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Dual Discounting Functions for Internet Shopping Optimization Problem

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1 Introduction

Electronic Commerce is definitely one of the youngest computer science branches. Moreover, we could claim that apart from being a cutting-edge discipline, it is also one of the fastest developing parts of computer science. Revenue of the e-commerce market grows rapidly. One can say that the multidisciplinary nature (connecting many of the well-known computer science branches like operations research, combinatorial optimization, algorithms, with other sciences - e.g. logistics, marketing, and many others) lies behind the success of the e-commerce science. One of the very important topics of research in that area is, motivated by practical applications, Internet shopping which is becomming more and more popular with every upcoming year. Products available in online stores are often cheaper than the ones offered by regular local retailers and a wide choice of offers is available just a click away from the customer. A crucial aspect of online shopping is the time spent on comparing offers and a comfortable way of shopping regardless of shop location.

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2 Internet Shopping Optimization Problem

Internet Shopping Optimization Problem (ISOP) tackles the situation when a single buyer is looking for a multiset of products $N = \{1, \dots, n\}$ to buy in m shops, $M = \{1, \dots, m\}$. A multiset of available products N_i, a cost p_{ij} of each product $j \in N_i$, and a delivery cost d_i of any subset of the products from the shop to the buyer are associated with each shop $i \in M$, $i = 1, \dots, m$. It is assumed that $p_{ij} = \infty$ if $j \notin N_i$. The problem lies in finding a sequence of disjoint selections (or carts) of products $X = (X_1, \dots, X_m)$, which we call a *cart sequence*, such that $X_i \subseteq N_i$, $i = 1, \dots, m$, $\cup_{i=1}^{m} X_i = N$, and the total product and delivery cost, denoted as $F(X) := \sum_{i=1}^{m} \left(\delta(|X_i|) d_i + \sum_{j \in X_i} p_{ij} \right)$, is minimized. Here $|X_i|$ denotes the cardinality of the multiset X_i, and $\delta(x) = 0$ if $x = 0$ and $\delta(x) = 1$ if $x > 0$. We denote this problem as ISOP (Internet Shopping Optimization Problem), its optimal solution as X^*, and its optimal solution value as F^*. Follow [2,4,3] for detailed formulations, NP-hardness proof and heuristic algorithm definitions. Moreover, you can find a detailed description of a computational experiment and comments on its results. The most interesting (and the most complicated) specialization of the ISOP is the so called Internet Shopping with Price Sensitive Discounts[1,3] problem. For each Internet shop, standard prices for the products are known, as well as an increasing discounting function of total standard and delivery price. Buying all the reacquired products at the minimum total discounted price constitutes the problem. Its mathematical program can be written as follows:

$$\begin{aligned} \min \ & \sum_{i=1}^{m} f_i\Big(\sum_{j \in N_i} p_{ij} x_{ij}\Big) + \sum_{i=1}^{m} d_i y_i, && (1) \\ s.t. \ & \sum_{i \in M} x_{ij} = 1, \ j = 1, \dots, n, \\ & 0 \le x_{ij} \le y_i, \ i = 1, \dots, m, \ j = 1, \dots, n, \\ & x_{ij} \in \{0, 1\}, \ y_i \in \{0, 1\}, \ i = 1, \dots, m, \ j = 1, \dots, n. \end{aligned}$$

where $f_i(T)$ - discounting function for the final sum of standard price T in shop i at all points $T > 0$, $f_i(0) = 0$.

3 Two price sensitive functions for ISOP

During MISTA conference a new optimization problem will be introduced - Dual Discounting Functions for Internet Shopping Optimization Problem. Moreover, its mathematical formulation as well as the first computational experiment results will be presented. Price sensitive shipping costs are often used in Internet shops to attract customers and to encourage them to buy more products. A typical example is the following advertisement in an Internet shop selling books and CD's: If the value of your purchase is at least 25 euros, then we will ship products for a half of shipping cost; 35 euros - a half of price shipped by courier; 40 - free post office shipping; 50 - free courier shipping. Values and thresholds can vary depending on the seller. One can describe ISOP with two price sensitive functions as follows. We would like to buy a number of products (n). Products are offered by many stores (m) from different locations - some are distant, some are close to our position. The price of each product n in every store m could

Table 1 Example price structure for a delivery.

Delivery type	Payment method	$T_i < 25$	$25 \le T_i < 50$	$50 \le T_i < 75$	$T_i \ge 75$
Post office	Credit card	10	5	0	0
Post office	At pick-up	15	10	5	0
Courier	Credit card	15	15	5	0
Courier	At pick-up	20	20	10	0

be written as p_{ij} where $i = 1, \dots, m,\ j = 1, \dots, n$. To every store m corresponds a discounting function (on the base of the total cost of the shopping from the one store) $f_i(T_i)$. Our goal is to find out how to pay the lowest bill for all the products including delivery costs. Mathematical program formulation can be written as follows:

$$\begin{aligned} \min\ & \sum_{i=1}^{m} f_i\Big(\sum_{j\in N_i} p_{ij}x_{ij}\Big) + \sum_{i=1}^{m} d_i\Big(\sum_{j\in N_i} p_{ij}x_{ij}\Big), \qquad (2) \\ s.t.\ & \sum_{i\in M} x_{ij} = 1,\ j = 1, \dots, n, \\ & x_{ij} \in \{0, 1\},\ i = 1, \dots, m,\ j = 1, \dots, n. \end{aligned}$$

The discounting function could look as follows (an example):

$$f_i(T_i) = \begin{cases} T_i & \text{if } 0 < T_i \le 50, \\ 50 + 0.95(T_i - 50) & \text{if } 50 < T_i \le 100, \\ 50 + 0.95 * 50 + 0.9(T_i - 100) & \text{if } 100 < T_i \le 150, \\ 50 + 0.95 * 50 + 0.9 * 100 + 0.85(T_i - 150) & \text{if } T_i > 150. \end{cases}$$

where T_i is the total standard price of books selected in bookstore i. The shipping cost function is more complicated for its shape depends additionally on a customer's further choices (delivery type - e.g. by post office, courier; payment method). We can define it as $d_i(T_i, dv_1, dv_2), i = 1, \dots, m$. Therefore, for the present discussion we can state that a customer makes two decisions - delivery type (dv_1) and payment method (dv_2). Shipping costs function example is shown in Table 1.

To solve this problem one can use some of the already prepared heuristics, however, before finishing computational experiments it cannot be said that they will be as efficient as for IS with Price Sensitive Discounts problem. Moreover, we can presume that new specific algorithms for this interesting case should be developed.

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References

1. Blazewicz J., Bouvry P., Kovalyov M.Y., Musial J., Internet Shopping with Price Sensitive Discounts, *4OR- A Quarterly Journal of Operations Research*, DOI: 10.1007/s10288-013-0230-7, (2013).
2. Blazewicz J., Kovalyov M.Y., Musial J., Urbanski A.P., Wojciechowski A., Internet Shopping Optimization Problem, *International Journal of Applied Mathematics and Computer Science*, Vol. 20(2), 385-390 (2010).
3. Musial J., Applications of Combinatorial Optimization for Online Shopping, NAKOM, Poznan (2012).
4. Wojciechowski A., Musial J., Towards Optimal Multi-item Shopping Basket Management: Heuristic Approach, *Lecture Notes in Computing Science*, Vol. 6428, 349-357 (2010).