Dataset with decision examples concerning ordinal classification

Student	Mathematics	Physics	Literature	Philosophy	Overall_Eval.
***S*1**	good	medium	bad	medium	bad
***S*2**	medium	medium	bad	bad	medium
***S*3**	medium	medium	medium	bad	medium
***S*4**	good	good	medium	medium	medium
***S*5**	good	good	medium	medium	good
***S*6**	good	medium	good	good	good
***S*7**	good	good	good	medium	good
***S*8**	bad	bad	bad	bad	bad
***S*9**	bad	bad	medium	bad	bad
***S*10**	good	medium	medium	bad	medium

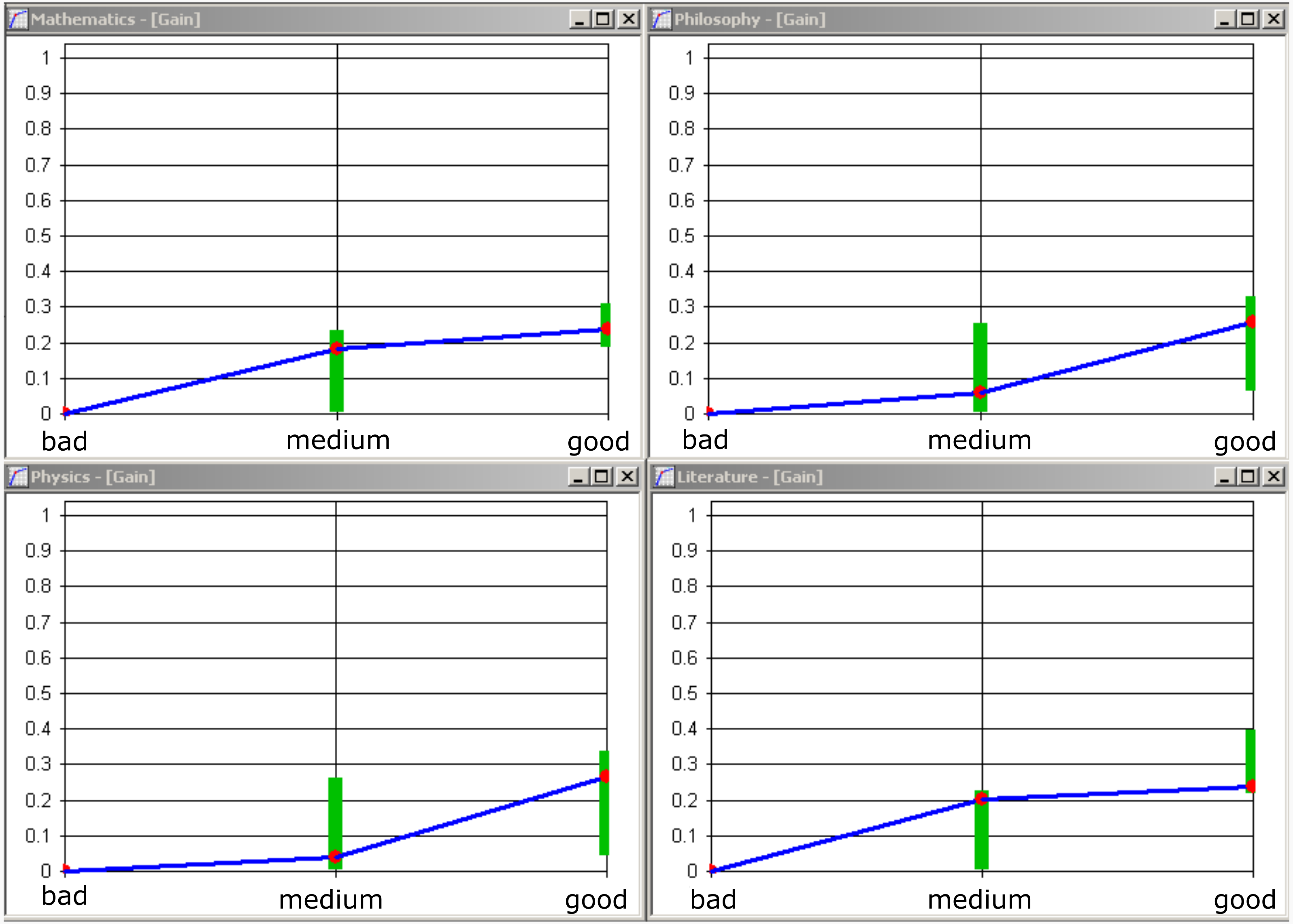
S7: 0.80
S6: 0.77
S5: 0.76

S4: 0.76
S10: 0.48
S3: 0.42

S1: 0.33
S2: 0.22
S9: 0.20
S8: 0.00

0.0 bad 0.34 medium 0.76 good 1.0 U

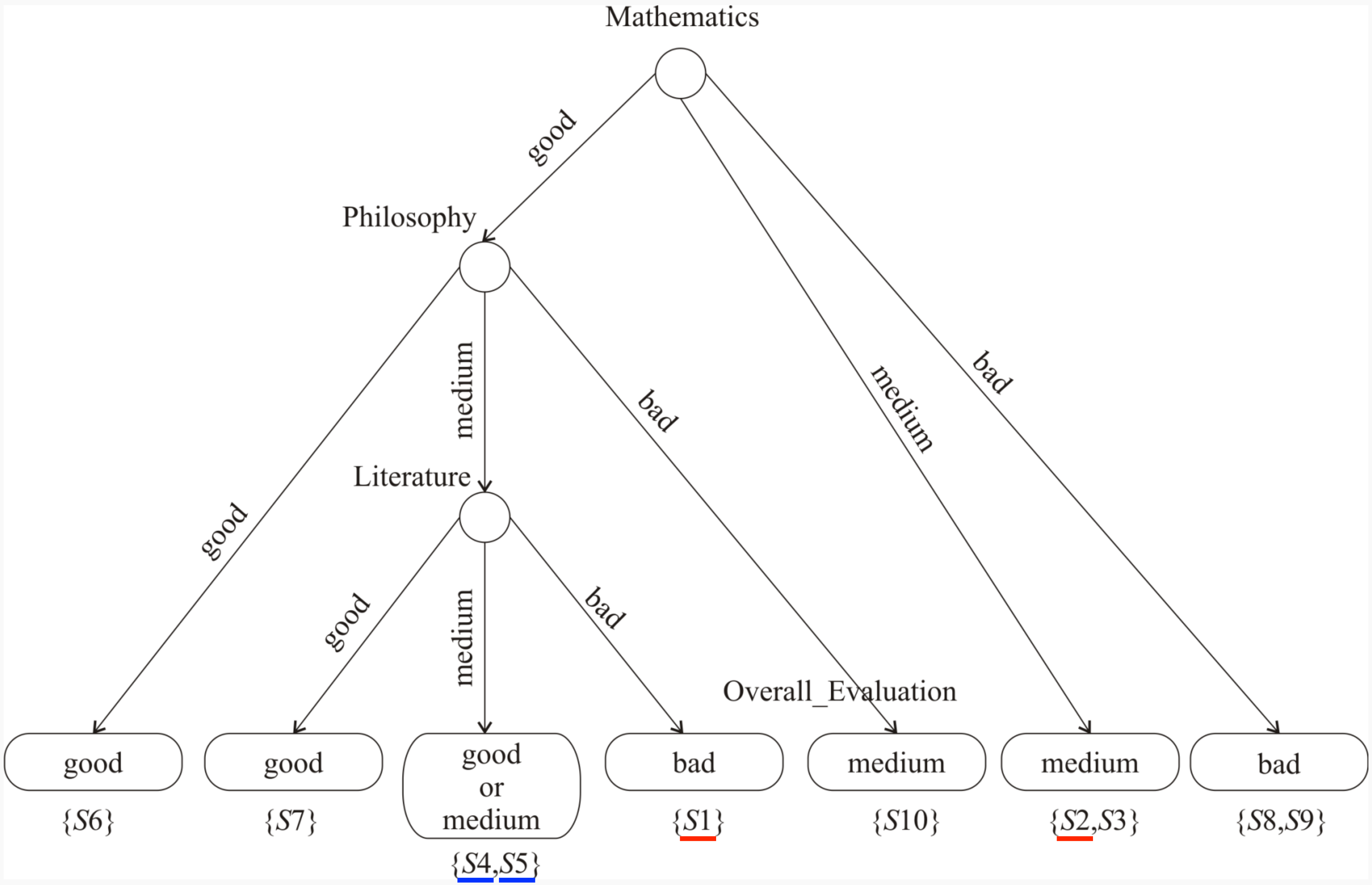
value function $U(a) = \sum_{i=1}^{n} u_i[g_i(a)]$



Dataset with decision examples concerning classification

Student	Mathematics	Physics	Literature	Philosophy	Overall_Eval.
S1	good	medium	bad	medium	bad
S2	medium	medium	bad	bad	medium
S3	medium	medium	medium	bad	medium
S4	good	good	medium	medium	medium
S5	good	good	medium	medium	good
S6	good	medium	good	good	good
S7	good	good	good	medium	good
S8	bad	bad	bad	bad	bad
S9	bad	bad	medium	bad	bad
S10	good	medium	medium	bad	medium

Decision tree



Decision rules

If Lit $\succeq$ good, *then* student $\succeq$ good {*S*6,*S*7}

If Phys $\succeq$ medium & Lit $\succeq$ medium, *then* student $\succeq$ medium
{*S*3,*S*4,*S*5,*S*6,*S*7,*S*10}

If Phys $\succeq$ good & Lit $\preceq$ medium, *then* student is medium <u>or</u> good
{<u>*S*4</u>,<u>*S*5</u>}

If Math $\succeq$ medium & Lit $\preceq$ bad, *then* student is bad <u>or</u> medium
{<u>*S*1</u>,<u>*S*2</u>}

If Lit $\preceq$ bad, *then* student $\preceq$ medium {*S*1,*S*2,*S*8}

If Philo $\preceq$ bad, *then* student $\preceq$ medium {*S*2,*S*3,*S*8,*S*9,*S*10}

If Phys $\preceq$ bad, *then* student $\preceq$ bad {*S*8,*S*9}

Inconsistencies in Data – Rough Set Theory

Inconsistencies in data – Rough Set Theory

- Zdzisław Pawlak (1926 – 2006)

Student	Mathematics	Physics	Literature	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- One wants to characterize sets of objects from a universe (classes, concepts)

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- One wants to characterize sets of objects from a universe (classes, concepts)

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- One wants to characterize sets of objects from a universe (classes, concepts)

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Objects with the same description are indiscernible and create granules

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- The granules of indiscernible objects are used to approximate classes

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Inconsistencies in data – Rough Set Theory

- Lower approximation of class „good”

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Lower Approximation

Inconsistencies in data – Rough Set Theory

- Lower and upper approximation of class „good"

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**medium**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Upper Approximation

Lower Approximation

CRSA – decision rules induced from rough approximations

- Certain decision rule supported by objects from lower approximation of class „good" (discriminant rule)

  If Lit=good, *then* Student is certainly good
  {S5,S6}

- Possible decision rule supported by objects from upper approximation of class „good" (partly discriminant rule)

  If Phys=medium & Lit=medium, *then* Student is possibly good
  {S3,S4}

- Approximate decision rule supported by objects from the boundary of class „medium" or „good"

  If Phys=medium & Lit=medium, *then* Student is medium or good
  {S3,S4}

Classical Rough Set Approach (CRSA)

- Let U be a finite universe of discourse composed of objects (e.g. set A) described by a finite set of attributes (or criteria)
- Sets of objects indiscernible w.r.t. attributes create granules of knowledge (elementary sets)
- Any subset $X \subseteq U$ may be expressed in terms of these granules:
  - either precisely – as a union of the granules
  - or roughly – by two ordinary sets, called *lower* and *upper approximations*
- The lower approximation of X consists of all the granules included in X (interior of X)
- The upper approximation of X consists of all the granules having non-empty intersection with X (closure of X)

