AB, a^3bAb^2AB^3\rangle$$
$$\langle a,b|baBab^2AB, a^3bAb^2AB^3\rangle \xrightarrow[(baBab^2AB)^{-1}]{} \langle a,b|baB^2AbAB, a^3bAb^2AB^3\rangle$$
$$\langle a,b|baB^2AbAB, a^3bAb^2AB^3\rangle \xrightarrow[(baB^2AbAB)^b]{} \langle a,b|aB^2AbA, a^3bAb^2AB^3\rangle$$
$$\langle a,b|aB^2AbA, a^3bAb^2AB^3\rangle \xrightarrow[(a^3bAb^2AB^3)^a]{} \langle a,b|aB^2AbA, a^2bAb^2AB^3a\rangle$$
$$\langle a,b|aB^2AbA, a^2bAb^2AB^3a\rangle \xrightarrow[(aB^2AbA)^a]{} \langle a,b|B^2Ab, a^2bAb^2AB^3a\rangle$$
$$\langle a,b|B^2Ab, a^2bAb^2AB^3a\rangle \xrightarrow[(a^2bAb^2AB^3a)^{-1}]{} \langle a,b|B^2Ab, Ab^3aB^2aBA^2\rangle$$
$$\langle a,b|B^2Ab, Ab^3aB^2aBA^2\rangle \xrightarrow[(B^2Ab)^B]{} \langle a,b|BA, Ab^3aB^2aBA^2\rangle$$
$$\langle a,b|BA, Ab^3aB^2aBA^2\rangle \xrightarrow[(Ab^3aB^2aBA^2)^a]{} \langle a,b|BA, A^2b^3aB^2aBA\rangle$$
$$\langle a,b|BA, A^2b^3aB^2aBA\rangle \xrightarrow[(BA)^{-1}]{} \langle a,b|ab, A^2b^3aB^2aBA\rangle$$
$$\langle a,b|ab, A^2b^3aB^2aBA\rangle \xrightarrow[A^2b^3aB^2aBA*=ab]{} \langle a,b|ab, A^2b^3aB^2a\rangle$$
$$\langle a,b|ab, A^2b^3aB^2a\rangle \xrightarrow[(A^2b^3aB^2a)^A]{} \langle a,b|ab, Ab^3aB^2\rangle$$
$$\langle a,b|ab, Ab^3aB^2\rangle \xrightarrow[(Ab^3aB^2)^{-1}]{} \langle a,b|ab, b^2AB^3a\rangle$$
$$\langle a,b|ab, b^2AB^3a\rangle \xrightarrow[(ab)^{-1}]{} \langle a,b|BA, b^2AB^3a\rangle$$
$$\langle a,b|BA, b^2AB^3a\rangle \xrightarrow[(BA)^a]{} \langle a,b|AB, b^2AB^3a\rangle$$
$$\langle a,b|AB, b^2AB^3a\rangle \xrightarrow[(AB)^{-1}]{} \langle a,b|ba, b^2AB^3a\rangle$$
$$\langle a,b|ba, b^2AB^3a\rangle \xrightarrow[(ba)^B]{} \langle a,b|b^2aB, b^2AB^3a\rangle$$
$$\langle a,b|b^2aB, b^2AB^3a\rangle \xrightarrow[(b^2aB)^B]{} \langle a,b|b^3aB^2, b^2AB^3a\rangle$$
$$\langle a,b|b^3aB^2, b^2AB^3a\rangle \xrightarrow[b^3aB^2*=b^2AB^3a]{} \langle a,b|a, b^2AB^3a\rangle$$

Instance: **T76**. Trivialization sequence length: 10

$$\langle a,b|a^2babABAB, b^2abaBABA\rangle \xrightarrow[(a^2babABAB)^B]{} \langle a,b|b^2abaBABA, ba^2babABAB^2\rangle$$

$$\langle a,b|b^2abaBABA, ba^2babABAB^2\rangle \xrightarrow[ba^2babABAB^2*=b^2abaBABA]{} \langle a,b|ba, b^2abaBABA\rangle$$

$$\langle a,b|ba, b^2abaBABA\rangle \xrightarrow[(ba)^A]{} \langle a,b|ab, b^2abaBABA\rangle$$

$$\langle a,b|ab, b^2abaBABA\rangle \xrightarrow[b^2abaBABA*=ab]{} \langle a,b|ab, b^2abaBA\rangle$$

$$\langle a,b|ab, b^2abaBA\rangle \xrightarrow[b^2abaBA*=ab]{} \langle a,b|ab, b^2aba\rangle$$

