An algorithm for induction of decision rules consistent with the dominance principle

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Abstract. Induction of decision rules within the dominance-based rough set approach to the multiple-criteria sorting decision problem is discussed in this paper. We introduce an algorithm called DOMLEM that induces a minimal set of generalized decision rules consistent with the dominance principle. An extension of this algorithm for a variable consistency model of dominance based rough set approach is also presented.

1. Introduction

The key aspect of Multiple-Criteria Decision Analysis (MCDA) is consideration of objects described by multiple criteria representing conflicting points of view. Criteria are attributes with preference-ordered domains. For example, if decisions about cars are based on such characteristics as price and fuel consumption, these characteristics should be treated as criteria because a decision maker usually considers lower price as better than higher price and moderate fuel consumption more desirable than higher consumption. Regular attributes, such as e.g. colour and country of production are different from criteria because their domains are not preference-ordered.

As pointed out in [1,6] the Classical Rough Set Approach (CRSA) cannot be applied to *multipl-criteria decision problems*, as it does not consider criteria but only regular attributes. Therefore, it cannot discover another kind of *inconsistency* concerning violation of the *dominance principle*, which requires that objects having better evaluations (or at least the same evaluations) cannot be assigned to a worse class. For this reason, Greco, Matarazzo and Slowinski [1] have proposed an extension of the rough sets theory, called Dominance-based Rough Set Approach (DRSA), that is able to deal with this inconsistency typical to exemplary decisions in MCDA problems. This innovation is mainly based on substitution of the *indiscernibility relation* by a *dominance relation*. In this paper we focus our attention on one of the major classes of MCDA problems which is a counterpart of multiple-attribute classification problem within MCDA: it is called *multiple-criteria sorting problem*. It concerns an assignment of some objects evaluated by a set of criteria into some pre-defined and preference-ordered decision classes (categories).

Within DRSA, due to preference-order among decision classes, the sets to be approximated are, so-called, *upward* and *downward unions* of decision classes. For each decision class, the corresponding upward union is composed of this class and all better classes. Analogously, the downward union corresponding to a decision class is composed of this class and all worse classes. The consequence of considering criteria instead of regular attributes is the necessity of satisfying the dominance principle, which requires a change of the approximating items from indiscernibility sets to dominating and dominated sets. Given object x, dominating set is composed of all objects evaluated not worse than x on all considered criteria, while dominated set is composed of all objects evaluated not better than x on all considered criteria. Moreover, the syntax of DRSA decision rules is different from CRSA decision rules. In the condition part of these rules, the elementary conditions have the form: "evaluation of object x on criterion q is at least as good as a given level" or "evaluation of object x on criterion q is at most as good as a given level". In the decision part of these rules, the conclusion has the form: "object x belongs (or possibly belongs) to at least a given class" or " object x belongs (or possibly belongs) to at most a given class".

The aim of this paper is to present an algorithm for inducing DRSA decision rules. This algorithm, called DOMLEM, is focused on inducing a minimal set of rules that cover all examples in the input data. Moreover, we will show how this algorithm can be extended to induce decision rules in a generalization of DRSA, called Variable Consistency DRSA model (VC-DRSA). This generalization accepts a limited number of counterexamples in rough approximations and in decision rules [2].

The paper is organized as follows. In the next sections, the main concepts of DRSA are briefly presented. In section 3, the DOMLEM algorithm is introduced and illustrated by a didactic example. Extensions of the DOMLEM algorithm for VC-DRSA model are discussed in section 4. Conclusions are grouped in final section.

2. Dominance-based Rough Set Approach

Basic concepts of DRSA are briefly presented (for more details see e.g. [1]). It is assumed that examplary decisions are stored in a *data table*. By this table we understand the 4-tuple $S=\langle U,Q,V,f\rangle$, where U is a finite set of objects, Q is a finite set of *attributes*, $V=\bigcup_{q\in Q}V_q$ and V_q is a domain of the attribute q, and f: $U\times Q\rightarrow V$ is a total function such that $f(x,q)\in V_q$ for every $q\in Q$, $x\in U$. The set Q is, in general, divided into set C of *condition attributes* and set D of *decision attributes*.

