Multi-dimensional Index Structures

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Software Development Technologies
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Review of the previous lectures

- Mining of massive datasets
- Classification and regression
- Evolution of database systems
- MapReduce

Outline

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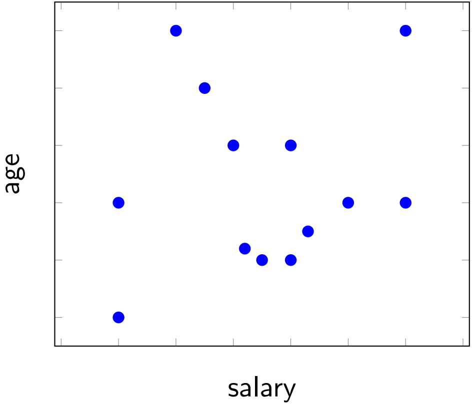
Multi-dimensional structures

- Conventional index structures are one dimensional and are not suitable for multi-dimensional search queries.



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  - Nearest-neighbor queries: find the closest one or more points to a given point.
  - Where-am-I queries: for a given point, where this point is located (in which shape).

Multi-dimensional queries with conventional indexes

- Consider a range query:

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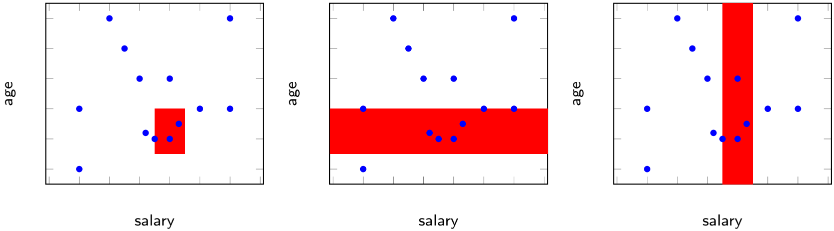

- To answer the query:
  - Scan along either index at once,
  - Intersect the elements returned by indexes
- This approach produces many false hits on each index!

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- Can we do better?

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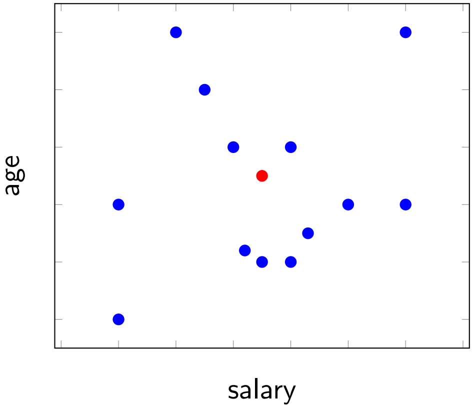

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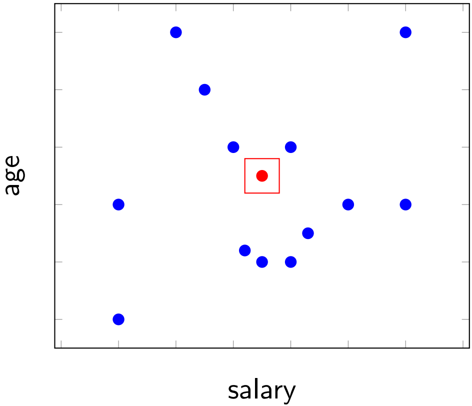

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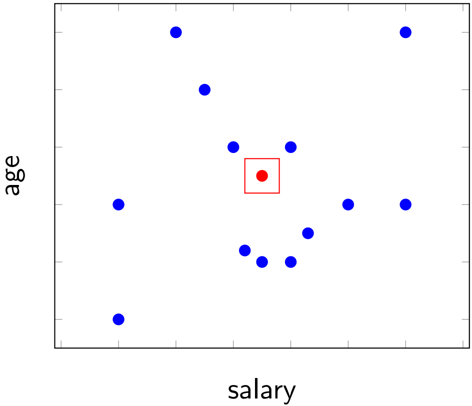

