Analysis of monotonicity properties of new normalized rule interestingness measures

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Presentation plan

- Introduction

- Basic quantitative characteristics of rules
- Normalization of interestingness measures
- Desired properties of interestingness measures

- Results of the conducted analysis
- Practical application of the results

- Conclusions

Introduction - motivations

The number of rules
induced from datasets is usually quite large

- **overwhelming for human comprehension,**
- **many rules are irrelevant or obvious**
(low practical value)

rule evaluation – interestingness (attractiveness) measures
(e.g. support, confidence, gain)

- **each measure was proposed to capture different characteristics of rules**
- **the number of proposed measures is very large**
- **easiness of interpretation of NORMALIZED measures**

Introduction - motivations

The choice of interestingness measure for a certain application is a difficult problem

- **the users expectations vary,**
- **the number of proposed measures is overwhelming**

properties of interestingness measures, which reflect users′ expectations towards the behavior of measures in particular situations

need to analyze measures with respect to their properties

Introduction – rule induction

- **Patterns in form of rules are induced from a data table**
- ***S*=⟨*U*, *A*⟩ – *data table*, where *U* and *A* are finite, non-empty sets**
  U* – *universe*; *A* – set of *attributes
- ***S*=⟨*U*, *C*, *D*⟩ – *decision table*, where C – set of *condition* *attributes*,**
  ***D* – set of *decision* *attributes*, *C*∩*D*=∅**

- ***Decision rule* or *association rule* induced from *S***
  is a *consequence relation*: f ® y *read as* *if* f *then* y
  where f is condition (evidence or premise)
  and y is conclusion (hypothesis or decision)
  formula built from attribute-value pairs (*q*,*v*)
- **If the division into independent and dependent attributes is fixed, then rules are regarded as decision rules, otherwise as association rules.**

Introduction – rule induction

Characterization of nationalities

U	*Height*	*Hair*	*Eyes*	*Nationality*	Support
1	*tall*	*blond*	*blue*	*Swede*	270
2	*medium*	*dark*	*hazel*	*German*	90
3	*medium*	*blond*	*blue*	*Swede*	90
4	*tall*	*blond*	*blue*	*German*	360
5	*short*	*red*	*blue*	*German*	45
6	*medium*	*dark*	*hazel*	*Swede*	45

C ***D***

- E.g. decision rules induced from „characterization of nationalities":
  1) **If (*Height=tall*), then (*Nationality=Swede*)**
  2) **If (*Height=medium*) & (*Hair=dark*), then (*Nationality=German*)**

Introduction – interestingness measures

- To measure the relevance and utility of rules, quantitative measures called attractiveness or interestingness measures, have been proposed (e.g. support, confidence, lift, gain, Piatetsky-Shapiro,...)
- Unfortunately, there is no evidence which measure(s) is (are) the best

- **Notation:**
  - $sup(\mathbf{0})$ is the number of all objects from *U*, having property °

    e.g. $sup(\phi)$, $sup(\psi)$

Basic quantitative characteristics of rules

- **Support of rule $\phi \rightarrow \psi$ in *S*:**

$$sup(\phi \rightarrow \psi) = sup(\phi \wedge \psi)$$

- **Anti-support of rule $\phi \rightarrow \psi$ in *S*:**

$$anti\text{-}sup(\phi \rightarrow \psi) = sup(\phi \wedge \neg\psi)$$

Basic quantitative characteristics of rules

- ***Rule Interest Function*** **(Piatetsky-Shapiro 1991)**

$$RI(\phi \to \psi) = sup(\phi \to \psi) - \frac{sup(\psi)sup(\phi)}{|U|}$$

- ***Gain measure*** **(Fukuda et al. 1996)**

$$gain(\phi \to \psi) = sup(\phi \to \psi) - \Theta sup(\phi)$$

where Θ is a fraction constant between 0 and 1

- ***Dependency Factor*** **(Popper 1959, Pawlak 2002)**

$$\eta(\phi \to \psi) = \frac{\frac{sup(\phi \to \psi)}{sup(\phi)} - \frac{sup(\psi)}{|U|}}{\frac{sup(\phi \to \psi)}{sup(\phi)} + \frac{sup(\psi)}{|U|}}$$

