Mining Association Rules in Preference-ordered Data

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Abstract. Problems of discovering association rules in data sets containing semantic information about preference orders on domains of attributes are considered. Such attributes are called criteria and they are typically present in data related to economic issues, like financial or marketing data. We introduce a specific form of association rules involving criteria. Discovering such rules requires new concepts: semantic correlation of criteria, inconsistency of objects with respect to the dominance, credibility index. Properties of these rules concerning their generality and interdependencies are studied. We also sketch the way of mining such rules.

1 Introduction

Mining association rules is an important research and application area in knowledge discovery, see e.g. review in [4]. Several algorithms for discovering association rules from data have been already proposed for various types of attributes (nominal, numerical, etc.) [1, 5, 6]. However, these algorithms do not take into account semantic information often present in data sets, that is information about *preference order* in attribute domains. For example, when considering persons applying for a credit to a bank, an attribute like "salary" has a clear preference scale – the more, the better. The attributes with preference–ordered domains are called *criteria*. When several criteria are present in data, any reasonable regularity to be discovered have to take into account a *semantic correlation* between criteria. For example, in decision about credit granting, where two criteria are considered, "month salary" and "evaluation of bank risk concerning payment of a credit", these criteria are semantically correlated in the following sense: an improvement of month salary should not deteriorate the evaluation of bank risk. This semantic information is often present in data related to cost or quality, like in financial data; however, it is neglected by today's data mining tools.

The paper introduces consideration of criteria in association rules. Some of basic concepts have been inspired by dominance-based decision rules of *multiple criteria classification problems* introduced in [3]. In multiple criteria classification problems criteria are semantically correlated with preference–ordered decision classes. This results in a special syntax and semantics of decision rules [3, 7].

The paper is organized as follows. In the next section we present basic concepts of the proposed approach. The syntax of association rules is defined in section 3. Quality measures and properties of association rules, concerning their generality and interdependencies, are also studied in this section. In section 4, we sketch the way of mining such rules. Final section presents conclusions.

2 Semantically Correlated Criteria in Data Tables

Formally, a *data table* is a pair $DT = (U, A)$, where U is a nonempty finite set of objects called *universe*, A is a nonempty finite set of *attributes* such that $a : U \to V_a$ for every $a \in A$. The set V_a is a domain of a. Let $a(x)$ denotes the value of attribute $a \in A$ taken by object $x \in U$. The domains of attributes may be preference–ordered or not. In the first case the attributes are called *criteria*, while in the latter case we call them *regular attributes*. The domains of criteria are called preference scales (increasing or decreasing). Scales may be either ordinal or cardinal depending whether the strength of preference is meaningful for the scale or not. A preference scale of each criterion a induces a weak preference relation being a complete preorder (i.e. a strongly complete and transitive binary relation) $\succeq_a$ on set V_a. For values $\alpha, \beta \in V_a$, $\alpha \succeq_a \beta$ means that α is at least as good as β. The asymmetric part of $\succeq_a$ is denoted by $\succ_a$, while the symmetric part is denoted by $\sim_a$. In other words, object x is *preferred* to object y on criterion a if $a(x) \succ_a a(y)$ and x is *indifferent* to y if $a(x) \sim_a a(y)$.

Definition 2.1:
Let P be a subset of criteria A. The object x *dominates* object y with respect to subset P (denotation $x \gg_P y$) iff $a(x) \succeq_a a(y)$ for all $a \in P$.

Definition 2.2:
Let $a_i, a_j \in A$, $x, y \in U$ and $a_i(x), a_i(y) \in V_{ai}$, $a_j(x), a_j(y) \in V_{aj}$. If the following conditions are supposed to hold:

$$a_i(x) \succeq_{ai} a_i(y) \Rightarrow a_j(x) \succeq_{aj} a_j(y)$$
$$a_i(x) \preceq_{ai} a_i(y) \Rightarrow a_j(x) \preceq_{aj} a_j(y)$$

then criteria a_i and a_j are in the relation of *semantic correlation* (denotation $a_i SC a_j$).

Notice that relation SC is not necessarily complete in the set of criteria A.