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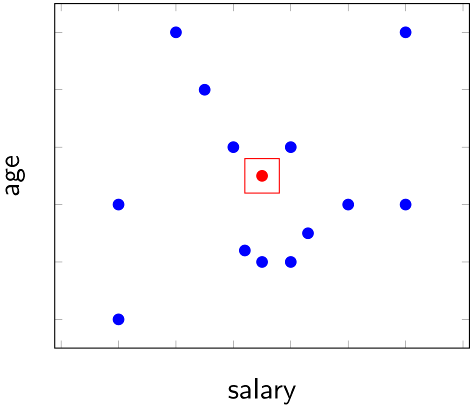

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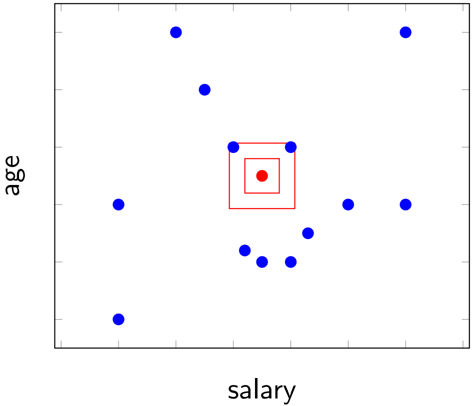

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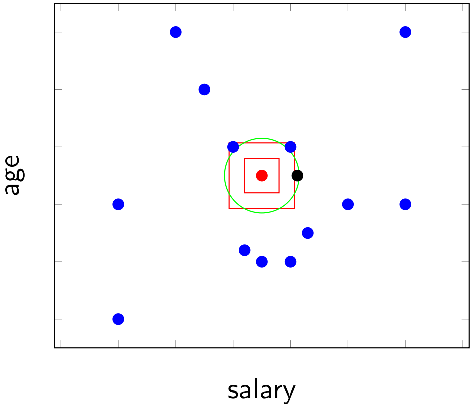

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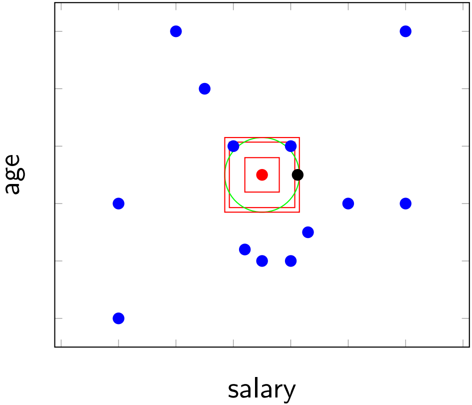

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  - Consider, whether there is the possibility that a closer point exists outside the range used. If so, increase appropriately the range once more and retrieve all points in the larger range to check.

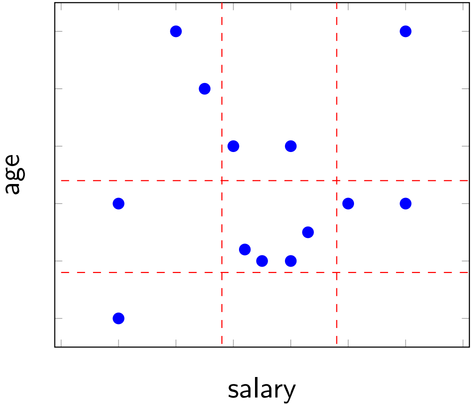
Multidimensional index structures

- Hash-table-like approaches
- Tree-like approaches

Outline

Grid files

- The space of points partitioned in a grid.