Normalization of interestingness measures

Normalization of interestingness measures

- **We propose to normalize measures, so that they would distinguish between two completely different situations:**
  - **situation α in which confirmation occurs:** $Pr(\psi|\phi) \geq Pr(\psi)$ **which, under „the closed world assumption", can be estimated as:**

$$\frac{sup(\phi \to \psi)}{sup(\phi)} \geq \frac{sup(\psi)}{|U|}$$

  - **situation β in which disconfirmation occurs:** $Pr(\psi|\phi) < Pr(\psi)$ **which, under „the closed world assumption", can be estimated as:**

$$\frac{sup(\phi \to \psi)}{sup(\phi)} < \frac{sup(\psi)}{|U|}$$

Normalization of interestingness measures

- Inspired by Crupi et al. 2007 we propose to normalize *RI*, *gain* and *dependency factor* by dividing them
  - by the maximum value they obtain in case of confirmation, and
  - by the absolute minimum value they obtain in case of disconfirmation.
- In this way, we will obtain interestingness measures taking values from the interval [–1, 1].

Normalization of interestingness measures

- There are many approaches to determining the maximum and minimum values (in case of confirmation and disconfirmation, respectively), which eventually lead to different normalizations.
- Inspired by the approach of (Nicod 1923):
  - we consider only cases in which the evidence is true,
  - while we ignore cases where there is no evidence.

- For example, in case of rule: *„if x is a raven, then x is black"*, the evidence is "*raven*" and the hypothesis is "*black*".

  In this situation, a *raven* which is *black* supports the rule,
  a *raven* which is *not black* is against this rule,
  and everything which is *not a raven* can be ignored.

Normalization of interestingness measures – „Nicod approach"

- E.g. consider rule $\phi \rightarrow \psi$: ***if x is a raven then x is black***

  a=$sup(\phi \rightarrow \psi)$ – the number of objects in ***U*** which are black ravens

  b=$sup(\neg\phi \rightarrow \psi)$ – the no. of objects in ***U*** which are black non-ravens

  c=$sup\ (\phi \rightarrow \neg\psi)$ – the no. of objects in ***U*** which are non-black ravens

  d=$sup(\neg\phi \rightarrow \neg\psi)$–the no. of objects in ***U*** which are non-black non-ravens

- The maximal support of the rule will be obtained when:
  - ***c*=:0** (i.e. there are no non-black ravens) and
  - ***a*** takes over all observations from ***c***

    (i.e. each raven is black, ***a*=:*a*+*c***)

Normalization of interestingness measures – „Nicod approach"

- **E.g. consider rule $\phi \rightarrow \psi$: *if x is a raven then x is black***

  a=$sup(\phi \rightarrow \psi)$ – **the number of objects in *U* which are black ravens**

  b=$sup(\neg\phi \rightarrow \psi)$ – **the no. of objects in *U* which are black non-ravens**

  c=$sup(\phi \rightarrow \neg\psi)$ – **the no. of objects in *U* which are non-black ravens**

  d=$sup(\neg\phi \rightarrow \neg\psi)$–**the no. of objects in *U* which are non-black non-ravens**

- **The minimal support of the rule will be obtained when:**
  - ***a*=:0 (i.e. there are no black ravens) and**
  - ***c* takes over all observations from *a***

    (i.e. each raven is not black, *c*=:*a*+*c*)

Alternative approach to normalization – „likelihoodist”

- „Likelihoodist” approach to normalization:
  - the evidence confirms the hypothesis, if the evidence is more frequent with the hypothesis rather than with ¬hypothesis, and
  - the evidence disconfirms the hypothesis, if the evidence is more frequent with ¬hypothesis rather than with the hypothesis.

Alternative approach to normalization – „likelihoodist"

- The maximal support of the rule will be obtained when:
  - a takes over all observations from c
    (i.e. each raven is black, $a=:a+c$),
  - $b=:0$ (i.e. there are no black non-ravens),
  - $c=:0$ (i.e. there are no non-black ravens), and
  - d takes over all observations from b
    (i.e. each non-raven is not black, $d=:b+d$).