$$\langle a,b|ab, b^2aba\rangle \xrightarrow[(ab)^B]{} \langle a,b|ba, b^2aba\rangle$$

$$\langle a,b|ba, b^2aba\rangle \xrightarrow[(b^2aba)^{-1}]{} \langle a,b|ba, ABAB^2\rangle$$

$$\langle a,b|ba, ABAB^2\rangle \xrightarrow[ABAB^2*=ba]{} \langle a,b|ba, ABABa\rangle$$

$$\langle a,b|ba, ABABa\rangle \xrightarrow[(ABABa)^A]{} \langle a,b|ba, BAB\rangle$$

$$\langle a,b|ba, BAB\rangle \xrightarrow[BAB*=ba]{} \langle a,b|B, ba\rangle$$

Instance: **T81**. Trivialization sequence length: 19

$$\langle a,b|a^2bAbABaB, b^2aBaBAbA\rangle \xrightarrow[(b^2aBaBAbA)^a]{} \langle a,b|Ab^2aBaBAb, a^2bAbABaB\rangle$$

$$\langle a,b|Ab^2aBaBAb, a^2bAbABaB\rangle \xrightarrow[(a^2bAbABaB)^a]{} \langle a,b|Ab^2aBaBAb, abAbABaBa\rangle$$

$$\langle a,b|Ab^2aBaBAb, abAbABaBa\rangle \xrightarrow[(Ab^2aBaBAb)^B]{} \langle a,b|abAbABaBa, bAb^2aBaBA\rangle$$

$$\langle a,b|abAbABaBa, bAb^2aBaBA\rangle \xrightarrow[bAb^2aBaBA*=abAbABaBa]{} \langle a,b|bAbaBa, abAbABaBa\rangle$$

$$\langle a,b|bAbaBa, abAbABaBa\rangle \xrightarrow[(abAbABaBa)^A]{} \langle a,b|bAbaBa, a^2bAbABaB\rangle$$

$$\langle a,b|bAbaBa, a^2bAbABaB\rangle \xrightarrow[a^2bAbABaB*=bAbaBa]{} \langle a,b|a^2b, bAbaBa\rangle$$

$$\langle a,b|a^2b, bAbaBa\rangle \xrightarrow[(bAbaBa)^b]{} \langle a,b|a^2b, AbaBab\rangle$$

$$\langle a,b|a^2b, AbaBab\rangle \xrightarrow[(a^2b)^{-1}]{} \langle a,b|BA^2, AbaBab\rangle$$

$$\langle a,b|BA^2, AbaBab\rangle \xrightarrow[AbaBab*=BA^2]{} \langle a,b|BA^2, AbaBA\rangle$$

$$\langle a,b|BA^2, AbaBA\rangle \xrightarrow[(BA^2)^{-1}]{} \langle a,b|a^2b, AbaBA\rangle$$

$$\langle a,b|a^2b, AbaBA\rangle \xrightarrow[(a^2b)^a]{} \langle a,b|aba, AbaBA\rangle$$

$$\langle a,b|aba, AbaBA\rangle \xrightarrow[AbaBA*=aba]{} \langle a,b|aba, Aba^2\rangle$$

$$\langle a,b|aba, Aba^2\rangle \xrightarrow[(Aba^2)^{-1}]{} \langle a,b|aba, A^2Ba\rangle$$

$$\langle a,b|aba, A^2Ba\rangle \xrightarrow[(A^2Ba)^A]{} \langle a,b|AB, aba\rangle$$

$$\langle a,b|AB, aba\rangle \xrightarrow[(aba)^A]{} \langle a,b|AB, a^2b\rangle$$

$$\langle a,b|AB, a^2b\rangle \xrightarrow[(a^2b)^{-1}]{} \langle a,b|AB, BA^2\rangle$$

$$\langle a,b|AB, BA^2\rangle \xrightarrow[(BA^2)^{-1}]{} \langle a,b|AB, a^2b\rangle$$

$$\langle a,b|AB, a^2b\rangle \xrightarrow[(AB)^A]{} \langle a,b|BA, a^2b\rangle$$

$$\langle a,b|BA, a^2b\rangle \xrightarrow[a^2b*=BA]{} \langle a,b|a, BA\rangle$$