Assuming that all condition attributes $q\in C$ are criteria, let S_q be an *outranking relation* on U with respect to criterion q such that xS_qy means "x is at least as good as y with respect to criterion q". Furthermore, assuming that the set of decision attributes D (possibly a singleton $\{d\}$) makes a partition of U into a finite number of classes, let $\boldsymbol{Cl}=\{Cl_t,\ t\in T\}$, $T=\{1,..., n\}$, be a set of these classes such that each $x\in U$ belongs to one and only one $Cl_t\in\boldsymbol{Cl}$. We suppose that the classes are ordered, i.e. for all $r,s\in T$, such that $r>s$, the objects from Cl_r are preferred to the objects from Cl_s. The above assumptions are typical for consideration of a *multiple-criteria sorting problem*.

The sets to be approximated are *upward union* and *downward union* of classes, respectively: $Cl_t^{\geq}=\bigcup_{s\geq t}Cl_s$, $Cl_t^{\leq}=\bigcup_{s\leq t}Cl_s$, t=1,...,n.

Then, the indiscernibility relation is substituted by a *dominance relation*. We say that x *dominates* y with respect to $P\subseteq C$, denoted by xD_Py, if xS_qy for all $q\in P$. The dominance relation is reflexive and transitive. Given $P\subseteq C$ and $x\in U$, the "granules of knowledge" used for approximation in DRSA are:

- a set of objects dominating x, called *P-dominating set*, $D_P^+(x)=\{y\in U: yD_Px\}$,
- a set of objects dominated by x, called *P-dominated set*, $D_P^-(x)=\{y\in U: xD_Py\}$.

Using $D_P^+(x)$ sets, *P-lower* and *P-upper approximation* of $Cl_t^{\geq}$ are defined as:

$$\underline{P}(Cl_t^{\geq})=\{x\in U: D_P^+(x)\subseteq Cl_t^{\geq}\},\quad \overline{P}(Cl_t^{\geq})=\bigcup_{x\in Cl_t^{\geq}} D_P^+(x),\quad \text{for } t=1,...,n.$$

Analogously, *P-lower* and *P-upper approximation* of $Cl_t^{\leq}$ are defined as:

$$\underline{P}(Cl_t^{\leq})=\{x\in U: D_P^-(x)\subseteq Cl_t^{\leq}\},\quad \overline{P}(Cl_t^{\leq})=\bigcup_{x\in Cl_t^{\leq}} D_P^-(x),\quad \text{for } t=1,...,n.$$

The *P-boundaries* (P-doubtful regions) of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are defined as: $Bn_P(Cl_t^{\geq})=\overline{P}(Cl_t^{\geq})-\underline{P}(Cl_t^{\geq})$, $Bn_P(Cl_t^{\leq})=\overline{P}(Cl_t^{\leq})-\underline{P}(Cl_t^{\leq})$, for $t=1,...,n$.

These approximations of upward and downward unions of classes can serve to induce generalized "*if... then...*" decision rules. For a given upward or downward union $Cl_t^{\geq}$ or $Cl_s^{\leq}$, $s,t\in T$, the rules induced under a hypothesis that objects belonging to $\underline{P}(Cl_t^{\geq})$ or to $\underline{P}(Cl_s^{\leq})$ are *positive* and all the others *negative*, suggest an assignment of an object to "at least class Cl_t" or to "at most class Cl_s", respectively. They are called *certain* $D_{\geq}$- (or $D_{\leq}$)-*decision rules* because they assign objects to unions of decision classes without any ambiguity. Next, if upper approximations differ from lower ones, *approximate* $D_{\geq\leq}$- *decision rules* can be induced under a hypothesis that objects belonging to the intersection $\overline{P}(Cl_s^{\leq})\cap\overline{P}(Cl_t^{\geq})$ ($s<t$) are *positive* and all the others *negative*. They suggest an assignment of objects to some classes between Cl_s and Cl_t. Yet another option is to induce $D_{\geq}$- (or $D_{\leq}$)-*possible decision rules* instead of approximate ones under the hypothesis that objects belonging to $\overline{P}(Cl_t^{\geq})$ or to $\overline{P}(Cl_t^{\leq})$ are *positive* and all the others *negative*. These rules suggest that an object *could belong* to "at least class Cl_t" or "at most class Cl_s", respectively.