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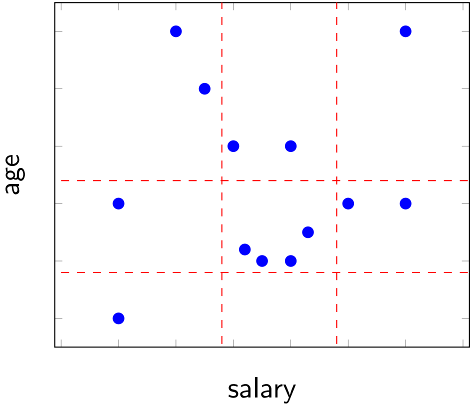

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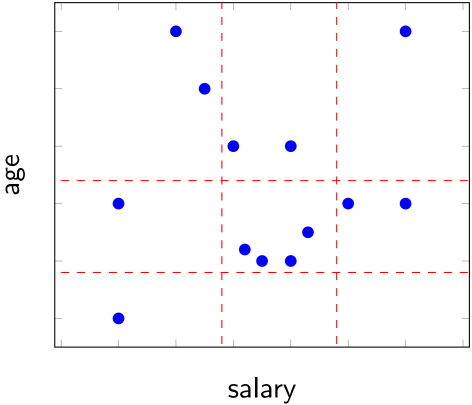

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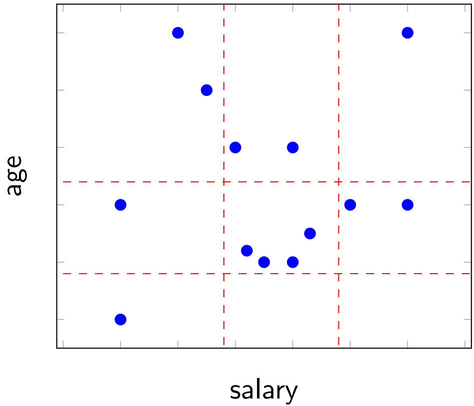

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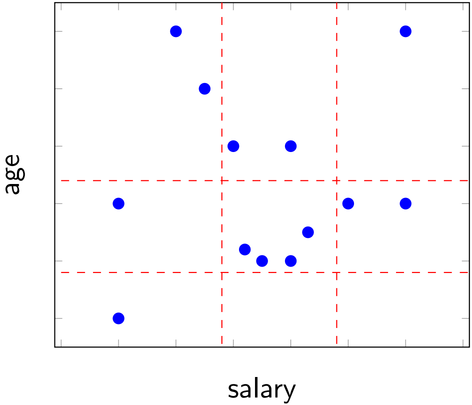
- Each region corresponds to a bucket.



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    - Reorganize the structure by adding or moving the grid lines.

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- If the matrix is sparse treat it as a relation whose attributes are corners of the nonempty buckets and a final attribute representing the pointer to the bucket.

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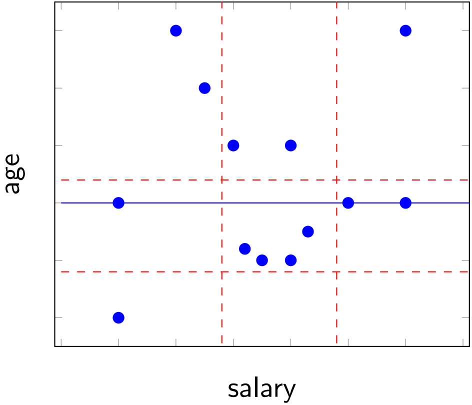
- Partial-match queries: We need to look at all the buckets in dimension not specified in the query



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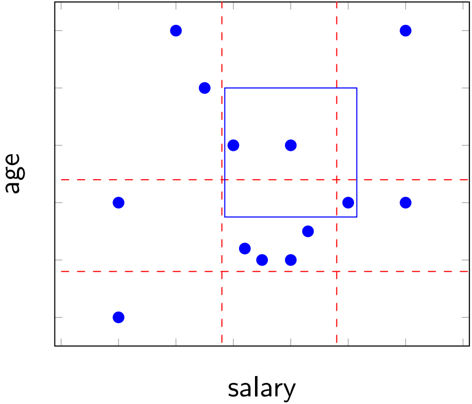
- Range queries: We need to look at all the buckets that cover the rectangular region defined by the query



