Heuristic Algorithm for Multi-item Internet Shopping with Price Sensitive Discounts. *

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1 Problem definition

We study an optimization aspect of Internet shopping with price sensitive discounts from customer perspective (which is a specific case of the Internet Shopping Optimization Problem [1, 5]). Specifically, we consider a problem in which a customer would like to buy products of a given set $N = \{1,\dots,n\}$ in a given set of Internet shops $M = \{1,\dots,m\}$ at the minimum total final price. There are the following given parameters and decision variables :

d_i - delivery price of all products from shop i to the customer ;

p_{ij} - standard price of product j in shop i, $p_{ij} = p_j$ if standard prices of product j are the same in all shops ;

N_i - subset of products of the set N in shop i (eligible products for shop i), $N_i \subseteq N$;

M_j - subset of shops in which product j can be bought (eligible shops for product j), $M_j \subseteq M$;

S_i - subset of products selected by the customer in shop i (basket of shop i, decision variable), $N = \cup_{i=1}^{m} S_i$ and $S_i \cap S_j = \phi$, $i \neq j$, for a feasible solution ;

$T_i(S_i) = d_i + \sum_{j \in S_i} p_{ij}$ - total delivery and standard price in shop i for a given set of products $S_i \subseteq N_i$; if there is no ambiguity, notation S_i in $T_i(S_i)$ can be omitted ;

$f_i(T)$ - discounting function for final price, a concave increasing differentiable or concave piecewise linear function of total delivery and standard price T in shop i at all points $T > 0$, $f_i(0) = 0$.

We denote the above problem as IS, where the abbreviation stands for Internet Shopping. Its mathematical program can be written as follows :

$$\min \quad \sum_{i=1}^{m} f_i(d_i y_i + \sum_{j \in N_i} p_{ij} x_{ij}), \tag{1}$$

$$\text{s.t.} \sum_{i \in M_j} x_{ij} = 1, \; j = 1,\dots,n, \tag{2}$$

$$0 \leq x_{ij} \leq y_i, \; i = 1,\dots,m, \; j = 1,\dots,n, \tag{3}$$

$$x_{ij} \in \{0,1\}, \; y_i \in \{0,1\}, \; i = 1,\dots,m, \; j = 1,\dots,n. \tag{4}$$

It is worth notice that no discounts version of the ISOP problem is equivalent to the well known FACILITY LOCATION PROBLEM (FLP). Discussions of FLPs can be found in [3].

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2 Heuristic algorithm

We developed and experimentally tested a simple algorithm for problem IS. Algorithm was based on the one proposed in [5]. In this algorithm, denoted as H, products are considered in a certain order. The algorithm is run for various product orders and the best solution found is presented to the customer. Let the products be ordered $1,\dots,n$. Values of total delivery and standard price for all shops are initially set as $T_i = d_i$, $i = 1,\dots,m$. In iteration j of algorithm H, product j is selected in its eligible shop $i \in M_j$ with minimum value $f_i(T_i + p_{ij})$, and the corresponding T_i-value is re-set : $T_i := T_i + p_{ij}$.

We performed computer experiments, in which solutions obtained by algorithm H were compared against optimal solutions and those provided by algorithm of *Price Comparison Sites* for the examples of problem IS, which are prepared on the basis of data from the online book industry reported in Clay et al. [2]. In these examples, $m \in \{10,15,20,25,30\}$, $n \in \{2,3,4,5\}$, and discounting functions

$$f_i(d_i + P) = \begin{cases} d_i + P, & \text{if } 0 < P \leq 50, \\ d_i + 50 + 0.95(P - 50), & \text{if } P > 50, \end{cases} \tag{5}$$

where P is the total standard price of books selected in bookstore i. It is assumed that each bookstore has all the required books. For each pair (n,m), 10 instances were generated. In each instance, the following values were randomly generated for all i and j in the corresponding ranges. Delivery price : $d_i \in \{5,10,15,20,25,30\}$, *publisher's recommended price* of book j : $r_j \in \{5,10,15,20,25\}$, and price of book j in bookstore i : $p_{ij} \in [a_{ij}, b_{ij}]$, where $a_{ij} \geq 0.69r_j$, $b_{ij} \leq 1.47r_j$, and the structure of intervals $[a_{ij}, b_{ij}]$ follow information in Table V in Clay et al. [2]. For each instance, algorithm H was run two times - for a sequence of books in the non-decreasing order of the recommended price and for the reverse sequence. In the worst case, solution found by algorithm H was 4.1% more expensive than the optimal solution, it was 36.1% cheaper than solutions provided by Price Comparison Sites without taking delivery prices into account. The average values of the above mentioned deviations are 2.3%, 45.9% respectively.

Key full paper improvements. New heuristic algorithm with forecasting H2 is under development. Changes we made will prevent to provide bad solutions for specific data (even if it is unrealistic) - it also should improve overall performance and provide even better solutions. One of the key aspects is to run a test considering real word data. The efficiency of the heuristics will be compared with other approaches. Problem size (number of stores, number of shops) will be greatly increased. The performance of the heuristic algorithms with respect to optimal and the other heuristic solutions (like Price Comparison Sites) will be explained more detailed and more precisely. Section 1 will be lengthen to provide smooth introduction to the problem.

Références

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