Instance: **T82**. Trivialization sequence length: 10

$$\langle a,b|a^2bABabAB, b^2aBAbaBA\rangle \xrightarrow[(a^2bABabAB)^B]{} \langle a,b|b^2aBAbaBA, ba^2bABabAB^2\rangle$$
$$\langle a,b|b^2aBAbaBA, ba^2bABabAB^2\rangle \xrightarrow[ba^2bABabAB^2*=b^2aBAbaBA]{} \langle a,b|ba, b^2aBAbaBA\rangle$$
$$\langle a,b|ba, b^2aBAbaBA\rangle \xrightarrow[(ba)^A]{} \langle a,b|ab, b^2aBAbaBA\rangle$$
$$\langle a,b|ab, b^2aBAbaBA\rangle \xrightarrow[b^2aBAbaBA*=ab]{} \langle a,b|ab, b^2aBAba\rangle$$
$$\langle a,b|ab, b^2aBAba\rangle \xrightarrow[(ab)^a]{} \langle a,b|ba, b^2aBAba\rangle$$
$$\langle a,b|ba, b^2aBAba\rangle \xrightarrow[(b^2aBAba)^{-1}]{} \langle a,b|ba, ABabAB^2\rangle$$
$$\langle a,b|ba, ABabAB^2\rangle \xrightarrow[(ba)^B]{} \langle a,b|b^2aB, ABabAB^2\rangle$$
$$\langle a,b|b^2aB, ABabAB^2\rangle \xrightarrow[ABabAB^2*=b^2aB]{} \langle a,b|ABa, b^2aB\rangle$$
$$\langle a,b|ABa, b^2aB\rangle \xrightarrow[(b^2aB)^A]{} \langle a,b|ABa, ab^2aBA\rangle$$
$$\langle a,b|ABa, ab^2aBA\rangle \xrightarrow[(ABa)^A]{} \langle a,b|B, abAB^2A\rangle$$

Instance: **T84**. Trivialization sequence length: 15

$$\langle a,b|a^2BabAbAB, b^2AbaBaBA\rangle \xrightarrow[(a^2BabAbAB)^a]{} \langle a,b|aBabAbABa, b^2AbaBaBA\rangle$$
$$\langle a,b|aBabAbABa, b^2AbaBaBA\rangle \xrightarrow[(b^2AbaBaBA)^b]{} \langle a,b|aBabAbABa, bAbaBaBAb\rangle$$
$$\langle a,b|aBabAbABa, bAbaBaBAb\rangle \xrightarrow[(bAbaBaBAb)^A]{} \langle a,b|aBabAbABa, abAbaBaBAbA\rangle$$
$$\langle a,b|aBabAbABa, abAbaBaBAbA\rangle \xrightarrow[abAbaBaBAbA*=aBabAbABa]{} \langle a,b|ab, aBabAbABa\rangle$$
$$\langle a,b|ab, aBabAbABa\rangle \xrightarrow[(ab)^a]{} \langle a,b|ba, aBabAbABa\rangle$$
$$\langle a,b|ba, aBabAbABa\rangle \xrightarrow[(aBabAbABa)^A]{} \langle a,b|ba, a^2BabAbAB\rangle$$
$$\langle a,b|ba, a^2BabAbAB\rangle \xrightarrow[a^2BabAbAB*=ba]{} \langle a,b|ba, a^2BabAb\rangle$$
$$\langle a,b|ba, a^2BabAb\rangle \xrightarrow[(ba)^{-1}]{} \langle a,b|AB, a^2BabAb\rangle$$
$$\langle a,b|AB, a^2BabAb\rangle \xrightarrow[(AB)^A]{} \langle a,b|BA, a^2BabAb\rangle$$
$$\langle a,b|BA, a^2BabAb\rangle \xrightarrow[a^2BabAb*=BA]{} \langle a,b|BA, a^2BabA^2\rangle$$
$$\langle a,b|BA, a^2BabA^2\rangle \xrightarrow[(BA)^{-1}]{} \langle a,b|ab, a^2BabA^2\rangle$$
$$\langle a,b|ab, a^2BabA^2\rangle \xrightarrow[(ab)^a]{} \langle a,b|ba, a^2BabA^2\rangle$$
$$\langle a,b|ba, a^2BabA^2\rangle \xrightarrow[(a^2BabA^2)^a]{} \langle a,b|ba, aBabA\rangle$$
$$\langle a,b|ba, aBabA\rangle \xrightarrow[(aBabA)^a]{} \langle a,b|ba, Bab\rangle$$
$$\langle a,b|ba, Bab\rangle \xrightarrow[(Bab)^B]{} \langle a,b|a, ba\rangle$$