Example:
Let us illustrate the above definitions. Consider a data table including 4 criteria $\{a_1, a_2, a_3, a_4\}$ and 10 objects $\{x_1, \ldots, x_{10}\}$ corresponding to persons applying for a credit to a bank. The criteria are: a_1 is the monthly salary with the increasing cardinal scale, e.g. counted in dollars; a_2 is an evaluation of the debt defined on the ordinal scale of three values: none $\succ$ small $\succ$ high; a_3 refers to amount of the requested credit with ordinal scale: small $\prec$ medium $\prec$ high; a_4 is an evaluation of bank risk concerning payment of a credit with ordinal scale: low $\succ$ medium $\succ$ high.

Table 1. Example of a bank data table

Object	a_1	a_2	a_3	a_4
x_1	3000	small	medium	low
x_2	5000	small	medium	medium
x_3	10000	none	high	low
x_4	7500	small	low	medium
x_5	5000	high	medium	high
x_6	10000	none	high	low
x_7	3000	high	low	high
x_8	7500	none	high	low
x_9	5000	high	low	high
x_{10}	7500	small	medium	medium

Remark that debt ratio a_2 is in SC relation with bank risk a_4. This means that with increase of the debt ratio we expect an increase of the bank risk. We assume that the following SC relations also hold: $a_1SCa_3, a_1SCa_4, a_2SCa_3, a_3SCa_4$.

3 Association Rules for Semantically Correlated Criteria

Let (a_i, l_i, u_i) denote an atomic formula defined in $A \times V_{a_i} \times V_{a_i}$, where criterion (or attribute) a_i takes value not worse than l_i and not better than u_i. If a_i is a regular attribute, then $l_i = u_i$. In this atomic formula a_i is treated as a variable.

Let $I_R = \{(a_i, l_i, u_i) : \exists x \in DT$ such that $a_i(x) \sim_a l_i$ and/or $a_i(x) \sim_a u_i$, for all $a_i \in A\}$, i.e. set I_R is composed of such atomic formulas that all values l_i and u_i exist in the data table DT for all $a_i \in A$.

Let C be a subset of I_R such that each criterion a_i is represented in at most one atomic formula. We will assume that in association rules formulas from C will be represented in a conjunction form. $[C] = \{a_i : (a_i, l_i, u_i) \in C\}$ is a set of criteria represented in C.

Definition 3.1:
Object $x \in U$ *supports* the conjunction of atomic formulas from set C if for all $(a_i, l_i, u_i) \in C$ $a_i(x) \succeq_{ai} l_i$ and $a_i(x) \preceq_{ai} u_i$.

Let *x_supp*(C) denote a set of objects x supporting C, while *support*(C) = card(*x_supp*(C)) will denote the cardinality of *x_supp*(C), shortly called *support* of C.

Definition 3.2:
An *association rule* is an implication of the form $C \to D$, where $C \subset I_R$, $D \subset I_R$ and $[C] \cap [D] = \emptyset$.

The rule $C \to D$ is characterized by support and confidence:

$$\begin{aligned} \textit{support}(C \to D) &= \textit{support}(C \cup D) \\ \textit{confidence}(C \to D) &= \textit{support}(C \cup D)/\textit{support}(C). \end{aligned}$$

In other words, the confidence of rule $C \to D$ is a percentage of objects in DT supporting C that also support D.

Remark that the syntax of association rules defined above is similar to generalized association rules proposed by Sirkant and Agrawal [6]. The main difference between our rules and the generalized association rules lies in their semantics. In our atomic formulas "$\preceq$" means a *preference relation*, while in generalized association rules "$\geq$" means simple relation between quantities.

Notice that atomic formulas $(a_i, l_i, u_i) \in C \cup D$ should be read as "$(a_i \succeq_{ai} l_i)$ and $(a_i \preceq_{ai} u_i)$". If l_i or u_i is an extreme value on preference scale of a_i and the other value is not, then the elementary condition boils down to either $(a_i \succeq_{ai} l_i)$ or $(a_i \preceq_{ai} u_i)$. For simplicity of further notation we skip index a_i in the symbol of the complete preorder $\succeq$.