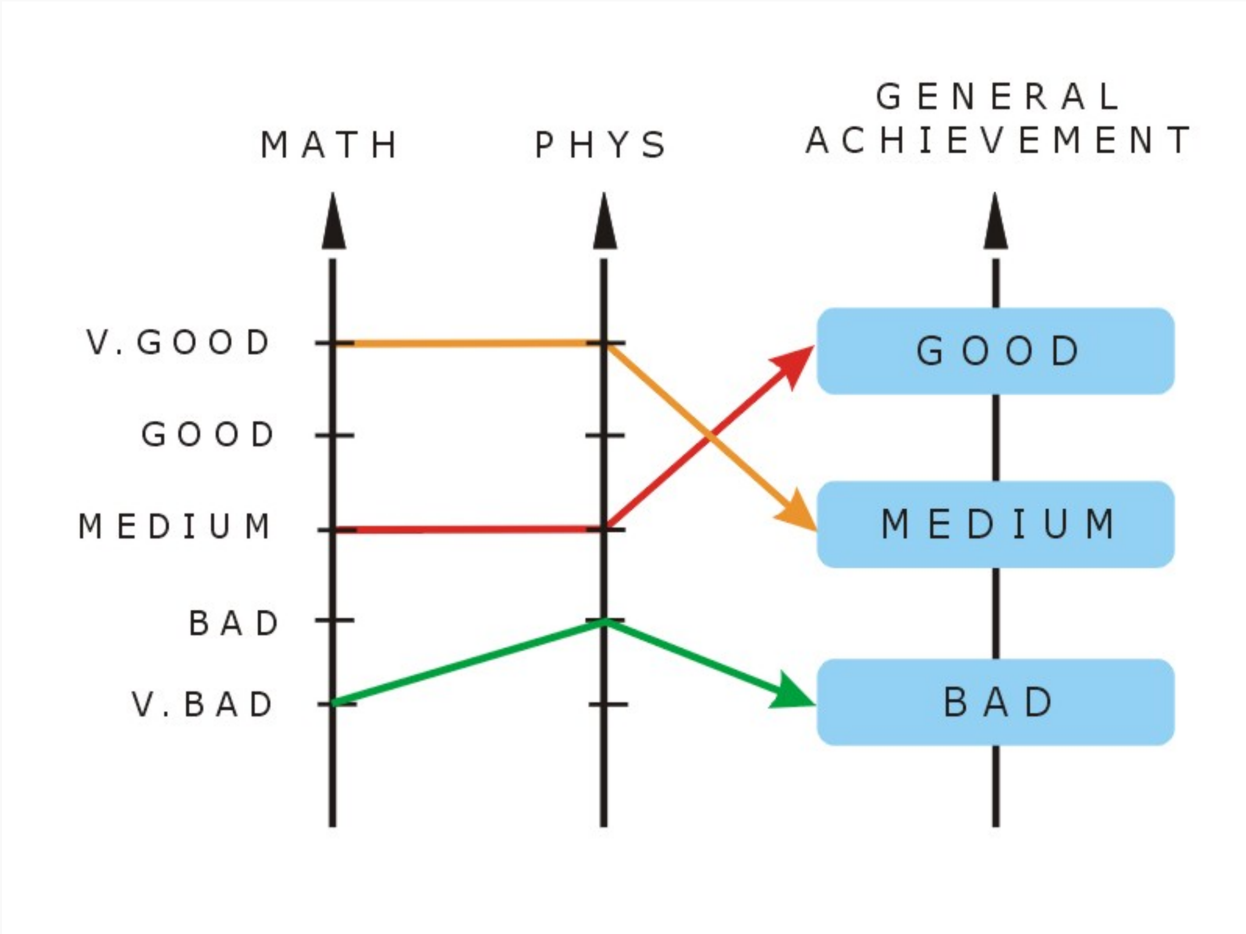
Classical Rough Set Approach has to be adapted to ordinal data

- Classical rough set approach does not properly handle decision examples

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**medium**	**medium**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**bad**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

Classical Rough Set Approach has to be adapted to ordinal data

- Inconsistency w.r.t. dominance principle (Pareto principle)



Dominance-based Rough Set Approach (DRSA)

Classical Rough Set Theory vs. Dominance-based Rough Set Theory

Classical Rough Set Theory

$\Downarrow$

Indiscernibility principle

If *x* and *y* are indiscernible with respect to all relevant **attributes**,

then *x* should classified to the same class as *y*

Dominace-based Rough Set Theory

$\Downarrow$

Dominance principle

If *x* is at least as good as *y* with respect to all relevant **criteria**,

then *x* should be classified at least as good as *y*

S.Greco, B.Matarazzo, R.Słowiński: Rough sets theory for multicriteria decision analysis. *European J. of Operational Research*, 129 (2001) no.1, 1-47

Dominance principle as monotonicity principle

- Interpretation of the dominance principle

  The better the evaluation of x with respect to considered criteria,
  the better its comprehensive evaluation

- Many other relationships of this type, e.g.:
  - The faster the car, the more expensive it is
  - The higher the inflation, the higher the interest rate
  - The larger the mass and the smaller the distance, the larger the gravity
  - The colder the weather, the greater the energy consumption

- The Dominance-based Rough Set Approach does not only permit representation and analysis of decision problems but, more generally, representation and analysis of all phenomena involving monotonicity

Dominance principle as monotonicity principle

Driving hypothesis:

- Relationship between different aspects of a phenomenon described by data can be represented by monotonicity relationship between specific <u>measures</u> or <u>perceptions</u>, e.g.

„the more a tomato is red, the more it is ripe"

„the more similar are the causes,
the more similar are the effects one can expect"

R.Słowiński, S.Greco, B.Matarazzo: Rough set based decision support. Chapter 16 [in]: E.K.Burke and G.Kendall (eds.), *Search Methodologies: Introductory Tutorials in Optimization and Decision Support Techniques*, Springer-Verlag, New York, 2005, pp. 475-527

Dominance-based Rough Set Approach (DRSA)

- Finite sets of condition (C) and decision (D) criteria are monotonically dependent
- $\succeq_q$ – weak preference relation (outranking) on U w.r.t. criterion $q \in \{C \cup D\}$ (complete preorder)
- $x_q \succeq_q y_q$: „x_q is at least as good as y_q on criterion q"
- xD_Py : x dominates y with respect to set of criteria $P \subseteq C$ in condition space $X_P = \prod_{q=1}^{|P|} V_q$ if $x_q \succeq_q y_q$ for all criteria $q \in P$
- $D_P = \bigcap_{q \in P} \succeq_q$ is a partial preorder
- Analogically, we define xD_Ry in decision space $X_R = \prod_{q=1}^{|R|} V_q$, $R \subseteq D$

Dominance-based Rough Set Approach (DRSA)

- For simplicity : $D=\{d\}$
- d makes a partition of U into decision classes $\boldsymbol{Cl}=\{Cl_t,\ t=1,\ldots,m\}$
- $[x \in Cl_r,\ y \in Cl_s,\ r>s] \Rightarrow x \succ y$ ($x \succeq y$ and *not* $y \succeq x$)
- In order to handle monotonic dependency between condition and decision criteria:

$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$ – upward union of classes, $t=2,\ldots,m$ („*at least*" class Cl_t)

$Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$ – downward union of classes, $t=1,\ldots,m-1$ („*at most*" class Cl_t)

- $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are positive and negative dominance cones in X_D, with D reduced to single dimension d

Granular computing with dominance cones

- Granules of knowledge are dominance cones in condition space X_P ($P \subseteq C$)

  $D_P^+(x) = \{y \in U: yD_Px\}$: *P-dominating set*

  $D_P^-(x) = \{y \in U: xD_Py\}$: *P-dominated set*

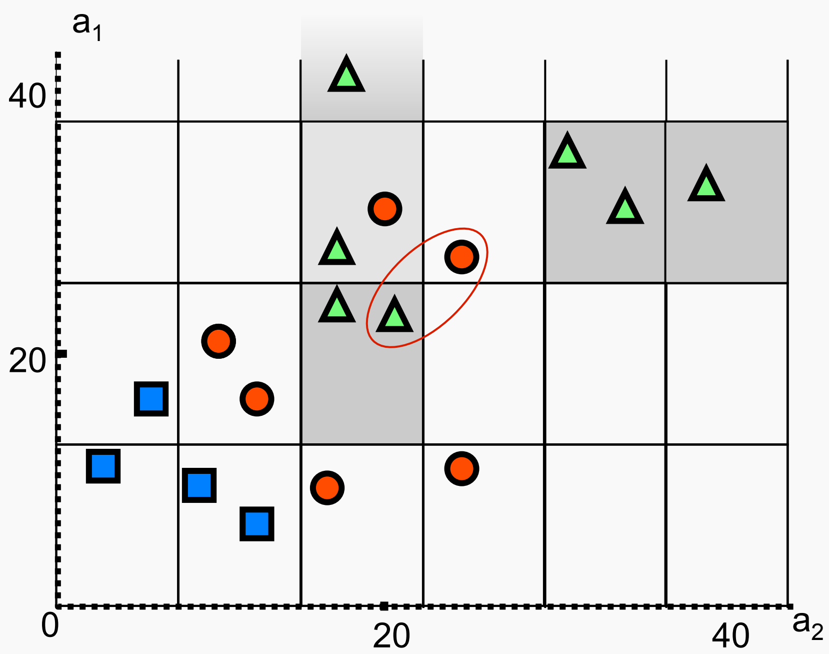
- *P*-dominating and *P*-dominated sets are positive and negative dominance cones in X_P

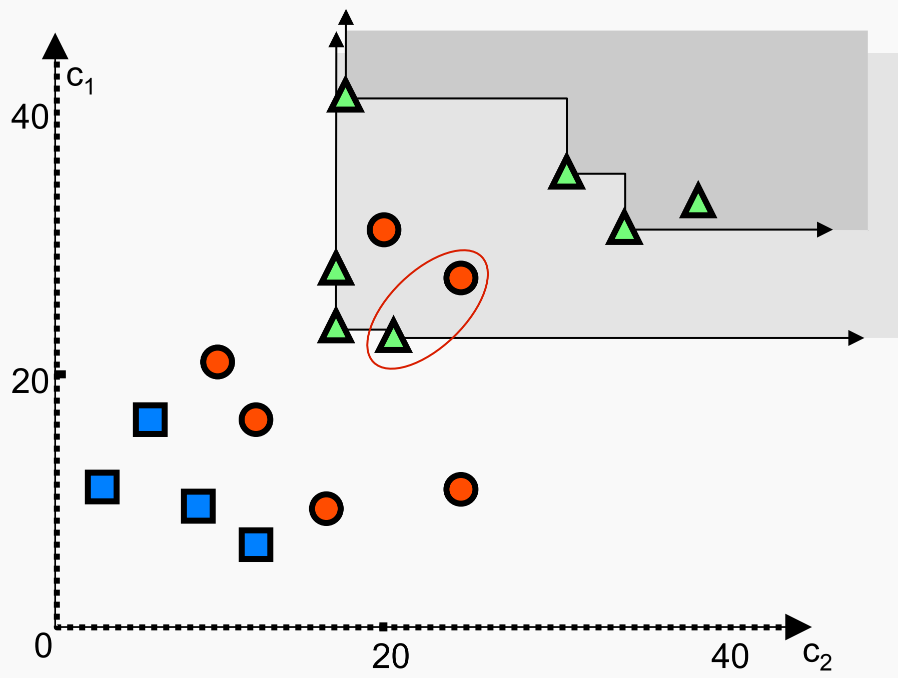
- Classification patterns (preference model) to be discovered are functions representing granules $Cl_t^{\geq}$, $Cl_t^{\leq}$, by granules $D_P^+(x)$, $D_P^-(x)$

Dominance-based Rough Set Approach vs. Classical RSA

Comparison of CRSA and DRSA

Classes: $\triangle \succ \bigcirc \succ \square$





$$\underline{P}(X)=\{x\in U: I_P(x)\subseteq X\}$$

$$\overline{P}(X)=\bigcup_{x\in X} I_P(x)$$

$$\underline{P}\left(Cl_t^{\geq}\right)=\left\{x\in U: D_P^+(x)\subseteq Cl_t^{\geq}\right\}$$

$$\overline{P}\left(Cl_t^{\geq}\right)=\bigcup_{x\in Cl_t^{\geq}} D_P^+(x)$$

Rough Set approach to ordinal classification

- Example of preference information about students:

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**good**	**good**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**bad**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

- Examples of classification of S1 and S2 are inconsistent

S.Greco, B.Matarazzo, R.Słowiński: Decision rule approach. Chapter 13 [in]: J.Figueira, S.Greco and M.Ehrgott (eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer-Verlag, New York, 2005, pp. 507-562

Rough Set approach to ordinal classification

- If we eliminate Literature, then more inconsistencies appear:

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**good**	**good**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**bad**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

- Examples of classification of S1, S2, S3 and S5 are inconsistent

Rough Set approach to ordinal classification

- Elimination of Mathematics does not increase inconsistencies:

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**good**	**good**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**bad**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

- Subset of criteria {Ph,L} is a reduct of {M,Ph,L}

Rough Set approach to ordinal classification

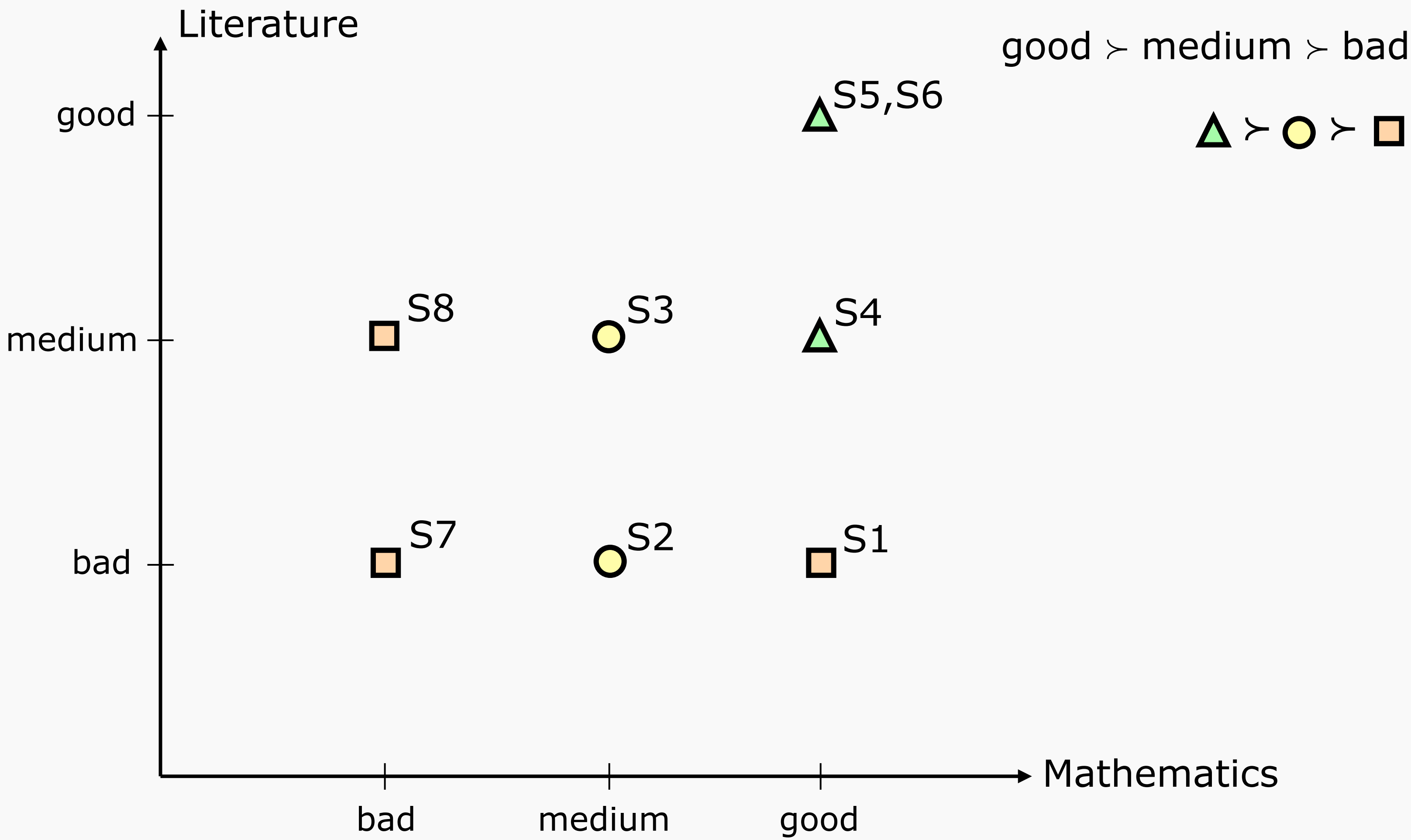
- Elimination of Physics also does not increase inconsistencies:

Student	Mathematics (**M**)	Physics (**Ph**)	Literature (**L**)	Overall class
S1	**good**	**medium**	**bad**	**bad**
S2	**medium**	**medium**	**bad**	**medium**
S3	**medium**	**medium**	**medium**	**medium**
S4	**good**	**good**	**medium**	**good**
S5	**good**	**medium**	**good**	**good**
S6	**good**	**good**	**good**	**good**
S7	**bad**	**bad**	**bad**	**bad**
S8	**bad**	**bad**	**medium**	**bad**

- Subset of criteria {M,L} is a reduct of {M,Ph,L}
- Intersection of reducts {M,L} and {Ph,L} gives the core {L}

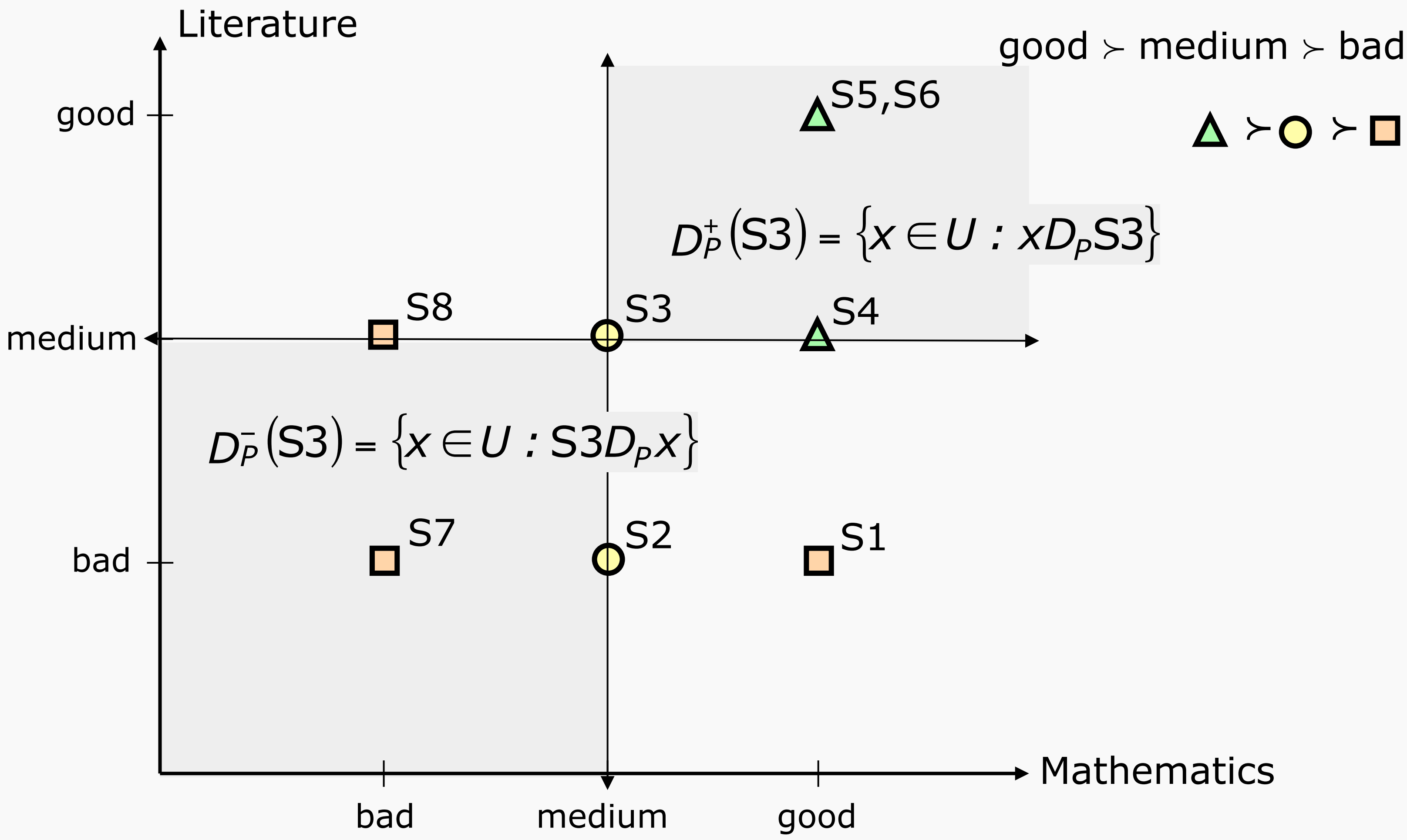
Rough Set approach to ordinal classification

- Let us represent the students in condition space {M,L} :



Rough Set approach to ordinal classification

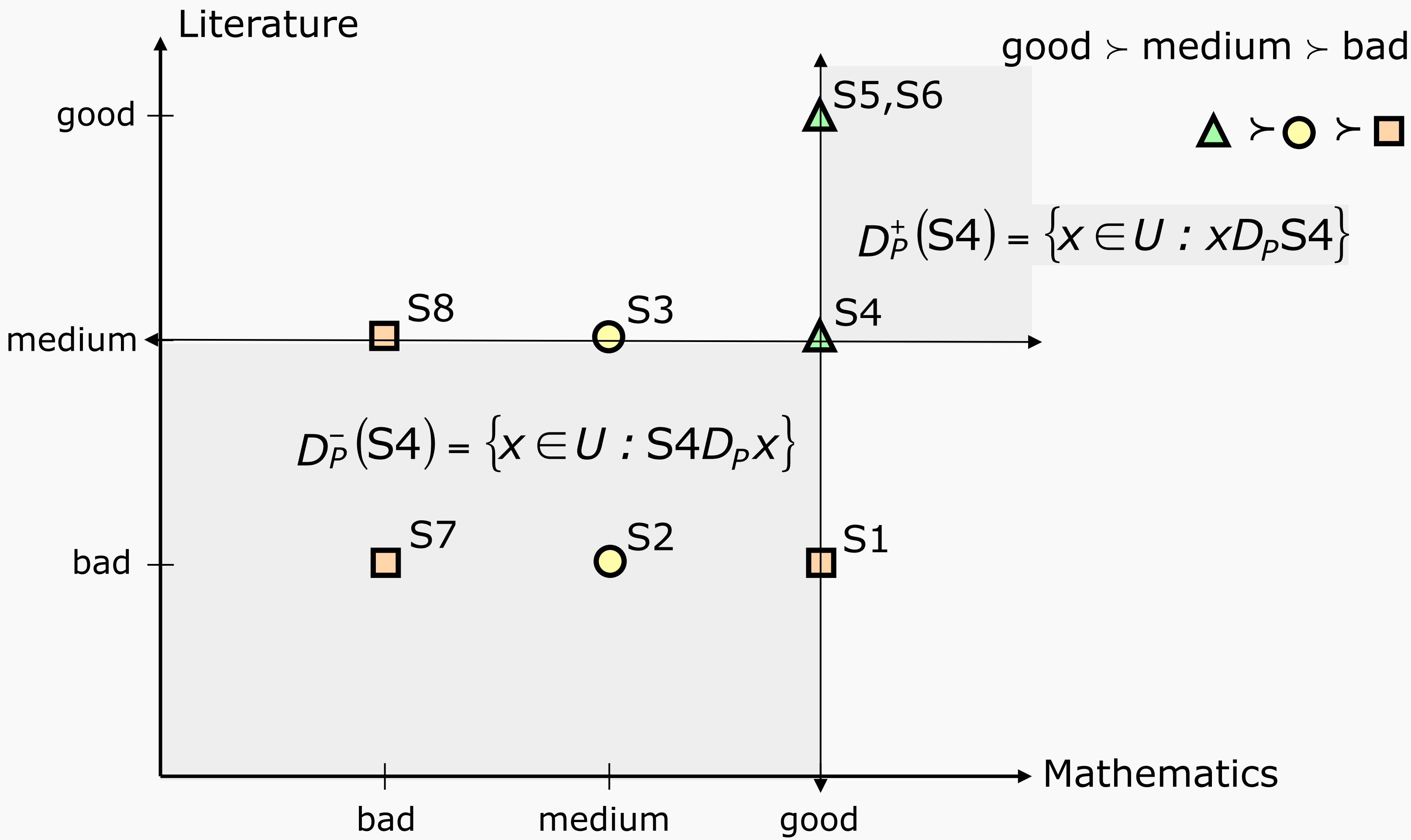
- Dominance cones in condition space {M,L} :



- P={M,L}

Rough Set approach to ordinal classification

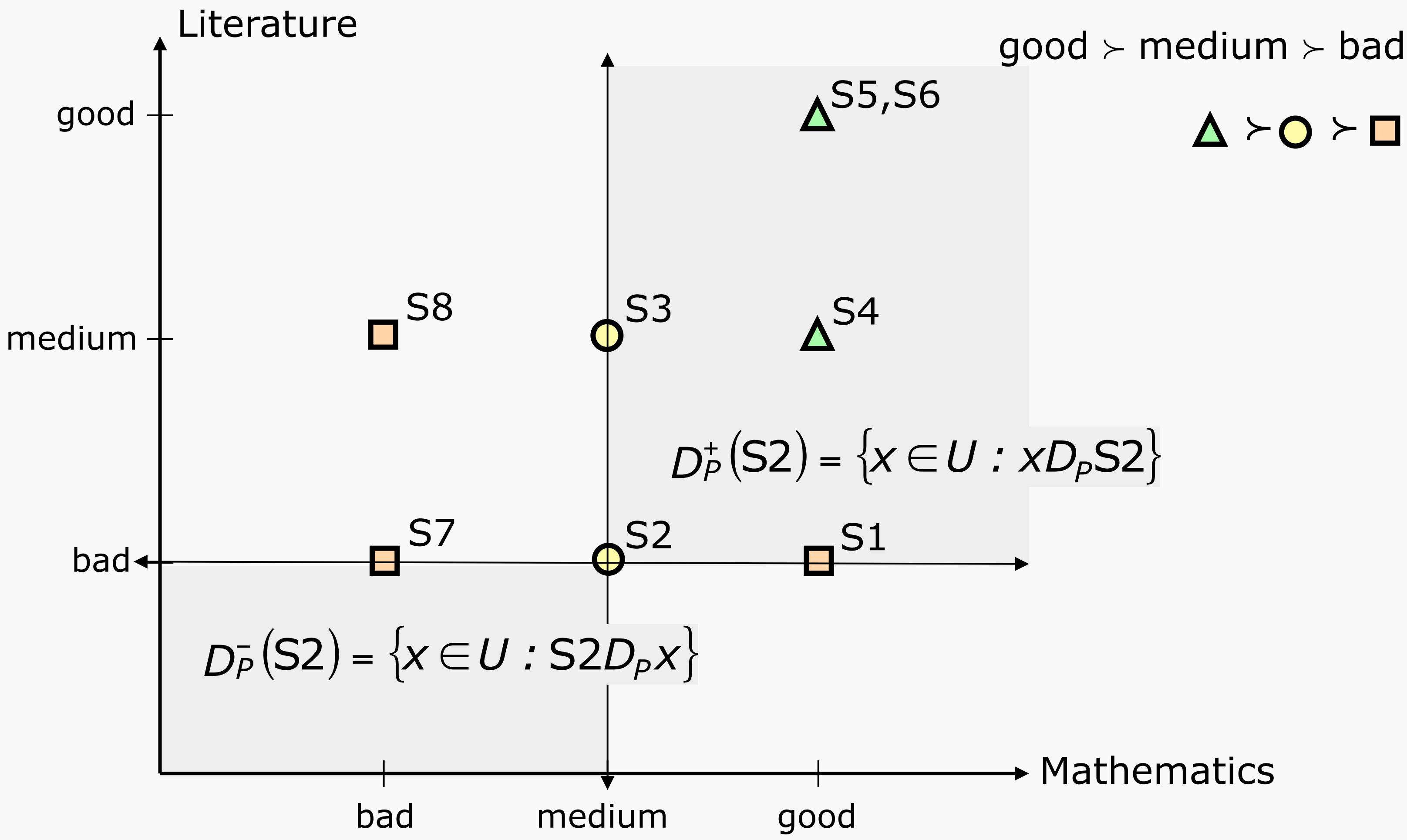
- Dominance cones in condition space {M,L} :