Assuming that for each criterion $q\in C$, $V_q\subseteq\mathbf{R}$ (i.e. V_q is quantitative) and that for each $x,y\in U$, $f(x,q)\geq f(y,q)$ implies xS_qy (i.e. V_q is preference-ordered), the following five types of decision rules can be considered:

1) *certain* $D_{\geq}$-*decision rules* with the following syntax:
   if $f(x,q_1)\geq r_{q1}$ *and* $f(x,q_2)\geq r_{q2}$ *and* $...f(x,q_p)\geq r_{qp}$, *then* $x\in Cl_t^{\geq}$,
2) *possible* $D_{\geq}$-*decision rules* with the following syntax:
   if $f(x,q_1)\geq r_{q1}$ *and* $f(x,q_2)\geq r_{q2}$ *and* $...f(x,q_p)\geq r_{qp}$, *then* x could belong to $Cl_t^{\geq}$,
3) *certain* $D_{\leq}$-*decision rules* with the following syntax:
   if $f(x,q_1)\leq r_{q1}$ *and* $f(x,q_2)\leq r_{q2}$ *and* $...f(x,q_p)\leq r_{qp}$, *then* $x\in Cl_t^{\leq}$,
4) *possible* $D_{\leq}$-*decision rules* with the following syntax:

if $f(x,q_1) \le r_{q1}$ *and* $f(x,q_2) \le r_{q2}$ *and ...* $f(x,q_p) \le r_{qp}$ *, then* x could belong to $Cl_t^{\le}$,

where $P=\{q_1,...,q_p\} \subseteq C$, $(r_{q1},...,r_{qp}) \in V_{q1} \times V_{q2} \times ... \times V_{qp}$ and $t \in T$;

5) *approximate* $D_{\ge\le}$*-decision rules* with the following syntax:

if $f(x,q_1) \ge r_{q1}$ *and* $f(x,q_2) \ge r_{q2}$ *and ...* $f(x,q_k) \ge r_{qk}$ *and* $f(x,q_{k+1}) \le r_{qk+1}$ *and ...* $f(x,q_p) \le r_{qp}$ *, then* $x \in Cl_s \cup Cl_{s+1} \cup \ldots \cup Cl_t$,

where $O'=\{q_1,...,q_k\} \subseteq C$, $O''=\{q_{k+1},...,q_p\} \subseteq C$, $P=O' \cup O''$, O' and O'' not necessarily disjoint, $(r_{q1},...,r_{qp}) \in V_{q1} \times V_{q2} \times ... \times V_{qp}$, $s,t \in T$ such that $s<t$. As it is possible that $\{q_1,...,q_k\} \cap \{q_{k+1},...,q_p\} \ne \emptyset$, in the condition part of a $D_{\ge\le}$-decision rule we can have "$f(x,q) \ge r_q$" and "$f(x,q) \le r'_q$", where $r_q \le r'_q$, for some $q \in C$. Moreover, if $r_q = r'_q$, the two conditions boil down to "$f(x,q)=r_q$".

The rules of type 1) and 3) represent certain knowledge extracted from the data table, while the rules of type 2) and 4) represent possible knowledge, and rules of type 5) represent ambiguous knowledge.

Moreover, each decision rule should be minimal. Since a decision rule is an implication, by a *minimal* decision rule we understand such an implication that there is no other implication with an antecedent of at least the same weakness (in other words, rule using a subset of elementary conditions or/and weaker elementary conditions) and a consequent of at least the same strength (in other words, rule assigning objects to the same union or sub-union of classes).