Let us assume, without loss of generality, that criteria from sets $[C]$ and $[D]$ are numbered consecutively from 1 to p and $p+1$ to r, respectively, $p < r$, i.e. $[C] = \{a_1, \ldots, a_p\}$ and $[D] = \{a_{p+1}, \ldots, a_r\}$. Also, from now, we assume that domains of these criteria are recorded with the same direction of preference. The association rule may have one of three following forms:

(i) $(a_1 \succeq l_{a_1}) \wedge \ldots \wedge (a_p \succeq l_{a_p}) \;\to\; (a_{p+1} \succeq l_{a_{p+1}}) \wedge \ldots \wedge (a_r \succeq l_{a_r})$
(ii) $(a_1 \preceq u_{a_1}) \wedge \ldots \wedge (a_p \preceq u_{a_p}) \;\to\; (a_{p+1} \preceq u_{a_{p+1}}) \wedge \ldots \wedge (a_r \preceq u_{a_r})$
(iii) $(l_{a_i} \preceq a_1 \preceq u_{a_1}) \wedge \ldots \wedge (l_{a_p} \preceq a_p \preceq u_{a_p}) \;\to\; (l_{a_{p+1}} \preceq a_{p+1} \preceq u_{a_{p+1}}) \wedge \ldots \wedge (l_{a_r} \preceq a_r \preceq u_{a_r})$

We will require the association rule to be *robust*. This means that for association rule (i), there must exist an object x in data table DT such that $a_1(x) \sim l_{a_1}, \ldots, a_p(x) \sim l_{a_p}, a_{p+1}(x) \sim l_{a_{p+1}}, \ldots a_r(x) \sim l_{a_r}$. Similarly, for association rule (ii) there must exist an object x in data table DT such that $a_1(x) \sim u_{a_1}, \ldots, a_p(x) \sim u_{a_p}, a_{p+1}(x) \sim u_{a_{p+1}}, \ldots a_r(x) \sim u_{a_r}$. Finally, for association rule (iii) there must exist two objects $x, y \in U$ such that $a_1(x) \sim l_{a_1}, \ldots, a_p(x) \sim l_{a_p}, a_{p+1}(x) \sim l_{a_{p+1}}, \ldots a_r(x) \sim l_{a_r}$ and $a_1(y) \sim u_{a_1}, \ldots, a_p(y) \sim u_{a_p}, a_{p+1}(y) \sim u_{a_{p+1}}, \ldots a_r(y) \sim u_{a_r}$. Such supporting objects will be called *basic objects* of the robust association rules.

Concepts of robust rules and their basic objects permits to interpret the syntax of rules in terms of objects existing in the data table rather than of some "artificial" ones obtained by combining the attribute values describing other objects from the data table.

If C and D do not contain semantically correlated criteria split between the two sets, then the association rule $C \to D$ does not need to fulfill any additional condition in comparison with the case of regular attributes.

If, however, rule $C \to D$ involves at least two semantically correlated criteria, one in C and another in D, then it has to be checked for inconsistency of supporting objects. More formally, let us consider rule $C \to D$, $a_i \in [C]$, $a_j \in [D]$ and $a_i SC a_j$. If the *confidence*$(C \to D) < 1$ (or 100%), then there is a difference between sets *x_supp*$(C \cup D)$ and *x_supp*(C) denoted by Y. In other words, Y is composed of objects y that support C but not $C \cup D$. We will now define the concept of inconsistency using the concept of dominance introduced in Definition 2.1.

Definition 3.3:
Object $y \in Y$ is *inconsistent* with an object x supporting rule $C \rightarrow D$ if either $(x \gg_{[C]} y$ and $y \gg_{[D]} x)$ or $(y \gg_{[C]} x$ and $x \gg_{[D]} y)$.

Inconsistency of some objects from Y with some other objects supporting the rule $C \rightarrow D$ means that there exist an object dominating the rule basic object on criteria from the antecedent part while being dominated by the basic object on criteria from the consequent part of the rule. Such an object clearly weakens the credibility of the regularity expressed by the rule.

For this reason we introduce a new quality measure for association rules called *credibility* and defined as follows:

$$credibility(C \rightarrow D) = confidence(C \cup D) - \alpha \cdot (INC\{W, Y\}/support(C))$$

where $\alpha \in [0, 1]$ is a parameter controlling the accepted level of inconsistency in the association rule, $W = x\_supp(C \cup D)$ and $INC(W, Y)$ is the cardinality of the set of objects from W being inconsistent with some objects from Y.