- P={M,L}

Rough Set approach to ordinal classification

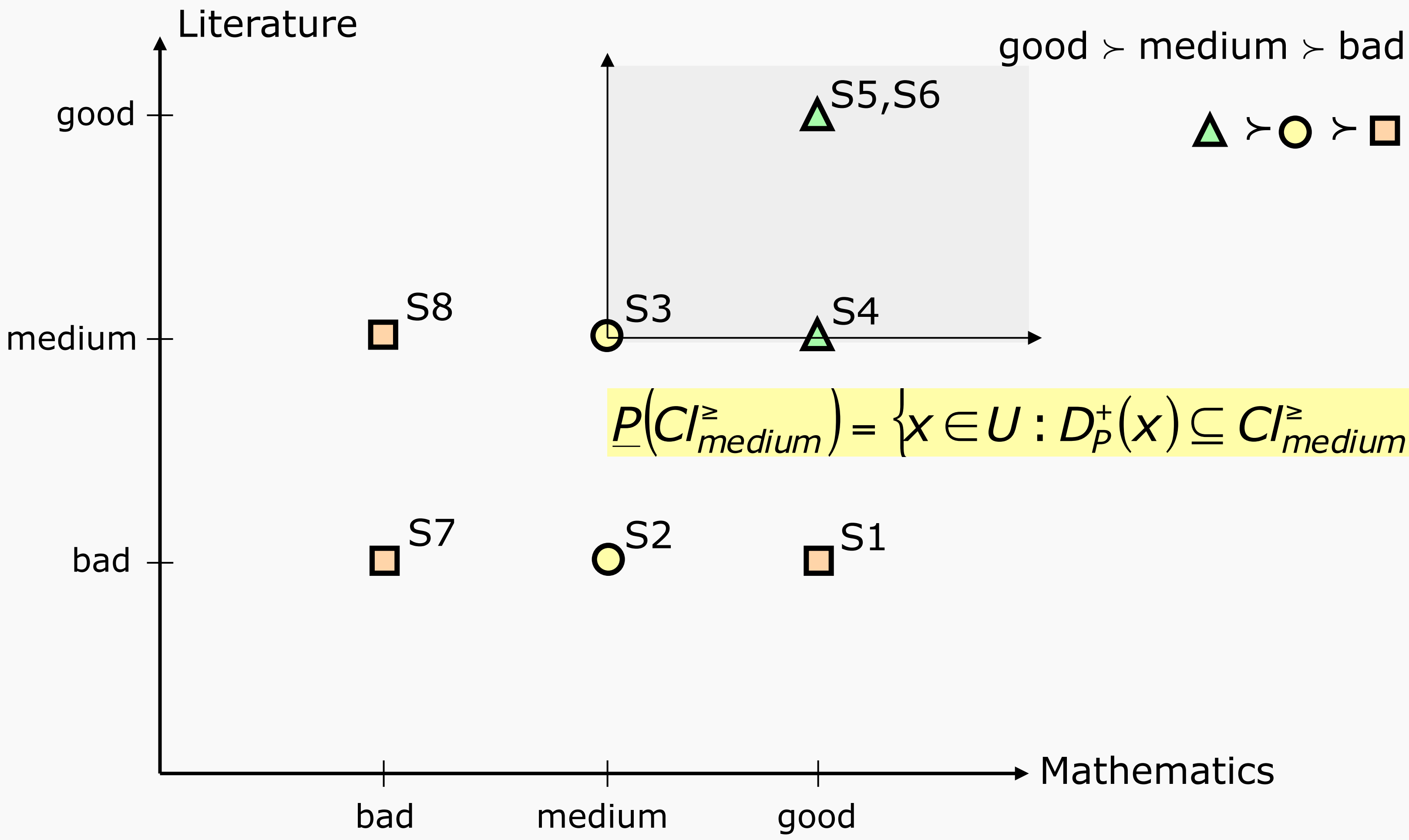
- Dominance cones in condition space {M,L} :



- P={M,L}

Rough Set approach to ordinal classification

- Lower approximation of at least medium students:

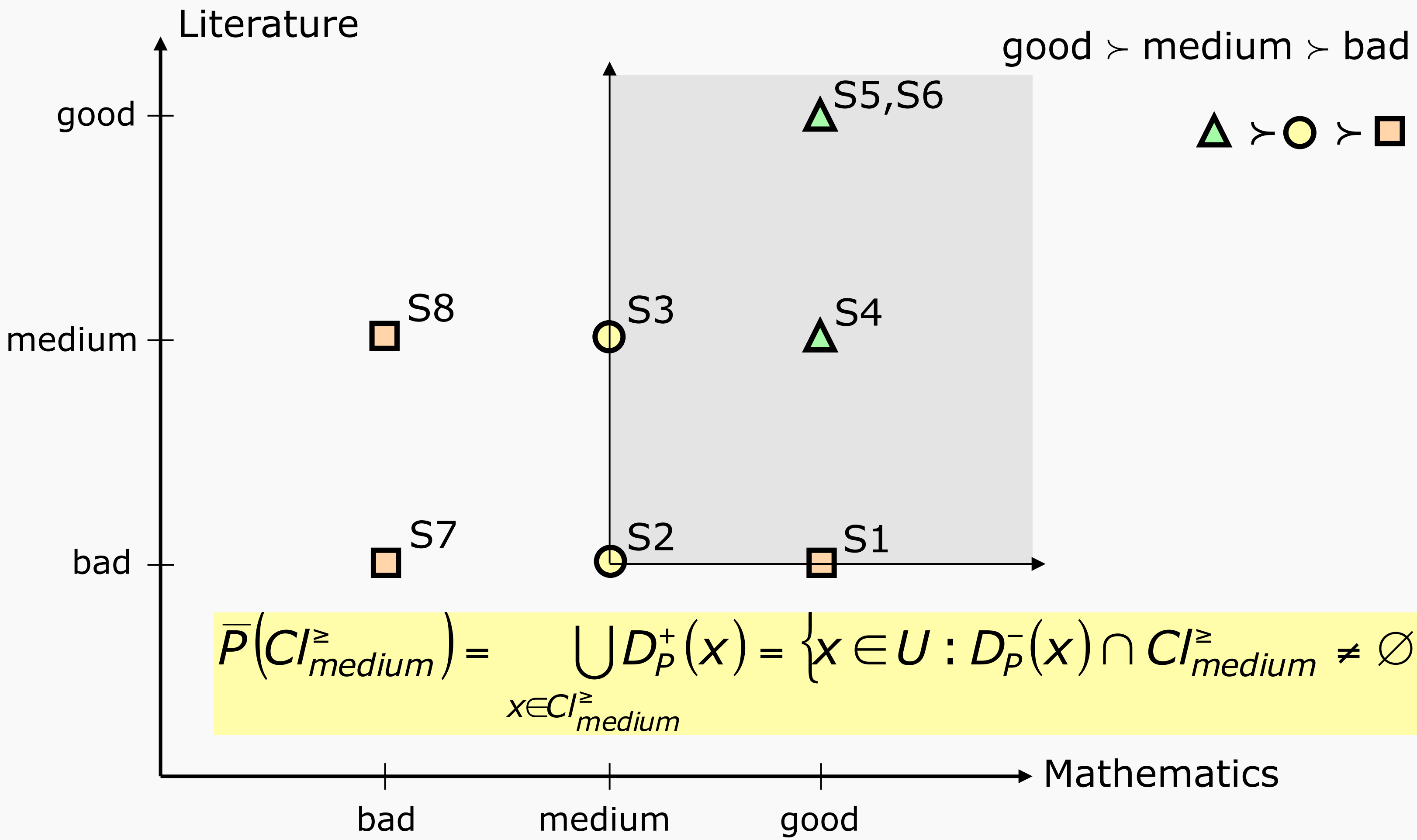


$$\underline{P}\left(Cl^{\geq}_{medium}\right) = \left\{x \in U : D^{+}_{P}(x) \subseteq Cl^{\geq}_{medium}\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Upper approximation of <u>at least</u> medium students:

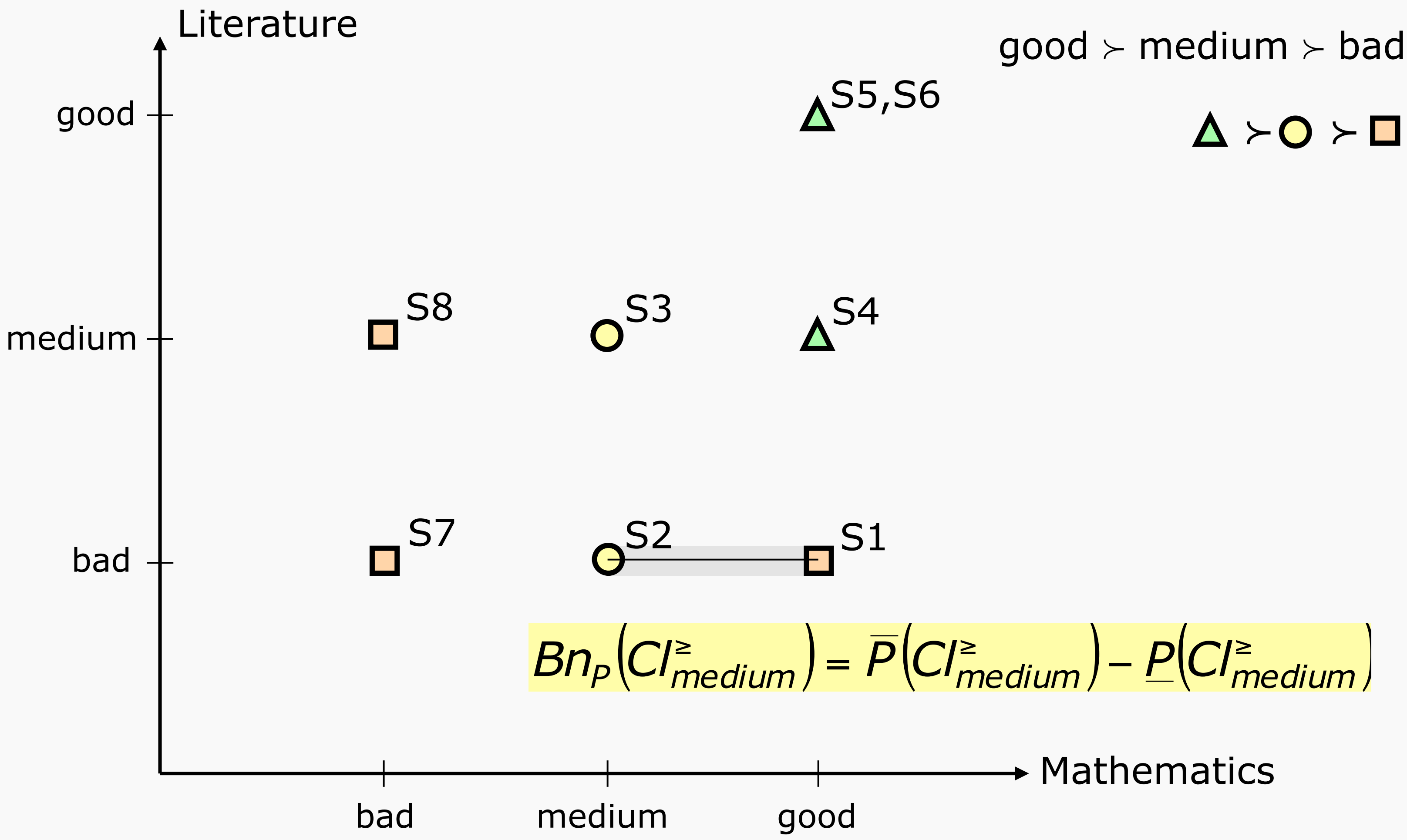


$$\overline{P}\left(Cl^{\geq}_{medium}\right) = \bigcup_{x\in Cl^{\geq}_{medium}} D_P^{+}(x) = \left\{x \in U : D_P^{-}(x) \cap Cl^{\geq}_{medium} \neq \emptyset\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Boundary region of <u>at least</u> medium students:

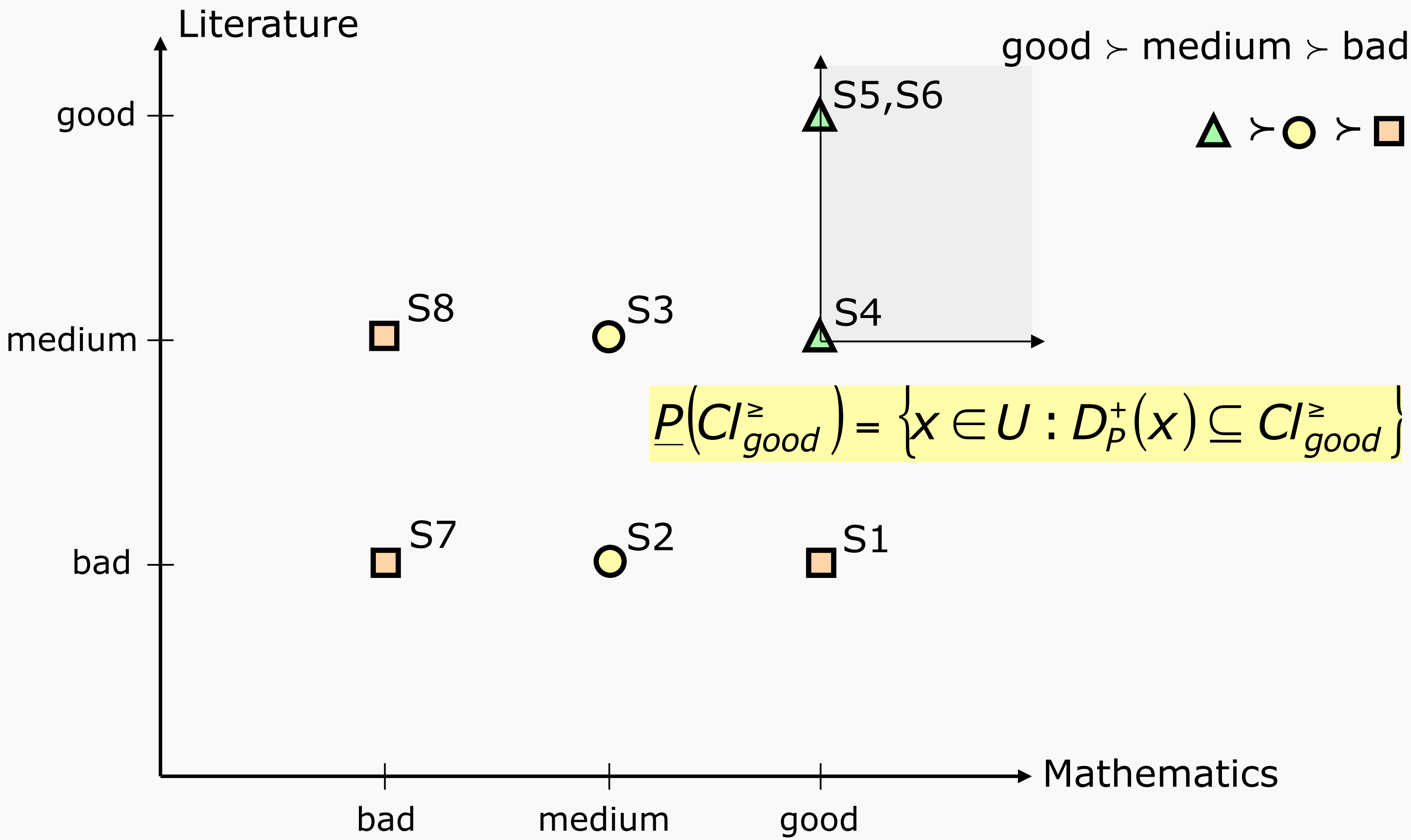


$$Bn_P\left(Cl^{\geq}_{medium}\right) = \overline{P}\left(Cl^{\geq}_{medium}\right) - \underline{P}\left(Cl^{\geq}_{medium}\right)$$