Alternative approach to normalization – „likelihoodist"

- The minimal support of the rule will be obtained when:
  - $a =: 0$ (i.e. there are no black ravens),
  - b takes over all observations from d

    (i.e. each non-raven is black, $b =: b + d$),
  - c takes over all observations from a

    (i.e. each raven is not black, $c =: a + c$), and
  - $d =: 0$ (i.e. there are no non-black non-ravens).

Alternative approach to normalization – „Bayesian"

- „Bayesian" approach to normalization:
  - the evidence confirms the hypothesis, if the hypothesis is more frequent with the evidence rather than with ¬evidence, and
  - the evidence disconfirms the hypothesis, if ¬hypothesis is more frequent with the evidence rather than with ¬evidence.

Alternative approach to normalization – „Bayesian"

- The maximal support of the rule will be obtained when:
  - *a* takes over all observations from *b*
    (i.e. each black object is a raven, $a=:a+b$),
  - $b=:0$ (i.e. there are no black non-ravens),
  - $c=:0$ (i.e. there are no non-black ravens), and
  - *d* takes over all observations from *c*
    (i.e. each non-black object is not a raven, $d=:c+d$).

Alternative approach to normalization – „Bayesian"

- The minimal support of the rule will be obtained when:
  - $a=0$ (i.e. there no black ravens),
  - b takes over all observations from a

    (i.e. each non-black object is not a raven, $b=a+b$),
  - c takes over all observations from d

    (i.e. each non-black object is a raven, $c=c+d$), and
  - $d=0$ (i.e. there no non-black non-ravens).

Normalized confirmation measures

- **Notation:** ***a*=**$sup(\phi\rightarrow\psi)$ **,** ***b*=**$sup(\neg\phi\rightarrow\psi)$**,** ***c*=**$sup\ (\phi\rightarrow\neg\psi)$**,** ***d*=**$sup(\neg\phi\rightarrow\neg\psi)$
- **Among confirmation measures that were normalized and analysed by Crupi et al. there are:**

$$D(\phi\rightarrow\psi)=\frac{a}{a+c}-\frac{(a+b)}{|U|}$$ **(Carnap 1950/1962)**

$$S(\phi\rightarrow\psi)=\frac{a}{a+c}-\frac{b}{b+d}$$ **(Christensen 1999)**

$$M(\phi\rightarrow\psi)=\frac{a}{a+b}-(a+c)$$ **(Mortimer 1988)**

$$N(\phi\rightarrow\psi)=\frac{a}{a+b}-\frac{c}{c+d}$$ **(Nozick 1981)**

$$C(\phi\rightarrow\psi)=\frac{a-(a+c)(a+b)}{a+b+c+d}$$ **(Carnap 1950/1962)**

$$R(\phi\rightarrow\psi)=\frac{a(a+b+c+d)}{(a+c)(a+b)}-1$$ **(Finch 1960)**

Z-measure

- **It can be observed that:**

$$D_{norm} = S_{norm} = M_{norm} = N_{norm} = C_{norm} = R_{norm} = G_{norm}.$$

- **Crupi et al. have therefore proposed to call them all by one name:**

_Z_-measure.

$$Z(\phi \rightarrow \psi) = \begin{cases} \dfrac{\dfrac{a}{a+c} - \dfrac{(a+b)}{a+b+c+d}}{\dfrac{c+d}{a+b+c+d}} = \dfrac{ad-bc}{(a+c)(c+d)} \\ \text{in case of confirmation} \\ \\ \dfrac{\dfrac{a}{a+c} - \dfrac{(a+b)}{a+b+c+d}}{\dfrac{a+b}{a+b+c+d}} = \dfrac{ad-bc}{(a+c)(a+b)} \\ \text{in case of disconfirmation} \end{cases}$$

Normalized Rule Interest Function

- **The normalized *Rule Interest Function*:**

$$RI_{norm} = \begin{cases} \dfrac{a - \dfrac{(a+b)(a+c)}{a+b+c+d}}{a+c-\dfrac{(a+b+c)(a+c)}{a+b+c+d}} & \text{in case of confirmation} \\[2em] \dfrac{a - \dfrac{(a+b)(a+c)}{a+b+c+d}}{\dfrac{b(a+c)}{a+b+c+d}} & \text{in case of disconfirmation} \end{cases}$$