Consider a $D_{\ge}$-decision rule "*if* $f(x,q_1) \ge r_{q1}$ *and* $f(x,q_2) \ge r_{q2}$ *and ...* $f(x,q_p) \ge r_{qp}$*, then* $x \in Cl_t^{\ge}$ ". If there exists an object $y \in \underline{P}(Cl_t^{\ge})$ such that $f(y,q_1)=r_{q1}$ *and* $f(y,q_2)=r_{q2}$ *and ...* $f(y,q_p)=r_{qp}$, then y is called *basis* of the rule. Each $D_{\ge}$-decision rule having a basis is called *robust* because it is "founded" on an object existing in the data table. Analogous definition of robust decision rules holds for the other types of rules.

We say that an object *supports* a decision rule if it matches both condition and decision parts of the rule. On the other hand, an object is *covered* by a decision rule if it matches the condition part of the rule.

A set of certain and approximate decision rules is *complete* if three following conditions are fulfilled: each $y \in \underline{C}(Cl_t^{\ge})$ supports at least one certain $D_{\ge}$-decision rule whose consequent is $x \in Cl_r^{\ge}$ ", with $r,t \in \{2,...,n\}$ and $r \ge t$; each $y \in \underline{C}(Cl_t^{\le})$ supports at least one certain $D_{\le}$-decision rule whose consequent is $x \in Cl_u^{\le}$ ", with $u,t \in \{1,...,n-1\}$ and $u \le t$; and each $y \in \overline{C}(Cl_s^{\le}) \cap \overline{C}(Cl_t^{\ge})$ supports at least one approximate $D_{\ge\le}$-decision rule whose consequent is $x \in Cl_v \cup Cl_{v+1} \cup \ldots \cup Cl_z$", with $s,t,v,z \in T$ and $s \le v < z \le t$.

In simple words, complete means that the set of rules is able to cover all objects from the data table in such a way that consistent objects are re-assigned to their original classes and inconsistent objects are assigned to clusters of classes referring to this inconsistency. An analogous definition of completeness can be formulated for a set of possible decision rules.

We call *minimal* each set of minimal decision rules that is complete and non-redundant, i.e. exclusion of any rule from this set makes it non-complete.

3. DOMLEM algorithm

Various algorithms have been proposed for induction of decision rules within CRSA (see e.g. [4,7,3] for review). Many of these algorithms tend to generate a minimal set of rules with the smallest number of rules. It is an NP-hard problem, so it is natural to use heuristic algorithms for rule induction, like LEM2 algorithm proposed by Grzymala [3]. In this paper, we approach the same problem with respect to DRSA.

The proposed rule induction algorithm, called DOMLEM, is built on the idea of MODLEM algorithm [8]. The latter, inspired by LEM2 [3], was designed to handle directly numerical attributes during rule induction.

The main procedure of DOMLEM is iteratively repeated for all lower or upper approximations of the upward (downward) unions of decision classes. Depending on the type of the approximation we are getting the corresponding type of decision rules,: e.g. of type1) from lower approximation of upward unions of classes, and of type 2) from upper approximation of upward unions of classes.

Moreover, taking into account the preference-order of decision classes and the requirement of minimality of decision rules, the procedure is repeated starting from the strongest union of classes, e.g. for type 1) decision rules the lower approximations of upward unions of classes should be considered in the decreasing order of the classes.

In the algorithm, $P \subseteq C$ and E denotes a complex (conjunction of elementary conditions e) being a candidate for a condition part of the rule. Moreover, $[E]$ denotes a set of objects matching the complex E. Complex E is accepted as a condition part of the rule iff $\emptyset \neq [E] = \bigcap_{e \in E} [e] \subseteq B$, where B is the considered approximation. For the sake of simplicity, in the following we present the general scheme of the DOMLEM algorithm only for a case of type 1) decision rules.