- P={M,L}

Rough Set approach to ordinal classification

- Lower approximation of <u>at least</u> good students:

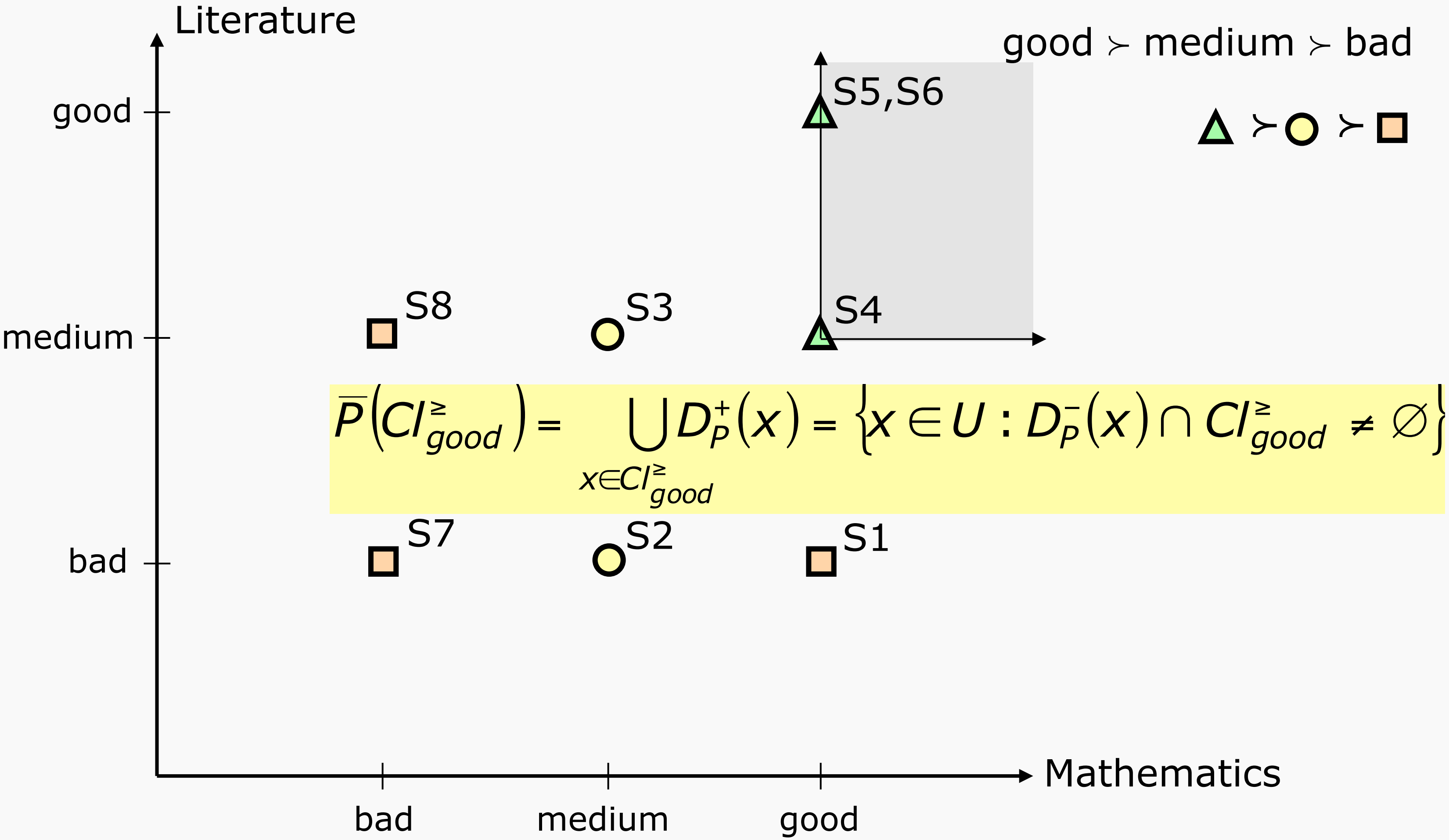


$$\underline{P}\left(Cl_{good}^{\geq}\right) = \left\{x \in U : D_P^{+}(x) \subseteq Cl_{good}^{\geq}\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Upper approximation of <u>at least</u> good students:

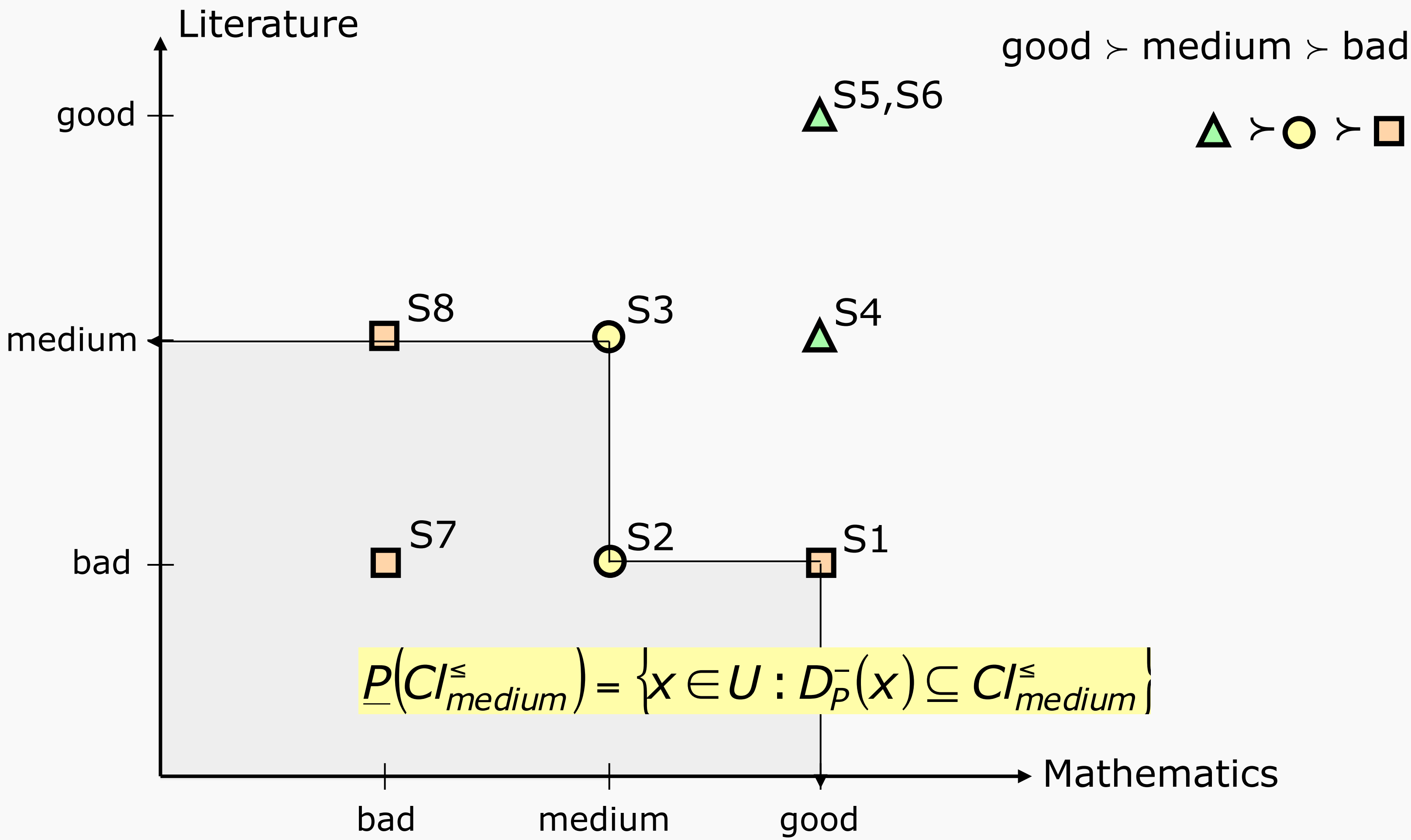


$$\overline{P}\left(Cl^{\geq}_{good}\right) = \bigcup_{x \in Cl^{\geq}_{good}} D^{+}_{P}(x) = \left\{x \in U : D^{-}_{P}(x) \cap Cl^{\geq}_{good} \neq \emptyset\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Lower approximation of at most medium students:

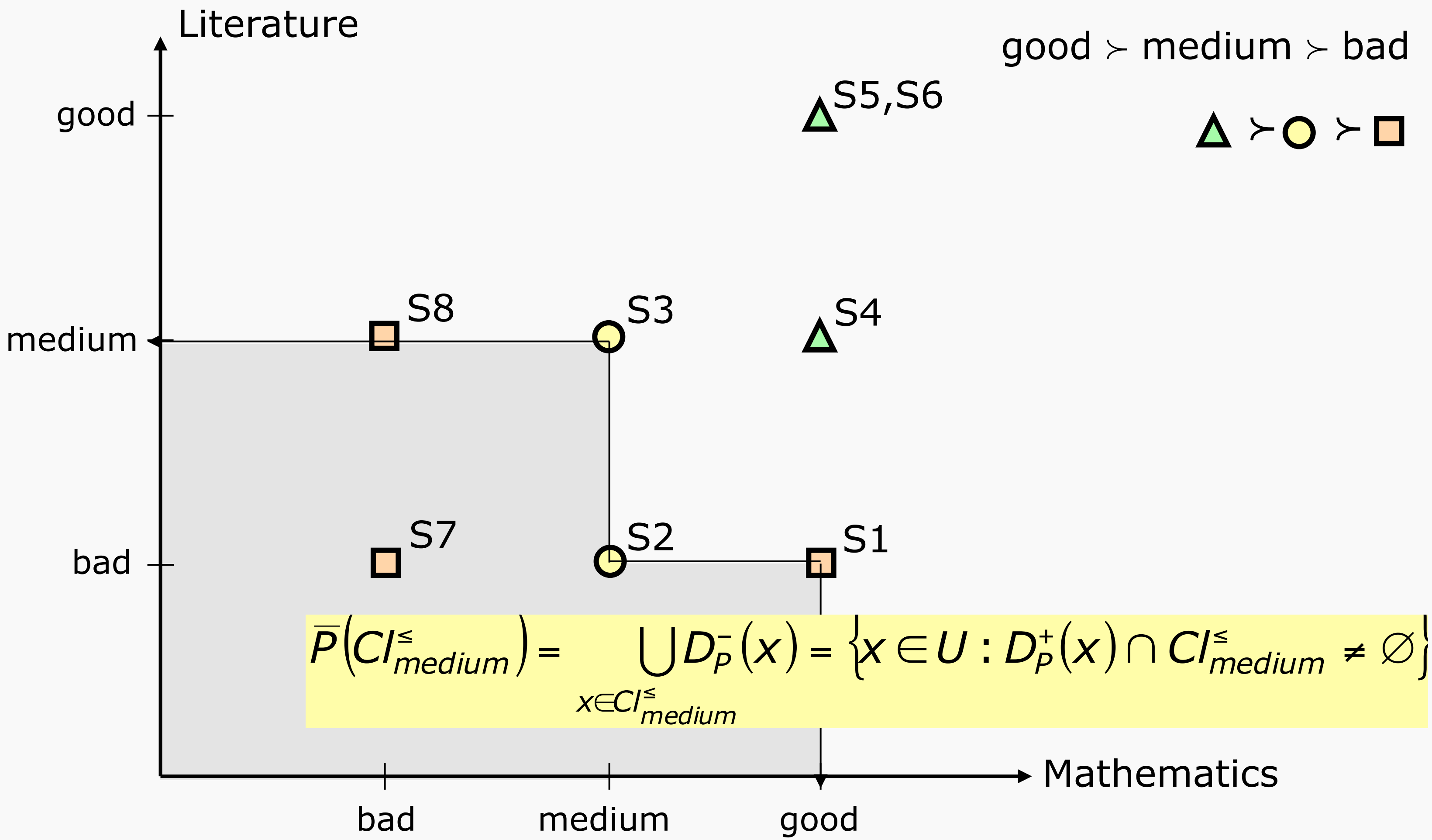


$$\underline{P}\left(Cl_{medium}^{\leq}\right)=\left\{x\in U: D_P^{-}(x)\subseteq Cl_{medium}^{\leq}\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Upper approximation of at most medium students:

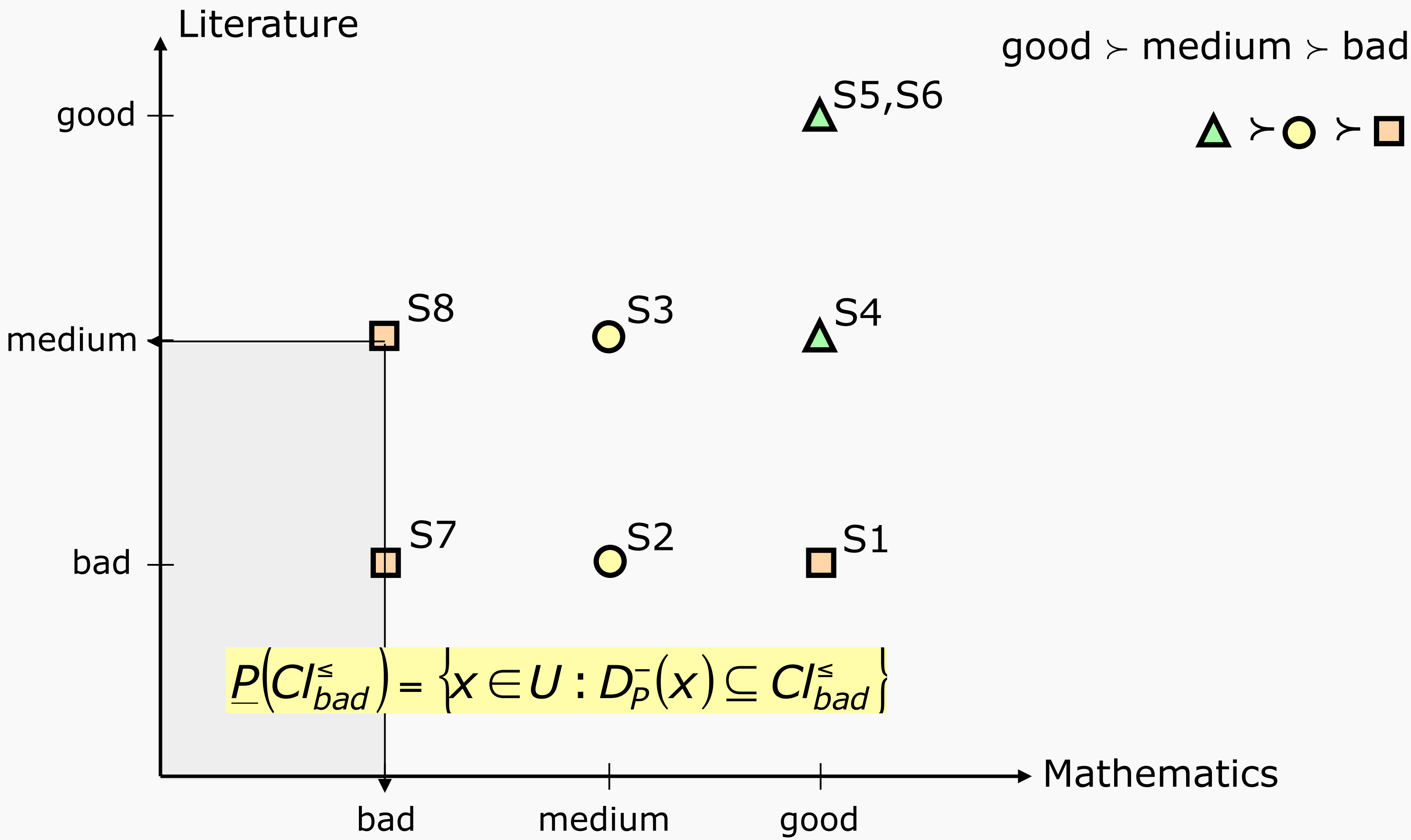


$$\overline{P}\left(Cl^{\le}_{medium}\right) = \bigcup_{x\in Cl^{\le}_{medium}} D_P^-(x) = \left\{x \in U : D_P^+(x) \cap Cl^{\le}_{medium} \ne \emptyset\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Lower approximation of at most bad students:

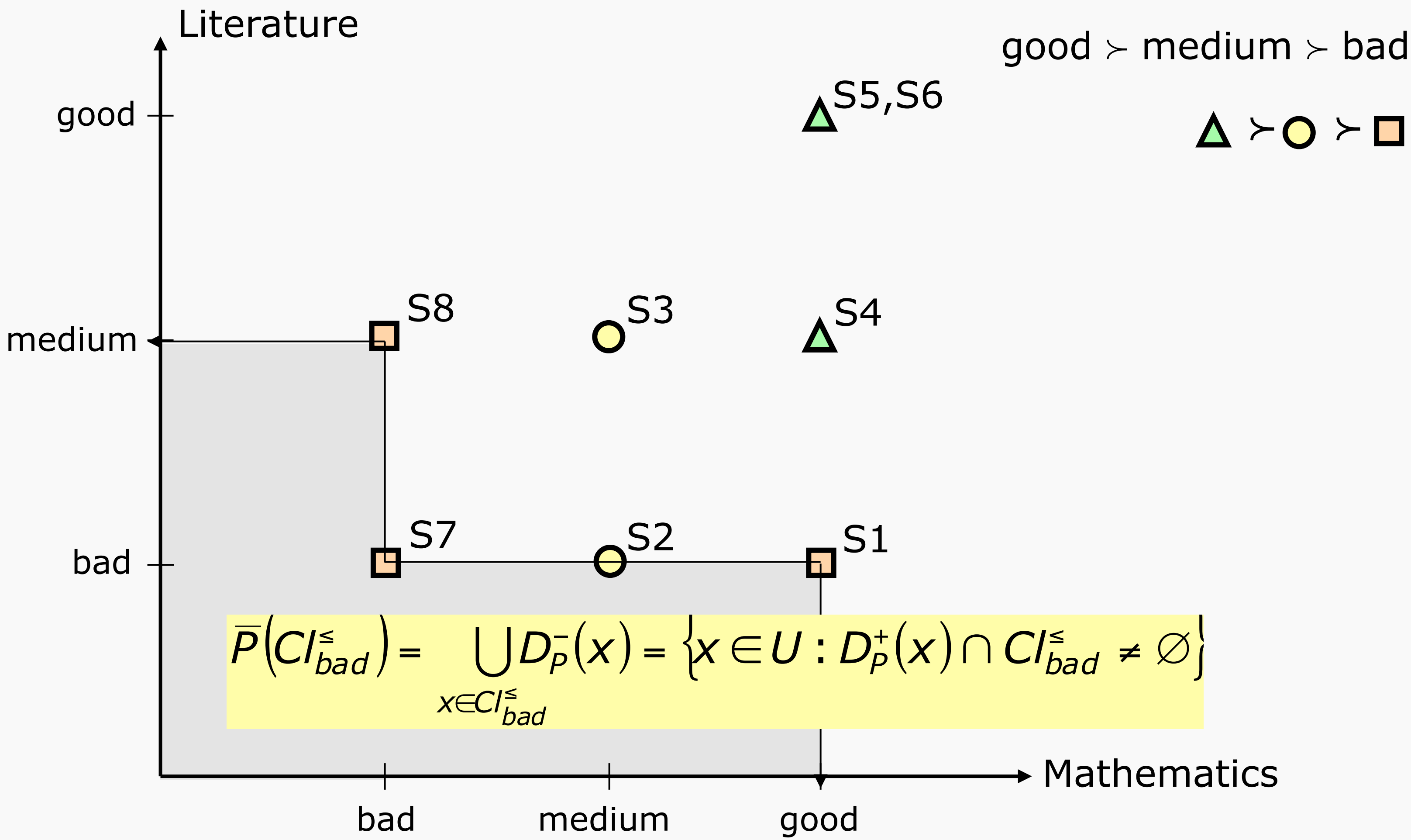


$$\underline{P}\left(Cl_{bad}^{\leq}\right)=\left\{x\in U: D_P^-(x)\subseteq Cl_{bad}^{\leq}\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Upper approximation of at most bad students:

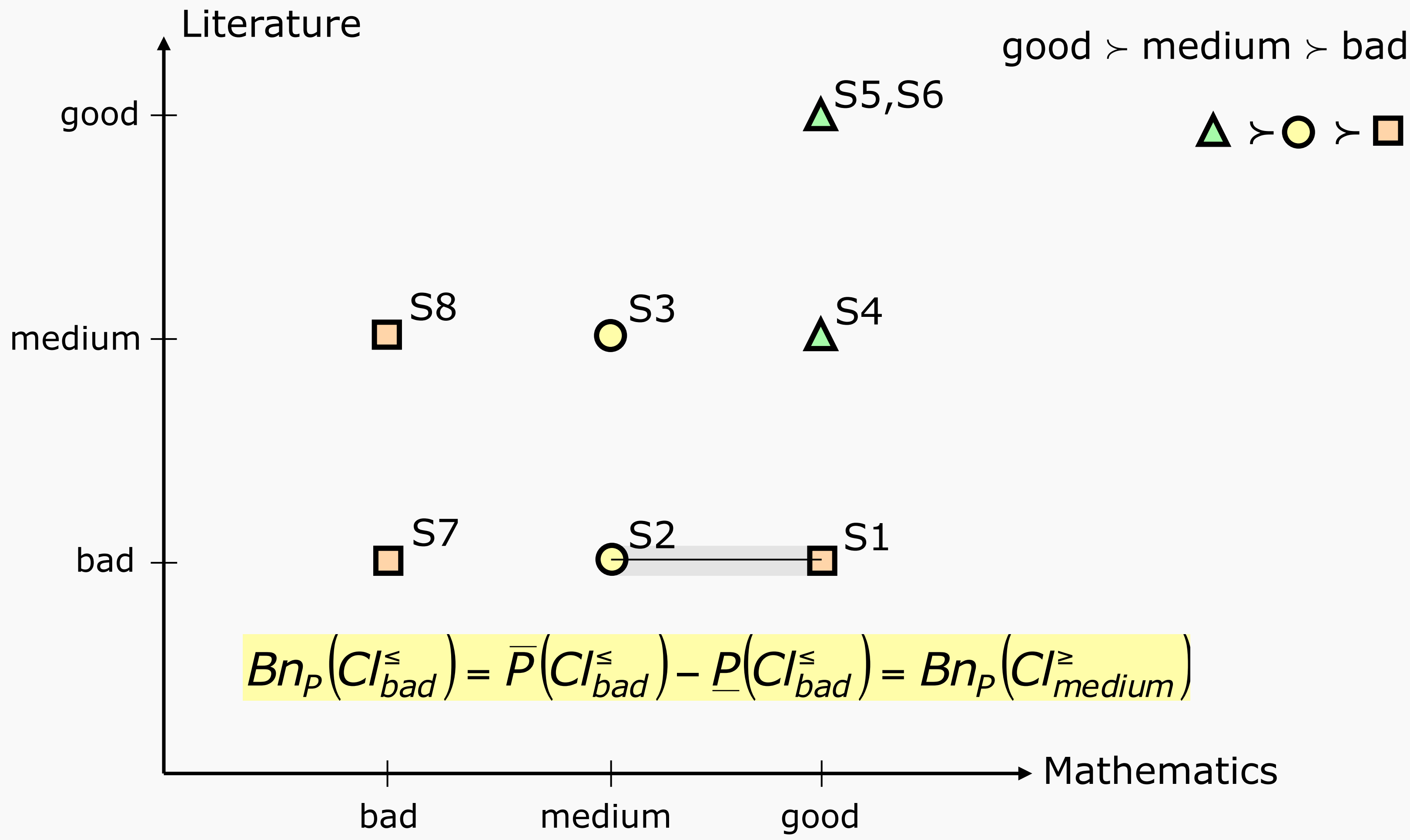


$$\overline{P}\left(Cl_{bad}^{\le}\right)=\bigcup_{x\in Cl_{bad}^{\le}} D_P^-(x)=\left\{x\in U: D_P^+(x)\cap Cl_{bad}^{\le}\neq\emptyset\right\}$$

- P={M,L}

Rough Set approach to ordinal classification

- Boundary region of <u>at most</u> bad students:



$$Bn_P\left(Cl_{bad}^{\leq}\right) = \overline{P}\left(Cl_{bad}^{\leq}\right) - \underline{P}\left(Cl_{bad}^{\leq}\right) = Bn_P\left(Cl_{medium}^{\geq}\right)$$

- P={M,L}

DRSA – properties

- Basic properies of rough approximations

$$\underline{P}\left(Cl_t^{\geq}\right) \subseteq Cl_t^{\geq} \subseteq \overline{P}\left(Cl_t^{\geq}\right) \qquad \underline{P}\left(Cl_t^{\leq}\right) \subseteq Cl_t^{\leq} \subseteq \overline{P}\left(Cl_t^{\leq}\right)$$

$$\underline{P}\left(Cl_t^{\geq}\right) = U - \overline{P}\left(Cl_{t-1}^{\leq}\right), \text{ for } t=2,\ldots,m$$

- Identity of boundaries $Bn_P\left(Cl_t^{\geq}\right) = Bn_P\left(Cl_{t-1}^{\leq}\right)$, for t=2,…,m

- Quality of approximation of classification $\boldsymbol{Cl}=\{Cl_t,\ t=1,\ldots m\}$ by set $P\subseteq C$

$$\gamma_P\left(\boldsymbol{Cl}\right) = \frac{\left|U - \bigcup_{t\in\{2,\ldots,m\}} Bn_P\left(Cl_t^{\geq}\right)\right|}{\left|U\right|}$$