Normalized gain measure

- **The normalized *gain* measure:**

$$gain_{norm} = \begin{cases} \dfrac{a - \Theta(a+c)}{(a+c) - (1-\Theta)} & \text{in case of confirmation} \\[2ex] \dfrac{a - \Theta(a+c)}{(a+c)\Theta} & \text{in case of disconfirmation} \end{cases}$$

Normalized dependency factor

- **The normalized *dependency factor*:**

$$\eta_{norm} = \begin{cases} \dfrac{\dfrac{a}{a+c} - \dfrac{a+b}{a+b+c+d}}{\dfrac{a}{a+c} + \dfrac{a+b}{a+b+c+d}} \times \dfrac{1 + \dfrac{a+b+c}{a+b+c+d}}{1 - \dfrac{a+b+c}{a+b+c+d}} \\ \textit{in case of confirmation} \\ \\ \dfrac{\dfrac{a}{a+c} - \dfrac{a+b}{a+b+c+d}}{\dfrac{a}{a+c} + \dfrac{a+b}{a+b+c+d}} \\ \textit{in case of disconfirmation} \end{cases}$$

Desired properties
of interestingness measures

Property M

- **Property M of monotonicity** **(Greco, Pawlak, Słowiński 2004)**
- **An interestingness measure $F(a, b, c, d)$ has the property M if it is a function non-decreasing with respect to a and d and non-increasing with respect to b and c**

where:

$a = sup(\phi \to \psi)$ **- the no. of objects in U for which ϕ and ψ hold together**

$b = sup(\neg\phi \to \psi)$,

$c = sup(\phi \to \neg\psi)$,

$d = sup(\neg\phi \to \neg\psi)$

Interpretation of the property M

- **E.g. consider rule $\phi \rightarrow \psi$:**

  if x is a raven then x is black

  - **non-decreasing with respect to *a* –**

    the more *black ravens* we observe,

    the more credible becomes the rule

  - **non-increasing with respect to *b* –**

    the more *black non-ravens* we observe,

    the less credible becomes the rule

  - **non-increasing with respect to *c***

  - **non-decreasing with respect to *d***

Results of the conducted analysis with respect to property M

- ***Theorem:***

  The normalized Rule Interest Function has the property M

- ***Theorem:***

  The normalized Gain measure has the property M

- ***Theorem:***

  The normalized Dependency factor does not have the property M

Results of the conducted analysis

- ***Theorem:*** **The normalized Rule Interest Function has the property M**
- ***Proof outline:***
  prove that the normalized *RI* both in case of confirmation and disconfirmation is:
  - ***non-decreasing*** **with respect to *a*,**
  - ***non-increasing*** **with respect to *b*,**
  - ***non-increasing*** **with respect to *c*,**
  - ***non-decreasing*** **with respect to *d***