Procedure DOMLEM
(**input:** L_{upp} – a family of lower approximations of upward unions of decision classes:
$\{ \underline{P}(Cl_t^{\geq}), \underline{P}(Cl_{t-1}^{\geq}), \ldots \underline{P}(Cl_2^{\geq}) \}$;**output:** $R_{\geq}$ set of $D_{\geq}$-decision rules);
begin
$R_{\geq} := \emptyset$;
for each $B \in L_{upp}$ **do**
begin
E:=find_rules(B);
for each rule $E \in$ **E do**
if E is a minimal rule **then** $R_{\geq} := R_{\geq} \cup E$;
end
end.

Function find_rules
(**input:** a set B; **output:** a set of rules **E** covering set B);
begin
$G := B$; {a set of objects from the given approximation}
E := $\emptyset$;

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while G ≠ ∅ do
begin
    E := ∅; {starting complex}
    S := G; {set of objects currently covered by E}
    while (E = ∅) or not ([E] ⊆ B) do
    begin
      best := ∅;  {best candidate for elementary condition}
      for each criterion q_i ∈ P do begin
        Cond:={(f(x,q_i)≥r_qi) : ∃x∈S  (f(x,q_i)=r_qi)};
        {for each positive object from S create an elementary condition}
        for each elem ∈ Cond do
      if evaluate({elem}∪E) is_better_than evaluate({best}∪E)  then best:=elem;
      end;{for}
      E := E ∪ {best}; {add the best condition to the complex}
      S := S ∩ [best];
    end; {while not ([E] ⊆ B)}
    for each elementary condition e ∈ E do
       if [E – {e}] ⊆ B then E := E – {e};
    create a rule on the basis of E;
    E := E ∪ {E};  {add the induced rule}
    G := B – ∪_{E∈E}[E];  {remove examples covered by the rule}
end; {while G ≠ ∅}
end {function}
```

Let us comment the choice of a best condition using function *evaluate(E)*. A candidate E for a condition part of a rule could be evaluated by various measures. In the current version of DOMLEM the complex E with the highest ratio $|[E] \cap G|/|[E]|$ is chosen. In case of a tie, the complex E with the highest value of $|[E] \cap G|$ is chosen.

In the case of other types of decision rules, the above scheme works with corresponding approximations and elementary conditions. For example, in the case of type 3) rules, the corresponding approximations are the lower approximations of the downward unions of classes, considered in the increasing order of preference, and the elementary conditions are of the form $f(x,q_i)\le r_{qi}$. In the case of type 5) rules, there are considered intersections of upper approximations of upward and downward unions of classes $\overline{P}(Cl_s^{\le}) \cap \overline{P}(Cl_t^{\ge})$, $s<t$, and the elementary conditions have the form $f(x,q)\ge r_q$ and $f(x,q')\le r'_{q'}$ for $q,q'\in C$; if $q=q'$, then $r_q\le r'_q$. Furthermore, because of testing minimality of rules, in the case of type 5) rules, it is useful to discover in a given intersection $K=\overline{P}(Cl_s^{\le}) \cap \overline{P}(Cl_t^{\ge})$, $s<t$, two subsets of objects, called "lower edge" and "upper edge" defined respectively as: the set of objects from K that do not dominate any other object from K having different evaluation on considered criteria, and the set of objects from K that are not dominated by any other object from K having different evaluation on considered criteria. Then combinations of conditions

based on object from the "lower edge" with conditions based on objects from the "upper edge" are the only candidates for entering a complex.