- ***Cl***-reducts and ***Cl***-core of $P\subseteq C$

$$CORE_{\boldsymbol{Cl}}\left(P\right) = \bigcap RED_{\boldsymbol{Cl}}\left(P\right)$$

DRSA – induction of decision rules from rough approximations

- Induction of decision rules from rough approximations
  - *certain* D$_{\geq}$-*decision rules*, supported by objects $\in Cl_t^{\geq}$ without ambiguity:

    if $x_{q1} \succeq_{q1} r_{q1}$ *and* $x_{q2} \succeq_{q2} r_{q2}$ *and* … $x_{qp} \succeq_{qp} r_{qp}$, *then* $x \in Cl_t^{\geq}$

  - *possible* D$_{\geq}$-*decision rules*, supported by objects $\in Cl_t^{\geq}$ with or without any ambiguity:

    if $x_{q1} \succeq_{q1} r_{q1}$ *and* $x_{q2} \succeq_{q2} r_{q2}$ *and* … $x_{qp} \succeq_{qp} r_{qp}$, *then* x possibly $\in Cl_t^{\geq}$

DRSA – induction of decision rules from rough approximations

- Induction of decision rules from rough approximations
  - *certain* $D_{\leq}$*-decision rules*, supported by objects $\in Cl_t^{\leq}$ without ambiguity:

    if $x_{q1} \preceq_{q1} r_{q1}$ *and* $x_{q2} \preceq_{q2} r_{q2}$ *and …* $x_{qp} \preceq_{qp} r_{qp}$, *then* $x \in Cl_t^{\leq}$

  - *possible* $D_{\leq}$*-decision rules*, supported by objects $\in Cl_t^{\leq}$ with or without any ambiguity:

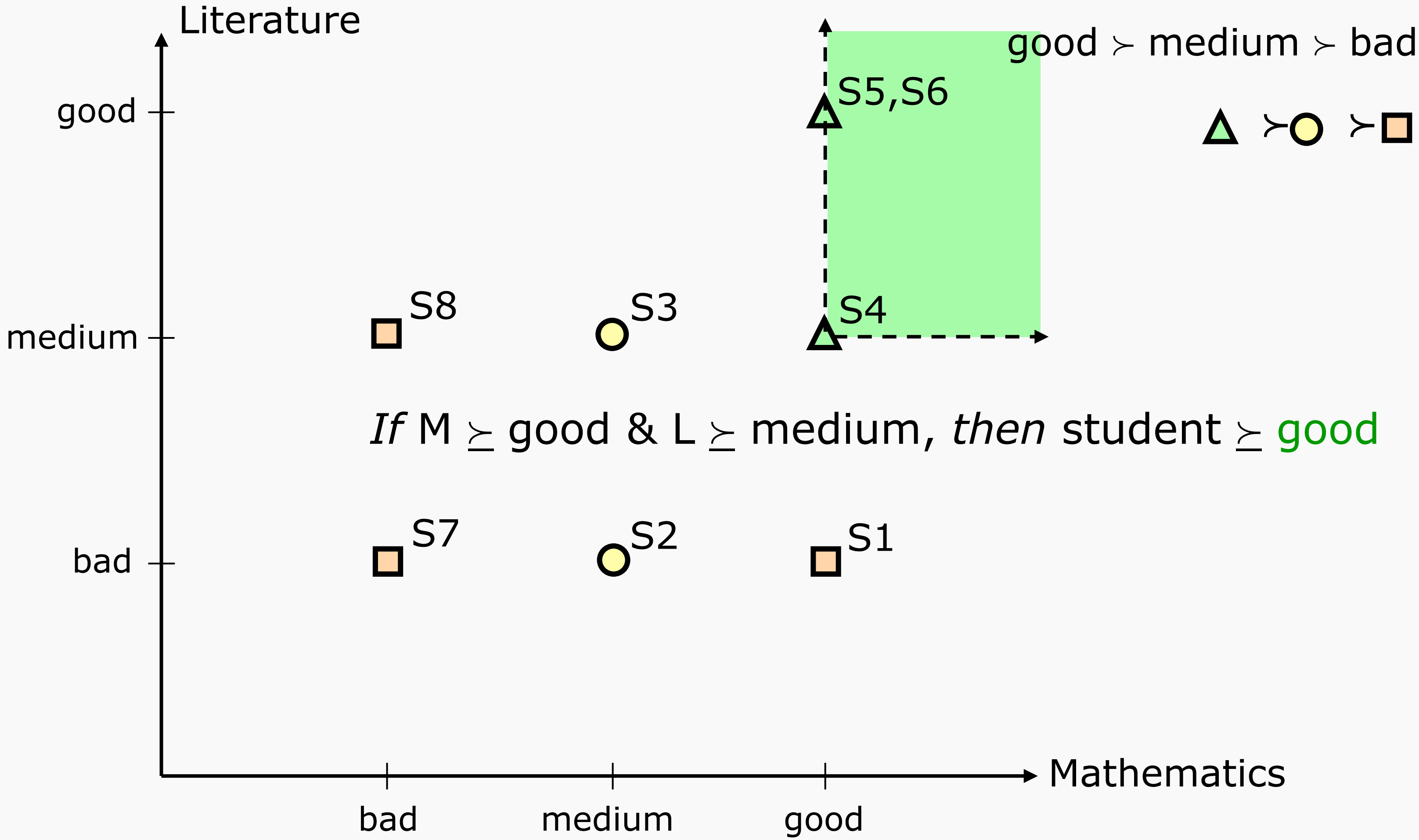
    if $x_{q1} \preceq_{q1} r_{q1}$ *and* $x_{q2} \preceq_{q2} r_{q2}$ *and …* $x_{qp} \preceq_{qp} r_{qp}$, *then x* possibly $\in Cl_t^{\leq}$

  - *approximate* $D_{\geq\leq}$*-decision rules*, supported by objects $\in Cl_s \cup Cl_{s+1} \cup \ldots \cup Cl_t$ without possibility of discerning to which class:

    if $x_{q1} \succeq_{q1} r_{q1}$ *and…* $x_{qk} \succeq_{qk} r_{qk}$ *and* $x_{qk+1} \preceq_{qk+1} r_{qk+1}$ *and …* $x_{qp} \preceq_{qp} r_{qp}$, *then* $x \in Cl_s \cup Cl_{s+1} \cup \ldots \cup Cl_t$.

Rough Set approach to ordinal classification

- Decision rules in terms of {M,L} :

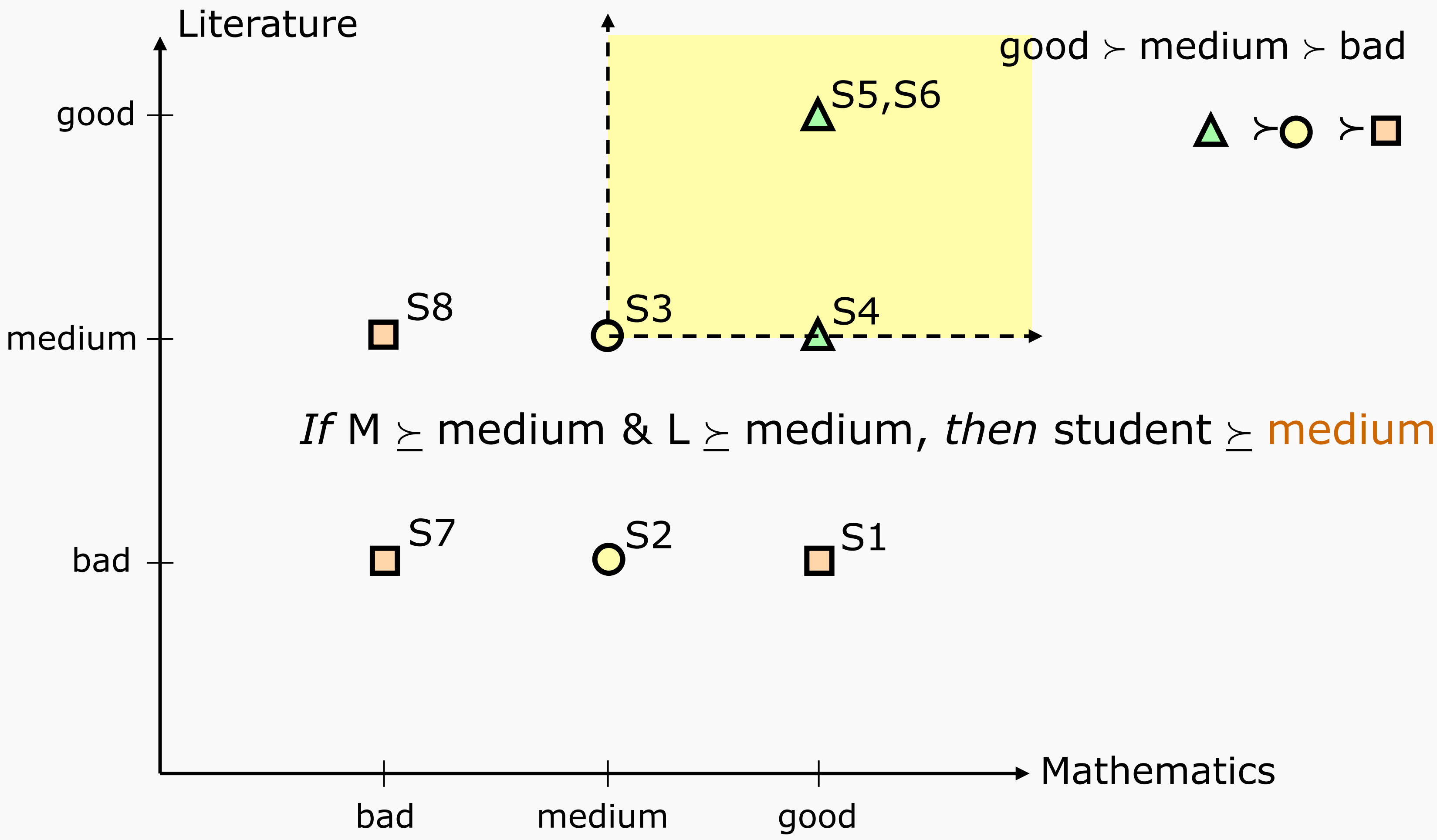


If M $\succeq$ good & L $\succeq$ medium, *then* student $\succeq$ good

- $D_{\geq}$ - certain rule

Rough Set approach to ordinal classification

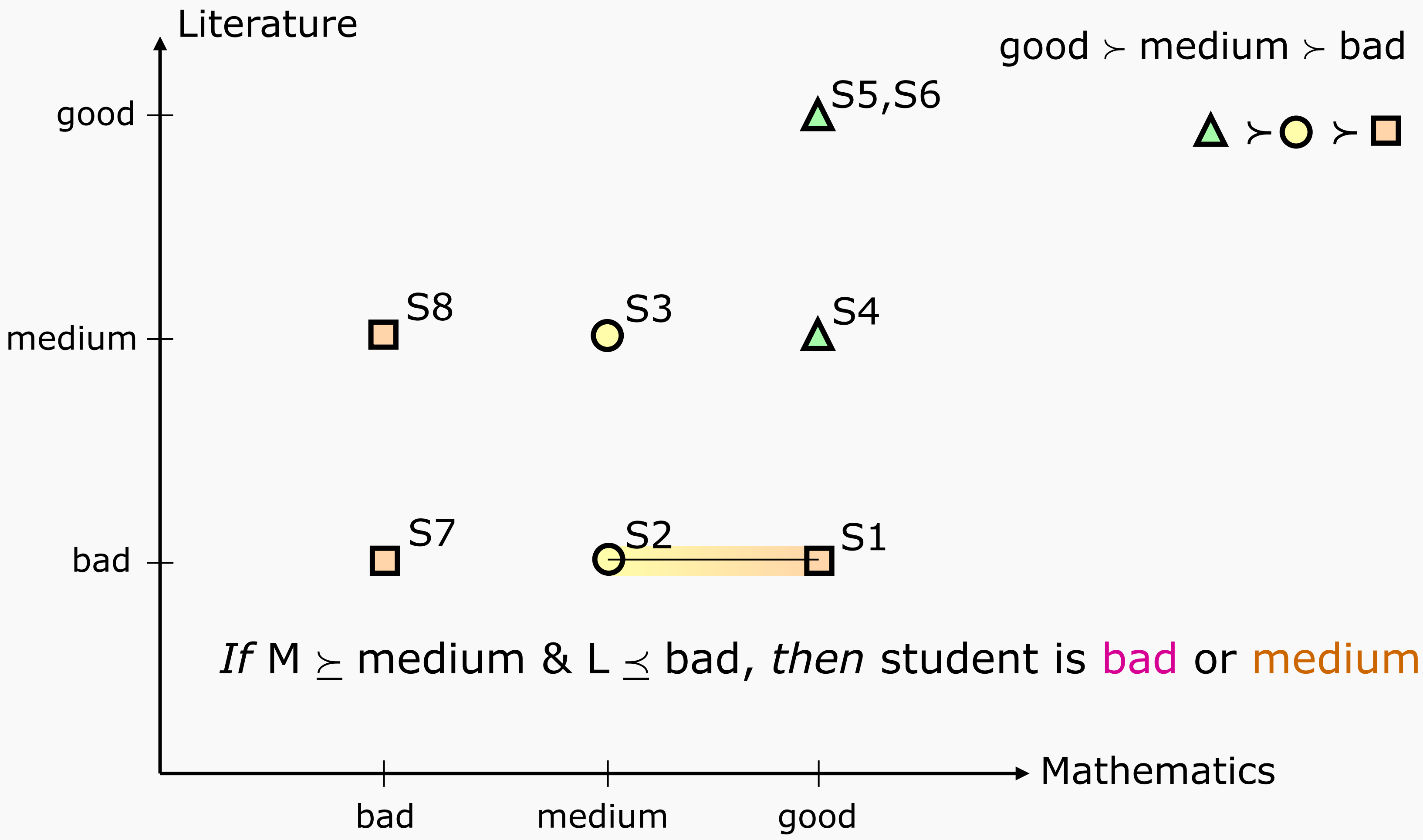
- Decision rules in terms of {M,L} :