Practical application of the results

Support - Anti-support Pareto border

- ***Theorem:***

  For a set of rules with the same conclusion,

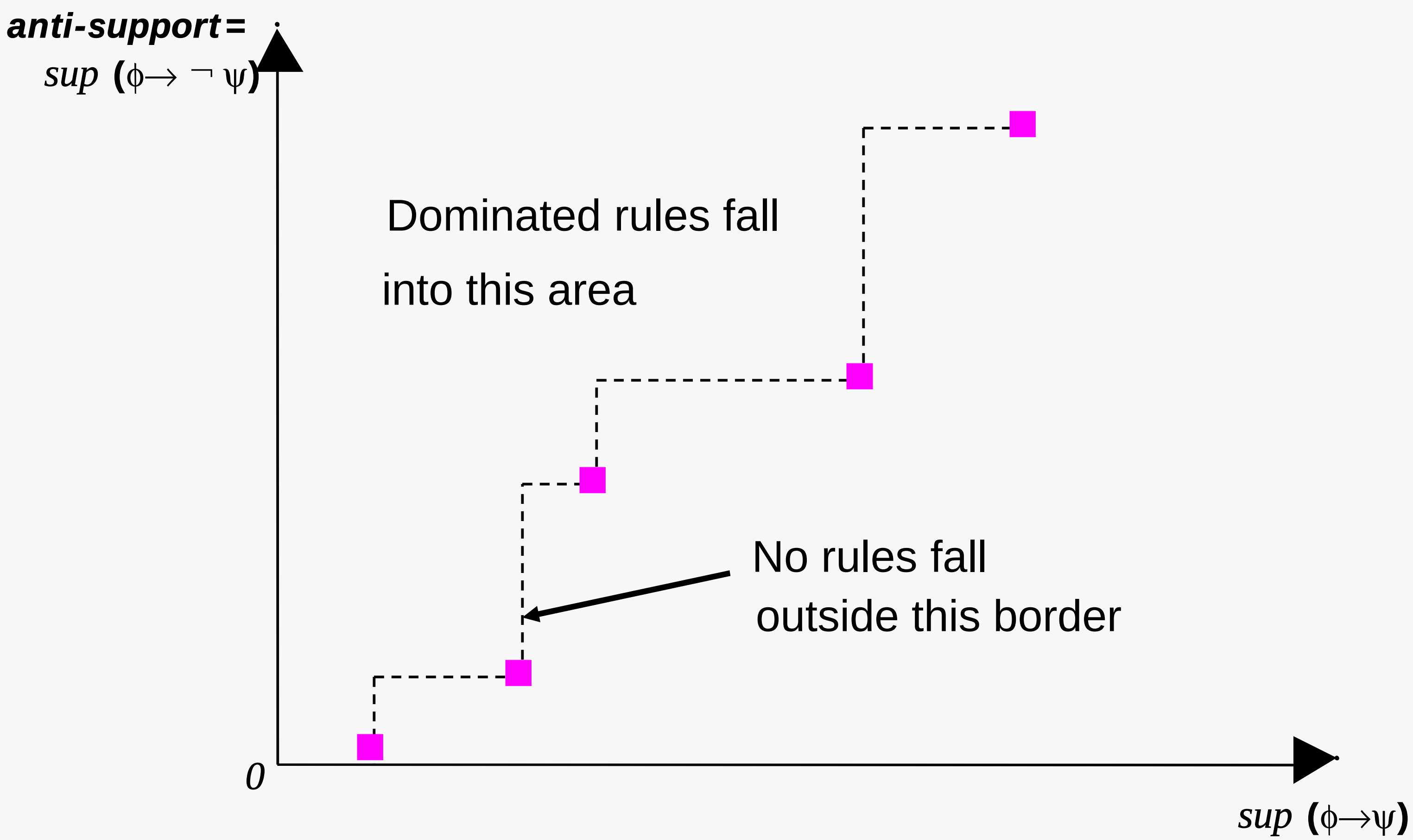
  due to (anti) monotonic dependencies between
  measures of support and anti-support on one hand
  and any interestingness measure with property M on the other hand

  the best rules according to any measure with the property M
  must reside on the support - anti-support Pareto optimal border

- **The support – anti-support Pareto border is a set of non-dominated rules with respect to those measures,**
  i.e. the set of rules for which there is no other rule
  with greater support and smaller anti-support

Brzezińska I., Greco S., Słowiński R.: "Mining Pareto-Optimal Rules with Respect to Support and Confirmation or Support and Anti-Support", Engineering Applications of Artificial Intelligence, vol. 20 no. 5 (2007) pp.587-600

Support - Anti-support Pareto border



The best rules according to any measure with the property M must reside on the support - anti-support Pareto border