Notice that requirement for inducing robust decision rules restricts the search space as only conjunctions of elementary conditions with thresholds referring to the same basis objects are allowed. Let us shortly discuss the computation complexity of the DOMLEM algorithm. We assume that the basic operation is checking which examples are covered by a complex (condition). Let m denotes the number of attributes, n is the number of objects. In the worst case each rule covers a single object using all criteria. In this case inducing robust rules requires at most $n(nm+3m-2)/2$ operations. On the other hand, while looking for non-robust rules one cannot restrict the search to conditions based on basic object only. Thus (assuming that each criterion is on average chosen once) we need at most $nm(n+1)(m+1)/4$ operations. So, the complexity of the algorithm is polynomial.

Illustrative example:

Consider the following example (see Table 1.). A set of 17 objects is described by the set of 3 criteria $C=\{q_1, q_2, q_3\}$ – all are to be maximized according to preference. The decision attribute d classifies objects into three decision classes Cl_1, Cl_2, Cl_3 which are preference-ordered according to increasing class number.

Table 1. Illustrative data table

Object	q_1	q_2	q_3	d
1	1.5	3	12	Cl_2
2	1.7	5	9.5	Cl_2
3	0.5	2	2.5	Cl_1
4	0.7	0.5	1.5	Cl_1
5	3	4.3	9	Cl_3
6	1	2	4.5	Cl_2
7	1	1.2	8	Cl_1
8	2.3	3.3	9	Cl_3
9	1	3	5	Cl_1
10	1.7	2.8	3.5	Cl_2
11	2.5	4	11	Cl_2
12	0.5	3	6	Cl_2
13	1.2	1	7	Cl_2
14	2	2.4	6	Cl_1
15	1.9	4.3	14	Cl_2
16	2.3	4	13	Cl_3
17	2.7	5.5	15	Cl_3

The downward and upward unions of classes are the following $Cl_1^{\leq}=\{3,4,7,9,14\}$, $Cl_2^{\leq}=\{1,2,3,4,6,7,9,10,11,12,13,14,15\}$, $Cl_2^{\geq}=\{1,2,5,6,8,10,11,12,13,15,16,17\}$, $Cl_3^{\geq}=\{5,8,16,17\}$. There are 5 inconsistent objects violating the dominance principle, i.e. 6,8,9,11,14. For instance, object # 9 dominates object # 6, because it is better on all criteria q_1, q_2, q_3, however, it is assigned to the decision class Cl_1 worse than Cl_2 to which belongs object # 6. So, the

C approximations of upward and downward unions of decision classes are: $\underline{C}(Cl_1^{\le})=\{3,4,7\}$, $\overline{C}(Cl_1^{\le})=\{3,4,6,7,9,14\}$, $Bn_C(Cl_1^{\le})=\{6, 9,14\}$, $\underline{C}(Cl_2^{\le})=\{1,2,3,4,6,7,9,10,12,13,14,15\}$, $\overline{C}(Cl_2^{\le})=\{1,2,3,4,6,7, 8,9,10,11,12, 13,14,15\}$, $Bn_C(Cl_2^{\le})=\{8,11\}$, $\underline{C}(Cl_2^{\ge})=\{1,2,5,8,10,11,12,13,15,16,17\}$, $\underline{C}(Cl_2^{\ge})=$ $\{1,2,5,6,8,9,10,11,12,13,14,15,16,17\}$, $Bn_C(Cl_2^{\ge})=\{6,9,14\}$, $\underline{C}(Cl_3^{\ge})=\{5,16,17\}$, $\overline{C}(Cl_3^{\ge})=\{5,8,11, 16,17\}$, $Bn_C(Cl_3^{\ge})=\{8,11\}$.

Let us illustrate in detail the induction of certain $D_{\ge}$-decision rules for the upward union $Cl_3^{\ge}$. The lower approximation $\underline{C}(Cl_3^{\ge})$ is an input set B to the DOMLEM function *find_rules*. The elementary conditions for objects {5,16,17} are as follows (reported elements mean: the condition e_i, the set of objects satisfying the condition e_i, the first evaluation measure $|[e_i]\cap G|/|[e_i]|$, the second evaluation measure $|[e_i]\cap G|$):

$e_1=(f(x,q_1)\ge 2.3)$, {5,8,11,16,17}, 0.6, 3; $e_5=(f(x,q_2)\ge 5.5)$, {17}, 1.0, 1;