The parameter $\alpha = 1$ corresponds to exclusion of all inconsistent objects from the support of the rule, while $\alpha = 0$ makes *credibility*$(C \rightarrow D) =$ *confidence*$(C \rightarrow D)$. The user should tune the value α with respect to his tolerance of inconsistent objects in the rule support. If the credibility of the association rule is smaller than user's defined threshold *mincredibility*, the rule will be deleted from the set of discovered rules.

Continuation of the example:
To illustrate the above definitions let us come back to the example. Consider the following conjunction C, which could be generated from data shown in Table 1: $(a_1 \succeq 5000)(a_2 \succeq small)(a_3 \succeq medium)$. The above conjunction can be expressed as a statement saying "*month salary at least \$5000, debt none or small, credit medium or high*". Its support is equal to 50% as $x\_supp(C \cup D) = \{x_2, x_3, x_6, x_8, x_{10}\}$.

Let us consider the following candidate for a rule $C \rightarrow D$: $(a_1 \succeq 5000)(a_2 \succeq small) \rightarrow (a_3 \succeq medium)$. This rule says *if an applicant have month salary at least \$5000 and debt none or small, then he gets credit medium or high.* It is supported by 50% objects. As $x\_supp(C) = \{x_2, x_3, x_4, x_6, x_8, x_{10}\}$, the rule confidence is equal to $5/6 \cdot 100\% = 83.3\%$. Let us remind that in this rule $a_1 SC a_3$ and $a_2 SC a_3$.

The object x_4 (with description \$7500, *small, low, medium*) supports the antecedent of the rule but does not support the consequence of this rule, therefore $Y = \{x_4\}$. One should check possible inconsistencies of object x_4 with all other objects from $x\_supp(C \rightarrow D)$. There are two inconsistencies of object x_4 with objects x_2 (with description \$5000, *small, medium, medium*) and object x_{10} (with description \$7500, *small, medium, medium*), i.e. $x_4 \gg_{[C]} x_2$ but $x_2 \gg_{[D]} x_4$, $x_4 \gg_{[C]} x_{10}$ but $x_{10} \gg_{[D]} x_4$. In other words, object x_4 is evaluated better than object x_2 or evaluated the same as object x_{10} with respect to amount of salary and debt but gets smaller credit than these objects. We intuitively may expect that the better assessment of object x_4 on the first criteria from $[C]$ should imply its better assessment of the criterion a_3. However, the examples $\{x_2, x_{10}\}$ show that these better values of a_1 and a_2 implied deterioration of value for a_3, what finally makes our doubtful as to the credibility of the presented rule. Therefore, if we want to consider complete influence of inconsistent objects x_2 and x_{10} on the quality of the rule, we determine $\alpha = 1$. In such a case, the rule credibility is equal to 83.3% – (2/6)*100% = 50%. □

Besides the above discussed inconsistency of objects supporting the rule, it is necessary to consider two next problems for discovered association rules, i.e.:

- redundancy of some rules,
- interdependency between two rules.

First problem refers to relations between two rules, where one rule contains more general antecedent and more precise consequent than the other rule. In such a case one is not interested in accepting both rules but prefers to stay with the more general (more informative) rule and to reject the less general one. The concept of more general rule is defined below.

Definition 3.4:
Given two rules $C \to D$ and $C' \to D'$, where $C \cup D = \{(a_1, l_1, u_1), \ldots, (a_m, l_m, u_m)\}$ and $C' \cup D' = \{(b_1, l_1^{'}, u_1^{'}), \ldots, (b_n, l_n^{'}, u_n^{'})\}$, the rule $C' \to D'$ is *more general* than the rule $C \to D$ if the following conditions are satisfied:

1. $[C'] \subseteq [C]$ and $[D] \subseteq [D']$;
2. $\forall a_i \in [C]$ and $\forall b_i \in [C']$ where $a_i = b_i$ we have $l_i^{'} \preceq l_i \preceq u_i \preceq u_i^{'}$;
3. $\forall a_i \in [D]$ and $\forall b_i \in [D']$ where $a_i = b_i$ we have $l_i \preceq l_i^{'} \preceq u_i^{'} \preceq u_i$;
4. $\mathit{confidence}(C \cup D) \leq \mathit{confidence}(C' \cup D')$.