- $D_{\geq}$ - certain rule

Rough Set approach to ordinal classification

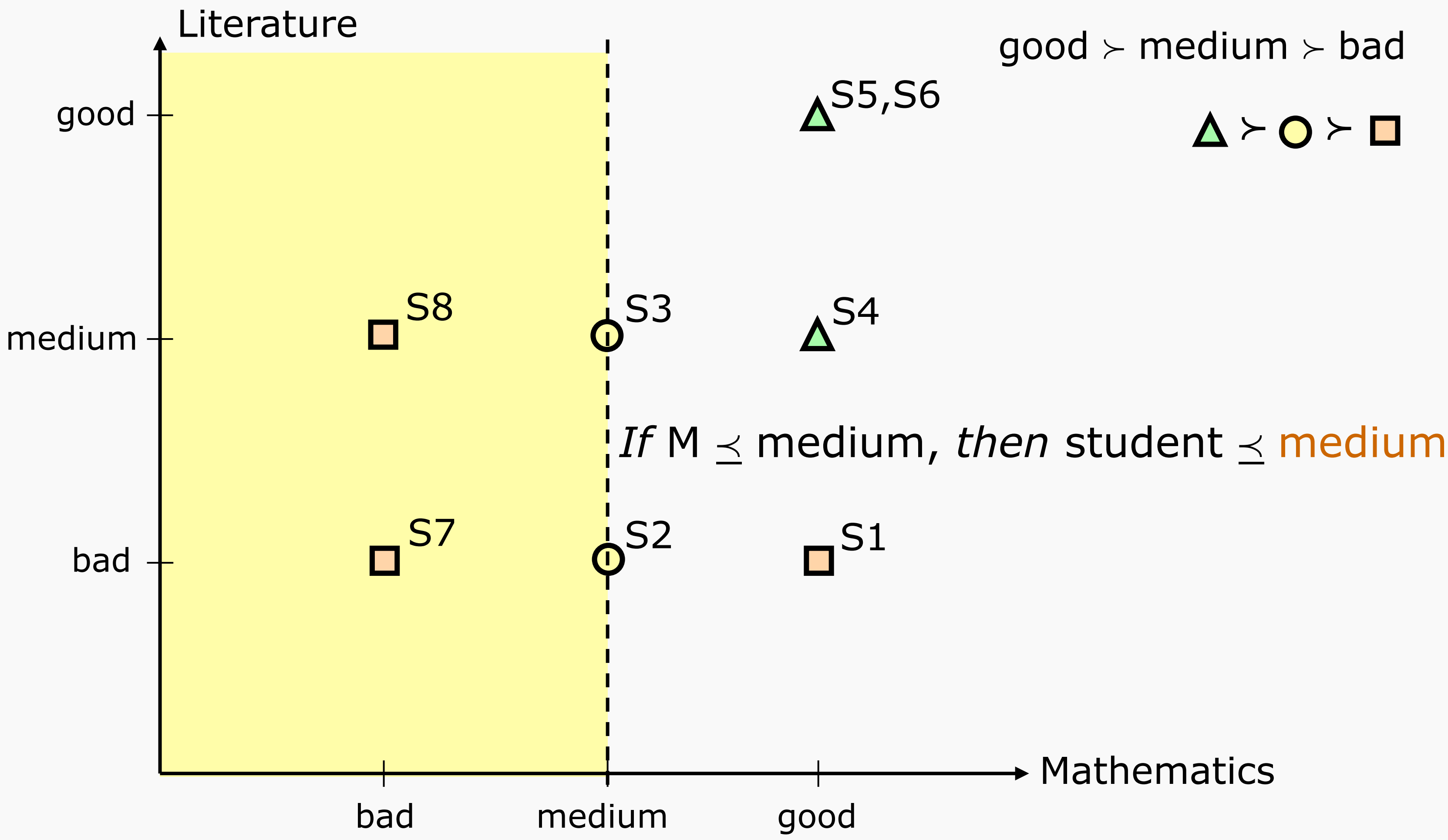
- Decision rules in terms of {M,L} :



- $D_{\geq\leq}$ - approximate rule

Rough Set approach to ordinal classification

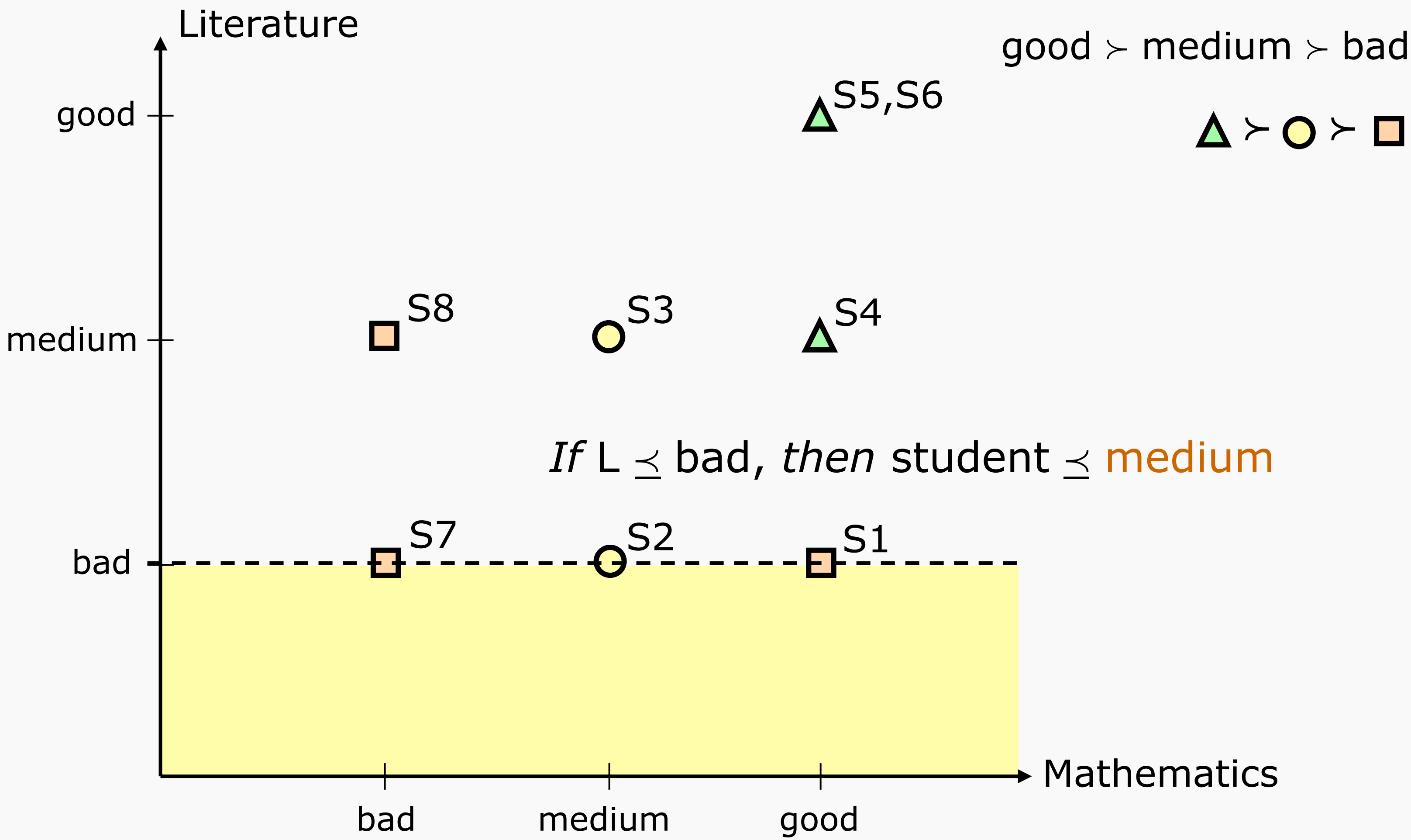
- Decision rules in terms of {M,L} :



- $D_{\leq}$ - certain rule

Rough Set approach to ordinal classification

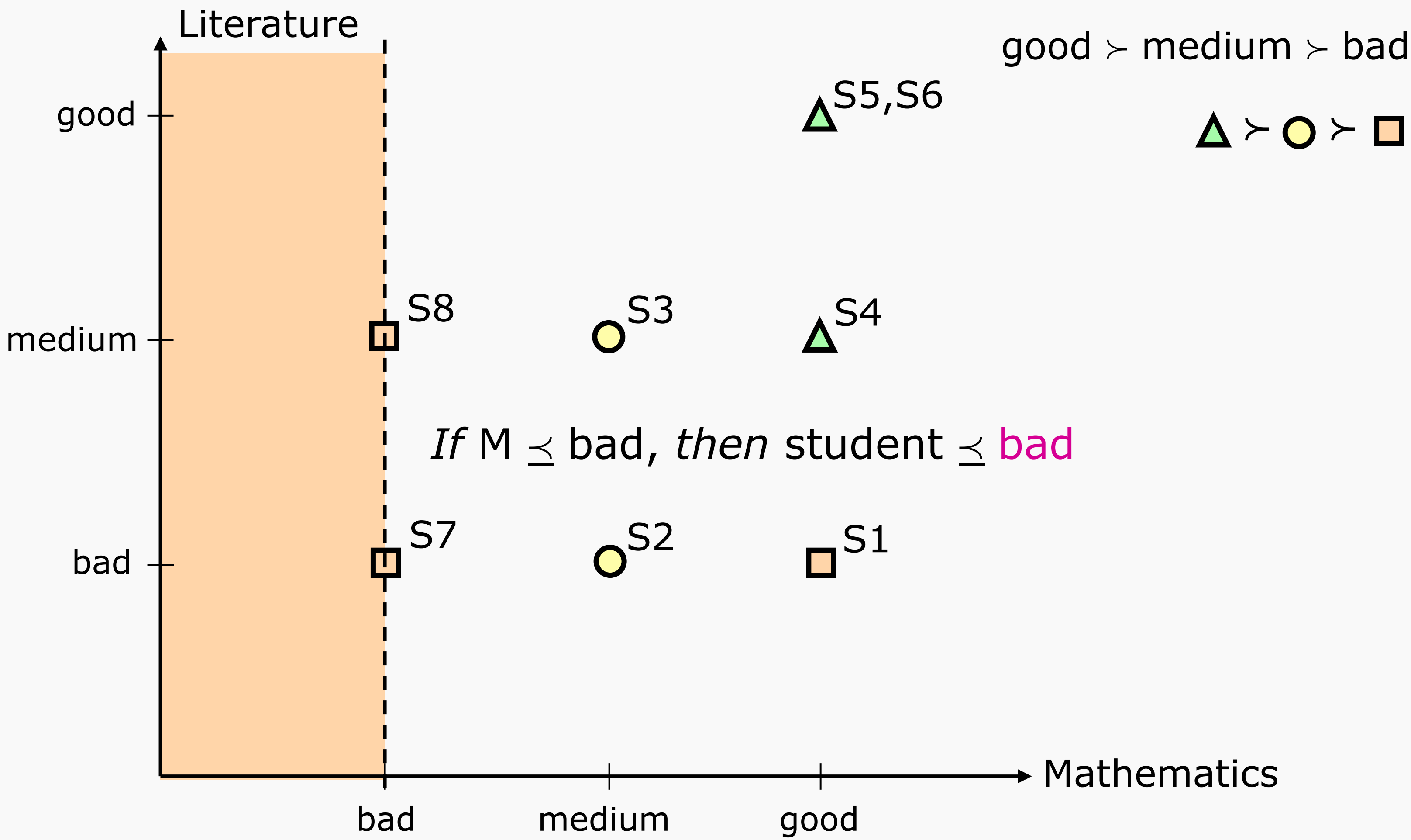
- Decision rules in terms of {M,L} :



- $D_{\leq}$ - certain rule

Rough Set approach to ordinal classification

- Decision rules in terms of {M,L} :



- $D_{\leq}$ - certain rule

Rough Set approach to ordinal classification

- Set of decision rules in terms of {M, L} representing preferences:

If M $\succeq$ good & L $\succeq$ medium, *then* student $\succeq$ good {S4,S5,S6}

If M $\succeq$ medium & L $\succeq$ medium, *then* student $\succeq$ medium {S3,S4,S5,S6}

If M $\succeq$ medium & L $\preceq$ bad, *then* student is bad or medium {S1,S2}

If M $\preceq$ medium, *then* student $\preceq$ medium {S2,S3,S7,S8}

If L $\preceq$ bad, *then* student $\preceq$ medium {S1,S2,S7}

If M $\preceq$ bad, *then* student $\preceq$ bad {S7,S8}

Rough Set approach to ordinal classification

- Set of decision rules in terms of {M,Ph,L} representing preferences:

If M $\succeq$ good & L $\succeq$ medium, *then* student $\succeq$ good {S4,S5,S6}

If M $\succeq$ medium & L $\succeq$ medium, *then* student $\succeq$ medium {S3,S4,S5,S6}

If M $\succeq$ medium & L $\preceq$ bad, *then* student is bad or medium {S1,S2}

If Ph $\preceq$ medium & L $\preceq$ medium *then* student $\preceq$ medium {S1,S2,S3,S7,S8}

If M $\preceq$ bad, *then* student $\preceq$ bad {S7,S8}

- The preference model involving all three criteria is more concise