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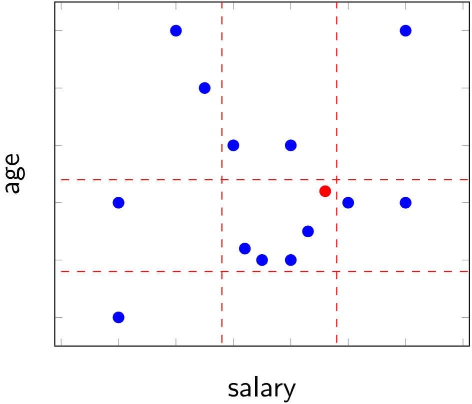

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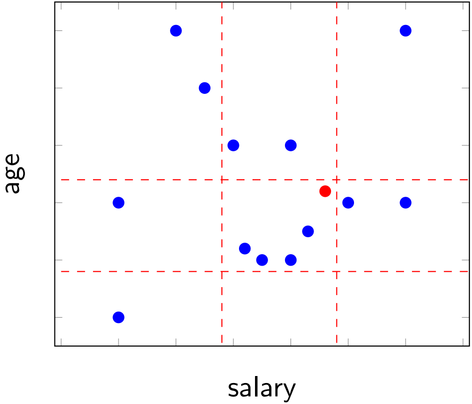

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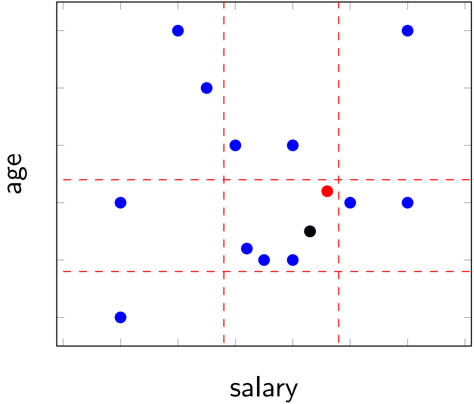

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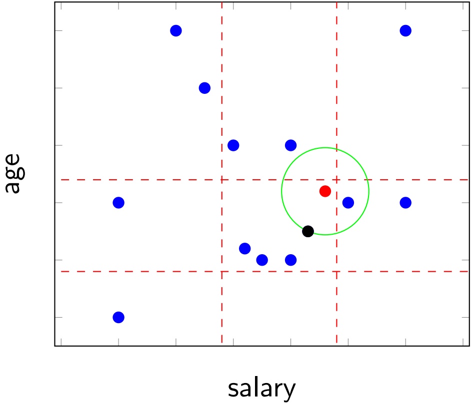

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- This is, however, useful only in the queries that specify values for both a and b.

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- The length of the hash is

$$\sum_{i=1}^{n} k_i = k$$

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    - For instance, for a value A of attribute a with $h_1 = 0101$, we know that the tuples with a-value A are in the 64 buckets whose numbers are of the form $0101 \cdots\cdots$.

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- Grid files will tend to leave many buckets empty if we deal with high dimensional and/or correlated data.
  - Hash tables are more efficient in this regard.

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  - More attributes in search key $\rightarrow$ index covers more queries, but takes up more disk space.
- **Example**: An index on attributes (a, b)
  - Search key is (a, b) combination.
  - Index entries sorted by a value.
  - Entries with same a value are sorted by b value, the so-called lexicographic sort.
  - A query `SELECT SUM(B) FROM R WHERE A=5` is covered by the index.