$e_2=(f(x,q_1)\ge 2.7)$, {5,17}, 1.0, 2; $e_6=(f(x,q_3)\ge 9)$, {1,2,5,8,11,15,16,17}, 0.38, 3;
$e_3=(f(x,q_2)\ge 4)$, {2,5,11,15,16,17}, 0.5, 3; $e_7=(f(x,q_3)\ge 13)$, {15,16,17}, 0.67, 2;
$e_4=(f(x,q_2)\ge 4.3)$, {2,5,15,17}, 0.5, 2; $e_8=(f(x,q_3)\ge 15)$, {17}, 1.0, 1;

The condition e_2 is found the best because its first measure is the highest and it covers more positive examples than e_5 and e_8. Moreover, as e_2 satisfies the inclusion $[e_2]\subseteq B$, it can be used to create a rule covering two objects # 5 and 17. They are removed from G and the last remaining positive example to be covered is 16. Now, there are available three elementary conditions: $e_9=(f(x,q_1)\ge 2.3)$, {8,11,16}, 0.33, 1; $e_{10}=(f(x,q_2)\ge 4)$ {2,11,15,16}, 0.25, 1; $e_{11}=(f(x,q_3)\ge 13)$, {15,16}, 0.5, 1.

The condition $e_{11}=(f(x,q_3)\ge 13)$ is chosen due the highest first evaluation measure. On the other hand, it is not sufficient to create a rule using only this condition because it covers object # 15 which is a negative example. So, in the next iteration one has to consider complexes $E=e_9\wedge e_{11}$ and $E= e_{10}\wedge e_{11}$. As the complex $E=e_9\wedge e_{11}$ has a higher first evaluation measure e_9 is chosen. Notice that $(f(x,q_3)\ge 13)$ and $(f(x,q_1)\ge 2.3)$ can be now accepted for the condition part of a rule as it covers objects # 16 and 17.

Proceeding in this way one obtains finally the minimal set of decision rules:

if $(f(x,q_3)\le 2.5)$, *then* $x\in Cl_1^{\le}$ {3, 4}
if $(f(x,q_2)\le 1.2)$, *and* $(f(x,q_1)\le 1.0)$ *then* $x\in Cl_1^{\le}$ {4, 7}
if $(f(x,q_1)\le 2.0)$, *then* $x\in Cl_2^{\le}$ {1, 2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 15}
if $(f(x,q_1)\ge 2.7)$, *then* $x\in Cl_3^{\ge}$ {5, 17}
if $(f(x,q_3)\ge 13.0)$ *and* $(f(x,q_1)\ge 2.3)$, *then* $x\in Cl_3^{\ge}$ {16, 17}
if $(f(x,q_1)\ge 1.2)$ *and* $(f(x,q_3)\ge 7.0)$, *then* $x\in Cl_2^{\ge}$ {1, 2, 5, 8, 11, 13, 15, 16, 17}
if $(f(x,q_2)\ge 2.8)$ *and* $(f(x,q_3)\ge 6.0)$, *then* $x\in Cl_2^{\ge}$ {1, 2, 5, 8, 11, 12, 15, 16, 17}
if $(f(x,q_2)\ge 2.8)$ *and* $(f(x,q_1)\ge 1.7)$, *then* $x\in Cl_2^{\ge}$ {2, 5, 8, 10, 11, 15, 16, 17}
if $(f(x,q_1)\ge 2.3)$ *and* $(f(x,q_2)\le 3.3)$, *then* $x\in Cl_2\cup Cl_3$ {8}
if $(f(x,q_1)\ge 2.5)$ *and* $(f(x,q_2)\le 4.0)$, *then* $x\in Cl_2\cup Cl_3$ {11}

if (f(x,q_3) ≤6.0) *and* (f(x,q_1) ≥2.0), *then* x∈$Cl_1 \cup Cl_2$ {14}
if (f(x,q_3) ≥4.5) *and* (f(x,q_3) ≤5.0), *then* x∈$Cl_1 \cup Cl_2$ {6, 9}

4. Decision rules in Variable Consistency model of Dominance-based Rough Set Approach

In [2] we proposed a generalization of DRSA to variable consistency model (VC-DRSA). It allows to define lower approximations of the unions of decision classes accepting limited number of negative examples controlled by pre-defined level of consistency $l \in (0, 1]$. Within VC-DRSA, given $P \subseteq C$ and consistency level l, the P-lower and P-upper approximations of the upward unions of classes are the following:

$$\underline{P}^{l}(Cl_t^{\geq}) = \{x \in Cl_t^{\geq} : \frac{card(D_P^{+}(x) \cap Cl_t^{\geq})}{card(D_P^{+}(x))} \geq l\},$$

$$\overline{P}^{l}(Cl_t^{\geq}) = Cl_t^{\geq} \cup \{x \in Cl_{t-1}^{\leq} : \frac{card(D_P^{-}(x) \cap Cl_{t-1}^{\leq})}{card(D_P^{-}(x))} < l\}.$$