Practical application of the results

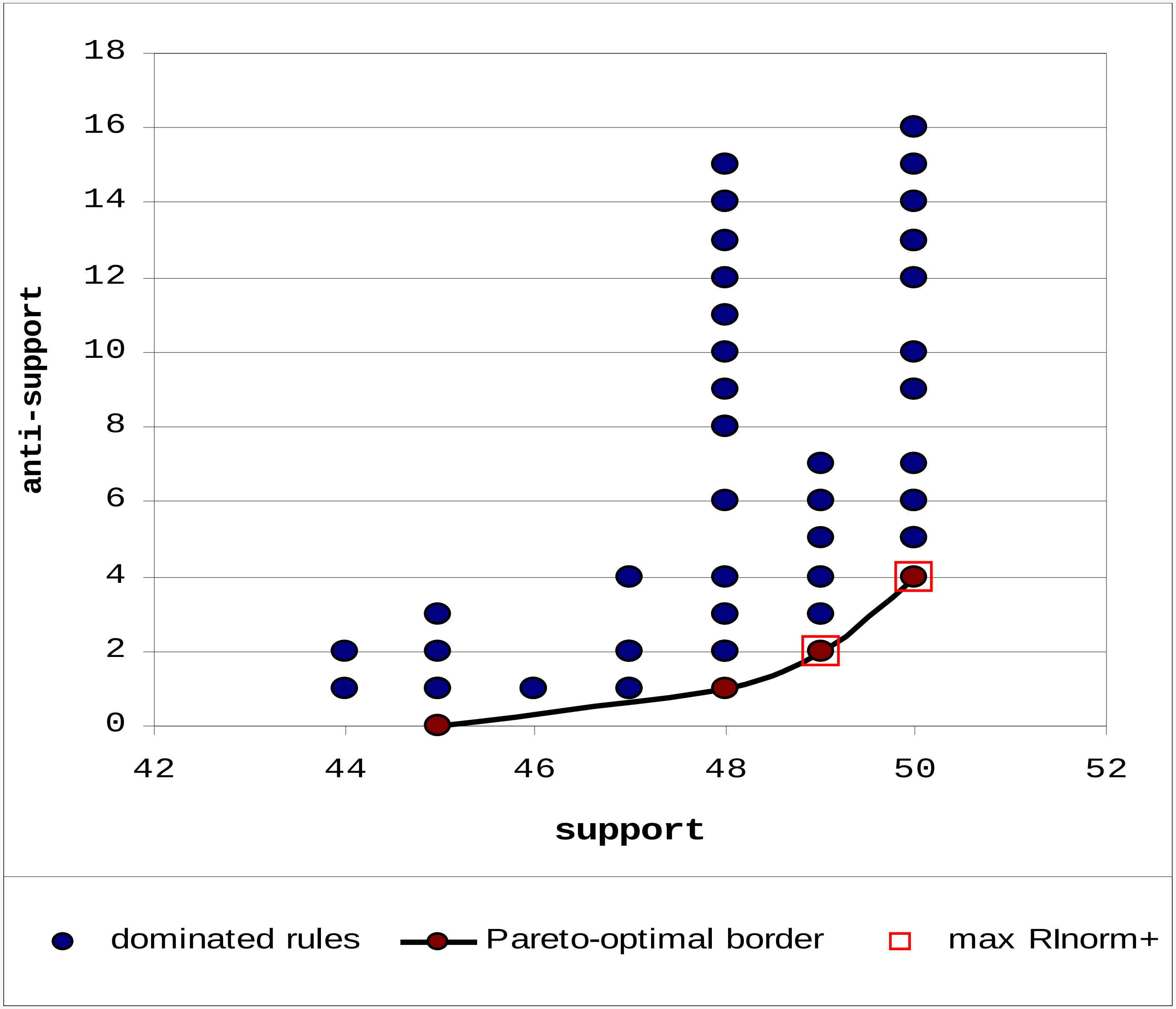
- Since the normalized *RI* and *gain* measures satisfy the property M we can conclude that
  rules optimal with respect to them will be found in the set of rules non-dominated according to support and anti-support.
  (considering rules with the same conclusion)

- Experiments illustrating the result:

  Dataset: *busses*, containing info. about technical state of buses

  Set of 85 rules with the same conclusion

Practical application of the results



Conclusions

Conclusions

Normalized interestingness measure	Property M
Rule Interest Function	YES
Gain	YES
Dependency Factor	NO

Conclusions

- **Properties explain how the measures behave in certain situations and thus, group them helping the user choose the measure relevant for his expectations**

  e.g. we know that the normalized *RI* is monotonically dependent on the number of objects supporting the rule
  or the number of objects supporting neither premise nor conclusion

Conclusions

- **Possession of property M implies potential efficiency improvement:**
  - **we can concentrate on mining only the support–anti-support Pareto set instead of conducting rule evaluation separately wrt to normalized *RI*, gain, or any other measure with property M**
  - **rules optimal wrt to normalized *RI*, gain or any other measure with property M can be mined from the support–anti-support Pareto set instead of searching the set of all rules**
  - **due to relationship between anti-support and any measure with property M, the rule order wrt anti-support (for fixed value of support) is the same for any other measure with M**

Thank you!