Multiple-key indexes

- Multiple-key index can be seen as a kind of an index of indexes, or a tree in which the nodes at each level are indexes for one attribute.
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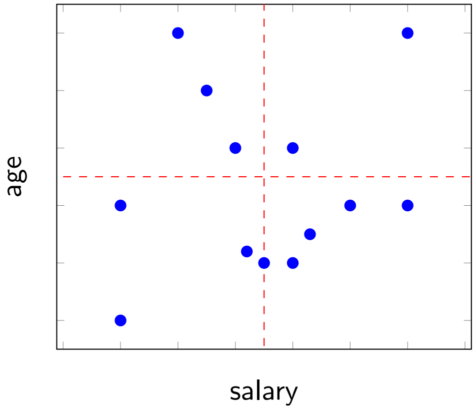
  - Entries with same a value are sorted by b value, the so-called lexicographic sort.
  - A query `SELECT SUM(B) FROM R WHERE A=5` is covered by the index.
  - But for a query `SELECT SUM(A) FROM R WHERE B=5` records with $B = 5$ are scattered throughout index.

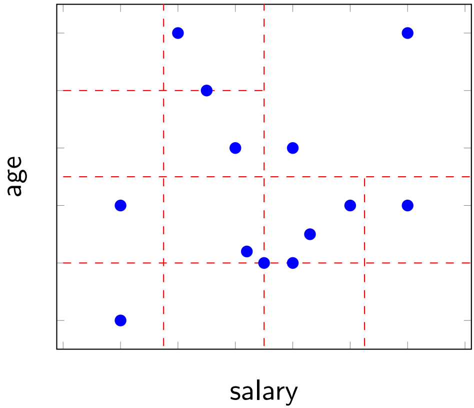
Quad trees

- Quad tree splits the space into 2^d equal sub-squares (cubes), where d is number of attributes.



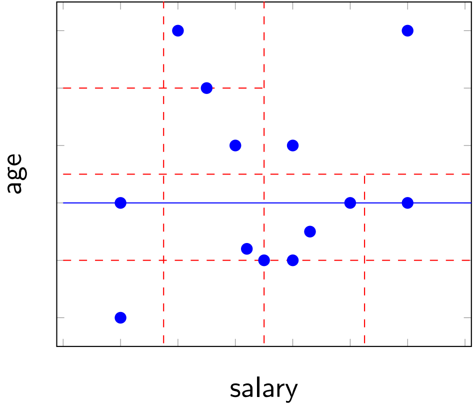
Quad trees

- Quad tree splits the space into 2^d equal sub-squares (cubes), where d is number of attributes.
- Repeat the partition until: only one pixel left; only one point left; only a few points left.



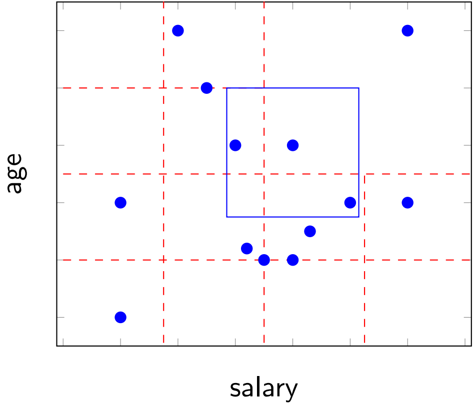
Quad trees

- Partial-match queries: We need to look at all cubes that intersect the condition of queries.



Quad trees

- Range queries: We need to look at all cubes that cover the region defined by the query



Quad trees

- Nearest neighbor search for point q:

```
Put  the root on the priority queue with the min distance = 0
Repeat {
    Pop the next node T from the priority queue
        if (min distance > r ) {
            the candidate is the nearest neighbor;
            break;
        }
        if (T is leaf) {
            examine point(s) in T and find the candidate;
            update r to be distance between q and the candidate;
        }
        else {
            for each child C of T {
                if( C intersects with the ball of radius r around q) {
                    compute the min distance from q to any point in C;
                    add C to the priority queue with the min distance;
                }
            }
    }
}
```

- Start search with $r = \infty$.
- Whenever a candidate point is found, update r.
- Only investigate nodes with respect to current r.