The definitions of approximations for downward unions are analogic – see [2]. These approximations are used for induction of decision rules having the same syntax as in DRSA. In the VC-DRSA context each decision rule is characterized by an additional parameter α called *confidence of the rule*. It is the ratio of the number of objects supporting the rule and the number of objects covered by the rules.

The induction of such rules can be done after simple modifications of the DOMLEM algorithm. First, the inputs B of the algorithm are the new P^l-approximations of upward or downward unions of decision classes. Notice that in DRSA the complex E was accepted as a condition part of a rule iff $[E] \subseteq B$. This corresponds to the requirement that $|[E] \cap B| / |[E]|$, should be equal to 1. The keypoint of VC-DRSA is a relaxation of this requirement permitting to build a rule based on a complex E having a confidence α not worse than the consistency level l. The rest of the algorithm remains unchanged.

Continuation of the example. Let us assume that the user considers only criteria $P=\{q_1,q_2\}$ and is interested in analysing upward union $Cl_2^{\geq}$. The DRSA leads to $\underline{P}(Cl_2^{\geq})=\{1,2,5,8,10,11,15,16,17\}$ and boundary $Bn_P(Cl_2^{\geq})=\{6,9,12,13,14\}$. The two following decision rules are induced to describe objects from $\underline{P}(Cl_2^{\geq})$:

if (f(x,q_1)≥ 1.7) *and* (f(x,q_2)≥ 2.8), *then* x∈ $Cl_2^{\geq}$, {2,5,8,10,11,15,16,17}

if (f(x,q_1)≥ 1.5) *and* (f(x,q_2)≥ 3), *then* x∈ $Cl_2^{\geq}$, {1,2,5,8,11,15,16,17}.

Let us assume now that the user works with VC-DRSA accepting consistency level l equal to 0.75. As P-dominating sets of objects # 6, 12 and 13 are contained in $Cl_2^{\geq}$ with a degree greater than the consistency level (0.83, 0,9 and 0.91, respectively) they can be added to the lower approximation $\underline{P}^{0.75}(Cl_2^{\geq})$. The boundary region is now composed of only two objects # 9 and 14. Further on, the following rules are induced from the lower approximation $\underline{P}^{0.75}\left(Cl_2^{\geq}\right)$ (within parentheses there are objectss supporting the corresponding rules and objects only covered by the corresponding rules but not satisfying their decision parts - the latter are marked by "*"):

if (f(x,q_1)≥1.2), *then* x∈ $Cl_2^{\geq}$ with confidence 0.91, {1,2,5,8,10,11,13,14*,15,16,17}

if (f(x,q_2)≥2), *then* x∈ $Cl_2^{\geq}$ with confid. 0.79, {1,2,3*,5,6,8,9*,10,11,12,14*,15,16,17}

One can also notice that these rules are supported by more examples (i.e. 10 and 11, respectively) than the previous ones (8 in both).

6. Conclusions

The paper addressed the important issue of inducing decision rules for multicriteria sorting problems. As none of already known rule induction algorithms can be directly applied to multicriteria sorting problems, we introduced a specific algorithm called DOMLEM. It produces a complete and non-redundant, i.e. minimal, set of decision rules. It heuristically tends to minimize the number of generated rules. It was also extended to produce decision rules accepting a limited number of negative examples within the variable consistency model of the dominance rough sets approach.

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