If one of the above conditions is not fulfilled, one cannot determine which rule is redundant and could be removed.

The second problem concerns *interdependency of rules*. Let us illustrate it by considering a simple example of two rules: $(a \preceq 3) \to (b \preceq 4)$ and $(a \succeq 2) \to (b \succeq 2)$. Informally speaking these rules have intersected elementary conditions. These rules can be synthesized into $(2 \preceq a \preceq 3) \to (2 \preceq b \preceq 4)$ if the support, confidence and credibility of the resulting rule is satisfactory.

Definition 3.5:
Two rules $C \to D$ and $C' \to D'$ are *interdependent*, if $[C] = [C']$ and $[D] = [D']$ and $\forall a_i \in [C] \cup [D]$ $l_i \preceq u_i^{'}$ or $l_i^{'} \preceq u_i$.

To deal with this situation we propose to add a new (intersected) rule with elementary conditions defined as $(a_i, max\{l, l^{'}\}, \min\{u, u^{'}\})$. Moreover, for the rule created in this way one has to check its support, confidence and credibility.

Continuation of the example.
Let us illustrate the problem of redundant rules by means of two rules, which could be generated from examples represented in Table 1:
r_1 : $(a_1 \preceq \$3000) \to (a_2 \preceq \text{small})$ with support=10% and confidence=100%
r_2 : $(a_1 \preceq \$5000) \to (a_2 \preceq \text{small})$ with support=50% and confidence=100%
The rule r_2 is more general than rule r_1 because the antecedent part of r_2 covers more examples than that of r_1, while the consequence parts of both rules and their confidence ratios are the same. Therefore, rule r_1 is removed.

Then, compare the rule r_2 with another rule r_3 of the following form:
r_3 : $(a_1 \preceq \$5000) \rightarrow (a_2 \preceq \text{high})$ with support=30% and confidence=60%
One can easily notice that rule r_3 is not more general than rule r_2 because both rules r_2 and r_3 satisfy conditions 1,2 and 3 of Definition 3.4, but they do not satisfy condition 4. In fact, rule r_3 has worse confidence, so one should stay with both rules.

Intersection of rules will be illustrated by means of two other rules that can be generated from examples presented in Table 1.
r_4 : $(a_2 \preceq \text{small}) \rightarrow (a_1 \leq \$7500) \wedge (a_3 \preceq \text{medium})$ with support = 70% and confidence = 100%
r_5 : $(a_2 \succeq \text{small}) \rightarrow (a_1 \geq \$5000) \wedge (a_3 \succeq \text{medium})$ with support = 50% and confidence = 71%
These two rules can be synthesized in a new rule:
r_6 : $(a_2, \text{small,small}) \rightarrow (a_1, \$5000, \$7500) \wedge (a_3, \text{medium,medium})$ or shortly written as $(a_2 \sim \text{small}) \rightarrow (\$5000 \preceq a_1 \preceq \$7500) \wedge (a_3 \sim \text{medium})$. □

Given a data table, the problem of mining association rules is now the following:

1. Find all association rules that have support, confidence and credibility at least equal to the user specified thresholds *minsupp*, *minconf* and *mincredbility* (minimum accepted support, confidence and credibility, respectively).
2. Check the redundancy and interdependency of these rules.

4 Algorithm of Mining Association Rules

We propose to induce the new kind of association rules by an algorithm which is an extension of "original" Agrawal et al. algorithm [2]. The problem of mining such rules is decomposed into two subproblems:

1. Find all conjunctions of elementary conditions (also called itemsets) that have support at least equal to the required *minsupp* parameter; they are shortly called frequent itemsets.
2. Use the frequent itemsets to generate the rules having evaluation measures not less than user's defined parameters *minsupp*, *minconf* and *mincredbility.*