Quad trees

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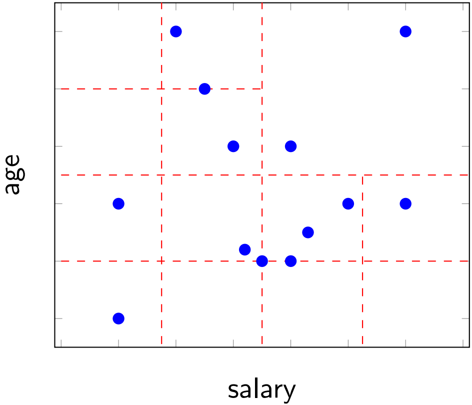

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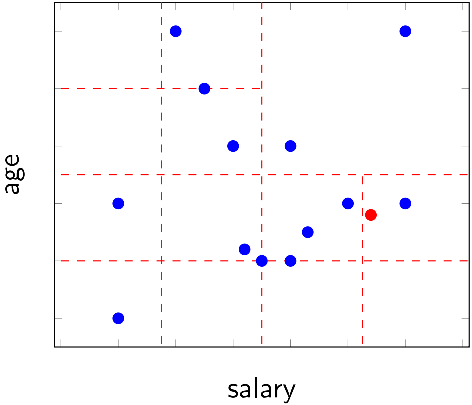

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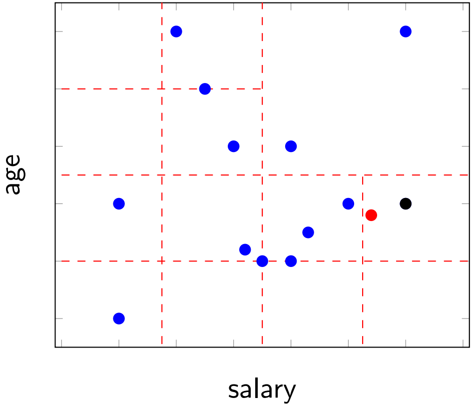

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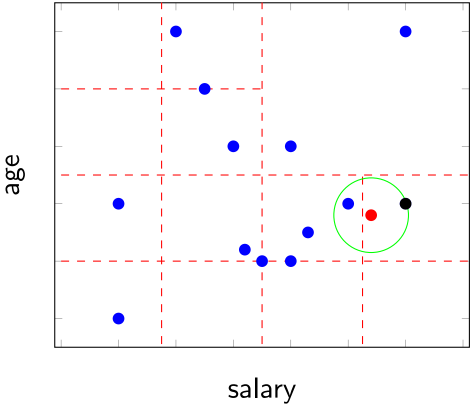

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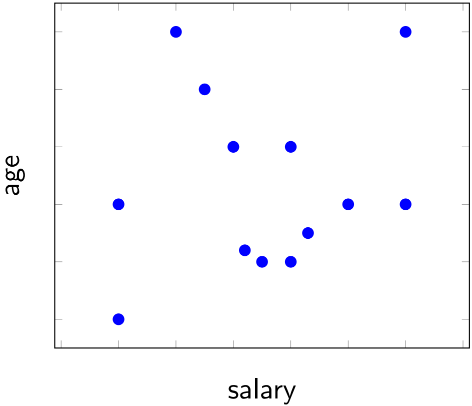
- Similar operations as for quad trees.
- Advantages: no (or less) empty spaces, only linear space.

kd-trees



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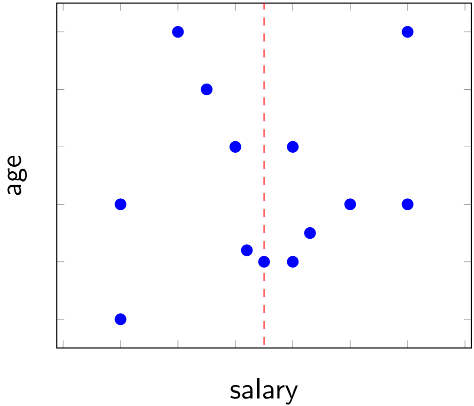