Instance: **T85**. Trivialization sequence length: 24

$\langle a,b|ababA^2BaB, babaB^2AbA\rangle \xrightarrow[(ababA^2BaB)^b]{} \langle a,b|BababA^2Ba, babaB^2AbA\rangle$

$\langle a,b|BababA^2Ba, babaB^2AbA\rangle \xrightarrow[(BababA^2Ba)^A]{} \langle a,b|aBababA^2B, babaB^2AbA\rangle$

$\langle a,b|aBababA^2B, babaB^2AbA\rangle \xrightarrow[babaB^2AbA*=aBababA^2B]{} \langle a,b|aBababA^2B, babaBabA^2B\rangle$

$\langle a,b|aBababA^2B, babaBabA^2B\rangle \xrightarrow[(aBababA^2B)^{-1}]{} \langle a,b|ba^2BABAbA, babaBabA^2B\rangle$

$\langle a,b|ba^2BABAbA, babaBabA^2B\rangle \xrightarrow[(babaBabA^2B)^b]{} \langle a,b|abaBabA^2, ba^2BABAbA\rangle$

$\langle a,b|abaBabA^2, ba^2BABAbA\rangle \xrightarrow[(abaBabA^2)^a]{} \langle a,b|baBabA, ba^2BABAbA\rangle$

$\langle a,b|baBabA, ba^2BABAbA\rangle \xrightarrow[(baBabA)^b]{} \langle a,b|aBabAb, ba^2BABAbA\rangle$

$\langle a,b|aBabAb, ba^2BABAbA\rangle \xrightarrow[ba^2BABAbA*=aBabAb]{} \langle a,b|aBabAb, ba^2BA^2b\rangle$

$\langle a,b|aBabAb, ba^2BA^2b\rangle \xrightarrow[(ba^2BA^2b)^A]{} \langle a,b|aBabAb, aba^2BA^2bA\rangle$

$\langle a,b|aBabAb, aba^2BA^2bA\rangle \xrightarrow[aba^2BA^2bA*=aBabAb]{} \langle a,b|aBabAb, aba^2BAbAb\rangle$

$\langle a,b|aBabAb, aba^2BAbAb\rangle \xrightarrow[(aba^2BAbAb)^B]{} \langle a,b|aBabAb, baba^2BAbA\rangle$

$\langle a,b|aBabAb, baba^2BAbA\rangle \xrightarrow[baba^2BAbA*=aBabAb]{} \langle a,b|babab, aBabAb\rangle$

$\langle a,b|babab, aBabAb\rangle \xrightarrow[(aBabAb)^{-1}]{} \langle a,b|babab, BaBAbA\rangle$

$\langle a,b|babab, BaBAbA\rangle \xrightarrow[(BaBAbA)^B]{} \langle a,b|babab, aBAbAB\rangle$

$\langle a,b|babab, aBAbAB\rangle \xrightarrow[aBAbAB*=babab]{} \langle a,b|babab, aBAb^2ab\rangle$

$\langle a,b|babab, aBAb^2ab\rangle \xrightarrow[(babab)^{-1}]{} \langle a,b|BABAB, aBAb^2ab\rangle$

$\langle a,b|BABAB, aBAb^2ab\rangle \xrightarrow[(aBAb^2ab)^a]{} \langle a,b|BABAB, BAb^2aba\rangle$

$\langle a,b|BABAB, BAb^2aba\rangle \xrightarrow[(BABAB)^B]{} \langle a,b|ABAB^2, BAb^2aba\rangle$

$\langle a,b|ABAB^2, BAb^2aba\rangle \xrightarrow[BAb^2aba*=ABAB^2]{} \langle a,b|BA, ABAB^2\rangle$

$\langle a,b|BA, ABAB^2\rangle \xrightarrow[(BA)^B]{} \langle a,b|AB, ABAB^2\rangle$

$\langle a,b|AB, ABAB^2\rangle \xrightarrow[(AB)^B]{} \langle a,b|bAB^2, ABAB^2\rangle$

$\langle a,b|bAB^2, ABAB^2\rangle \xrightarrow[(bAB^2)^{-1}]{} \langle a,b|b^2aB, ABAB^2\rangle$

$\langle a,b|b^2aB, ABAB^2\rangle \xrightarrow[ABAB^2*=b^2aB]{} \langle a,b|AB^2, b^2aB\rangle$

$\langle a,b|AB^2, b^2aB\rangle \xrightarrow[AB^2*=b^2aB]{} \langle a,b|B, b^2aB\rangle$

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