Due to the limited size of this paper we can only sketch this algorithm stressing the main extensions. In the first phase single conditions are defined as $(a_i \preceq u_i)$ or $(a_i \succeq l_i)$, respectively. If a criterion is defined on the ordered set of values $\{v_i^1, v_i^2, \ldots, v_i^{n-1}, v_i^n\}$, then the following elementary conditions are considered $(a_i \preceq v_i^1)$, $(a_i \preceq v_i^2)$, …, $(a_i \preceq v_i^{n-1})$ and $(a_i \succeq v_i^2)$, $(a_i \succeq v_i^3)$, …, $(a_i \succeq v_i^n)$. Only these elementary conditions that have support not less than *minsupp* become frequent itemsets of size 1 (denoted as L_1). Then, in a subsequent pass, say pass k, the frequent itemsets L_{k-1} (found in k-1 pass) are used to create candidate itemsets C_k (see the description of *Apriorigen* function in [2]). However, combinations of conditions referring to the same criterion are not allowed. Moreover, we require to select only robust itemsets. Any itemset defined by conditions $(a_i \succeq r_{ai})$ or $(a_i \preceq r_{ai})$, where $a_i \in C \subseteq A$, is robust iff $\exists x \in DT$

such that $\forall a_i\ a_i(x) \sim r_{ai}$. While combining conditions for criteria having the same direction of preference, we also require use of the same type of preference operator either $\preceq$ or $\succeq$. While performing *join* and *prune* steps of *Apriorigen* function we use proper lexicographic order of candidates C_k, where we may use properties of dominance between different combinations of conditions. It means that for instance given two combinations $(a_1 \succeq r_1)$, $(a_2 \succeq r_2)$, $\ldots$, $(a_k \succeq r_k)$ and $(a_1 \succeq p_1)$, $(a_2 \succeq p_2)$, $\ldots$, $(a_k \succeq p_k)$, where $\forall i\ \ p_i \succeq r_i$, if the first candidate is not frequent, then also the second will not be frequent.

The generating rule phase extends the schema of *ap_genrules* procedure defined in [2]. For every frequent itemset we generate rules starting from consequence with one element, then with two elements, etc. The modifications include operations performed in the following order:

1. For each rule with SC criteria and confidence below 100% check consistency of supporting objects. Calculate the credibility of the rule. Delete rules with credibility less than *mincredibility*.
2. For all rules test their generality.
3. Test possible interdependency of rules.

Let us comment that this extended algorithm is more time consuming than its original version. The most complex part is the operation of checking consistency of supporting objects. In the first phase of *Apriori* candidate generation procedure the number of considered values for each attribute is a crucial issue. If one has to consider too many values, the discretization of criterion scales may be applied in early phase of the algorithm - see also discussion of similar problems for mining quantitative association rules [5,6]. Here, we should stress the role of robust items which limit the allowed combinations of conditions to objects existing in data tables. We have also experimentally checked that it can significantly reduce the number of frequent itemsets and final rules. Obviously, the proper choice of minimal support, confidence and credibility, depending on the considered data, is quite important.

5 Conclusions

The novelty of the approach to Knowledge Discovery proposed in this paper is the handling of preference information in data. Preference information is a fundamental concept of Decision Theory and it is well known that, in particular, multiple criteria decision problems have no solution unless this information is taken into account. Nowadays methods of Knowledge Discovery do not take into account the preference information present in data and thus they may lead to wrong conclusions. For instance, they may discover a rule that says "the higher the month salary, the more risky the credit payment for the bank", because in the data there were objects supporting this statement and not identified as inconsistent.

The present proposal is based on a specific understanding of the association rules that handles inconsistency between objects with respect to semantically correlated criteria.

This proposal was applied to real data concerning technical diagnostics for motor vehicles. We compared it with typical approach for mining association rules that do not take into account criteria. For chosen values of rule parameters, we observed that 78% of typical association rules were not consistent if preference ordered scales of criteria and semantically correlation between criteria were considered.

The presented approach can be combined with methods handling "imperfect" character of data like missing or imprecise values of attributes and criteria. Moreover, other future research topics could concern defining specific rule interest measures for selecting the most relevant results among the quite large number of rules inferred from data. Yet another important problem is evaluation of the computation cost of the proposed approach on various data sets. We are also interested in developing applications of the association rules on preference ordered data in some real life problems (e.g. financial markets).

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