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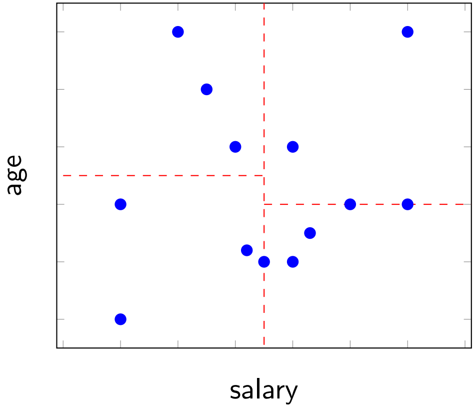

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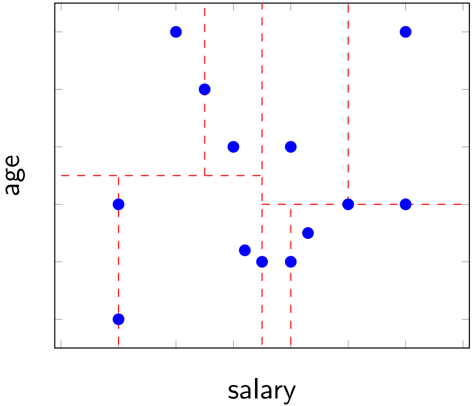
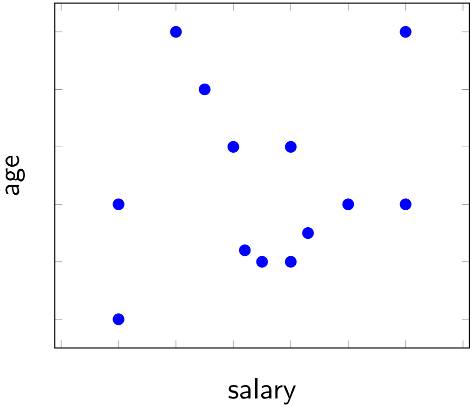


R-trees

- Similar in construction to B-trees.
- A kind of bottom-up approach (where kd-tree are top-down).
- Suitable for where-am-I queries, but also for the other types of queries (similar operations as before).
- Can deal with points and shapes.
- Avoid empty spaces.
- The regions may overlap.
- Work well in low dimensions, but may have problems with high. dimensions.

R-trees



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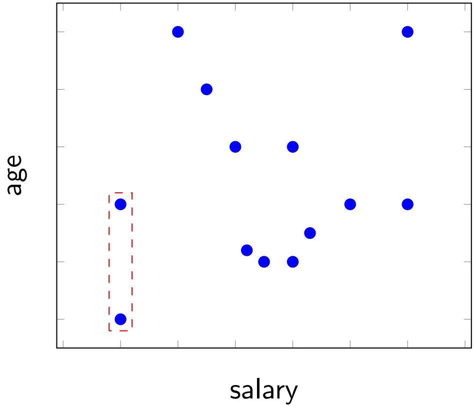

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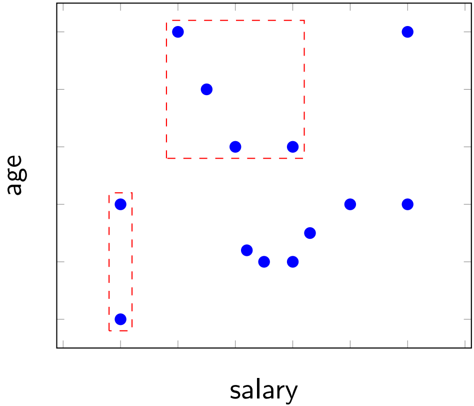

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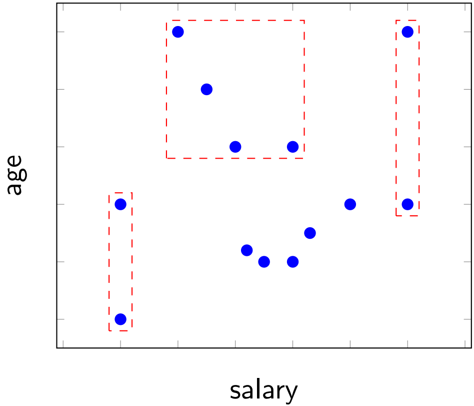

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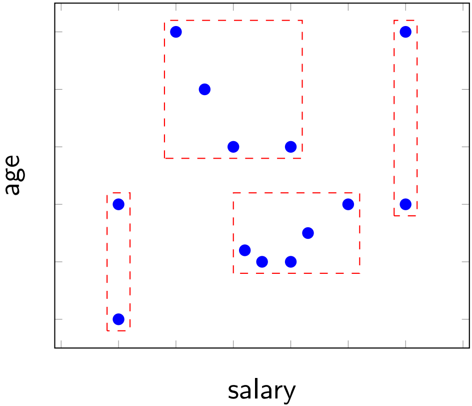

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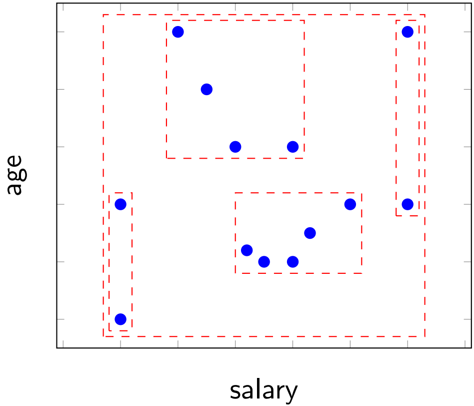

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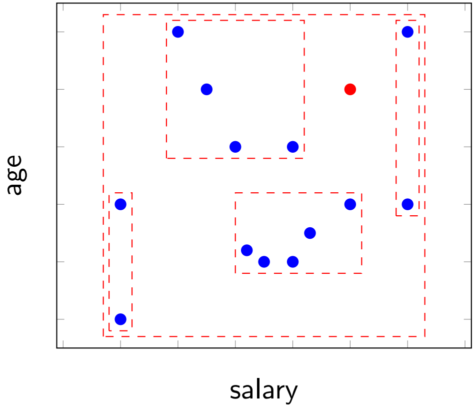

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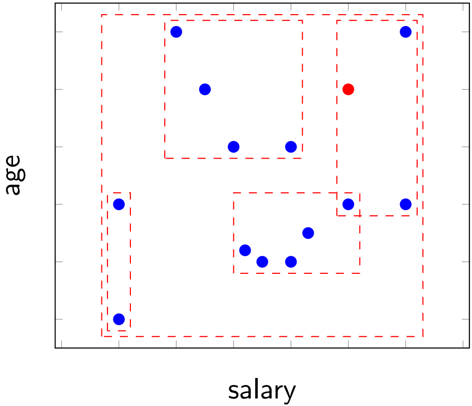

Additional aspects of multidimensional indexes

- Adaptation to secondary storage.
- Balancing of the tree structures.
- Storing data only in leaves or in internal nodes and leaves.
- Many variations of the structures presented.

Problems with nearest neighbor search

- Exponential query time
  - The query time is from $\log n$ to $\mathcal{O}(n)$, but can be exponential in d.
  - Tree structures are good when $n \gg 2^d$.
  - The curse of dimensionality.
- Solution: Approximate nearest neighbor search.

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  - In general, for d dimensions, we must go $(0.001)^{\frac{1}{d}}$.
  - For instance, for $d = 20$, it is 0.707, and for $d = 200$, it is 0.966.

Outline

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  - Applications: partial match queries, range queries, where-am-I-queries, nearest-neighbor search.
  - Approaches: hash table-based, tree-like structures.
  - Work good for low-dimensional problems – curse of dimensionality.

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