# Bayesian Confirmation Measures in Rule-based Classification

**Dariusz Brzeziński**, Zbigniew Grudziński, Izabela Szczęch

*Poznan University of Technology, Poland*

# Outline

- Rule-based classification
- Confirmation measures
- CM-CAR algorithm
- Experiments
- Conclusions

# Classification rules

- One the most popular classification models when working with human experts
- Consequence relation: *if E then H*

$$E \longrightarrow H$$

**Evidence** **Hypothesis**

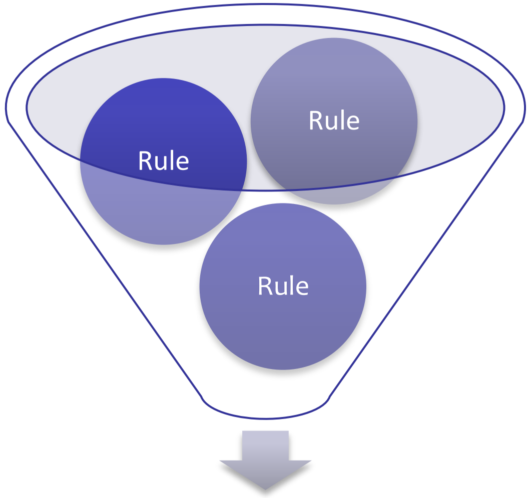
- In classification rules $H$ is a class label
- Ex: talk=*short* and slides=*funny* $\longrightarrow$ audience=*happy*

# Association rules

- Used to find associations rather than predict
- Same $E \longrightarrow H$ relation, but H can be any attribute
- Usually (too) many rules are found

**Common task**:

filter out only the most interesting rules



# Interestingness measures

| Height | Hair | Eyes | Nationality |
|---|---|---|---|
| tall | blond | blue | Swede |
| medium | dark | hazel | German |
| medium | blond | blue | Swede |
| tall | blond | blue | German |
| short | red | blue | German |
| medium | dark | hazel | Swede |

| | |
|---|---|
| ¬E | ¬H |
| ¬E | H |
| ¬E | ¬H |
| ¬E | H |
| E | H |
| ¬E | ¬H |

| | H | ¬ H |
|---|---|---|
| E | 1 | 0 |
| ¬ E | 2 | 3 |

*if (Hair = red) & (Eyes = blue) then (Nationality = German)*

*if Evidence then Hypothesis*

The contingency table is a form used to calculate the value of interestingness measures

$$sup(E \longrightarrow H) = a$$

$$conf(E \longrightarrow H) = \frac{a}{a+c}$$

| | $H$ | $\neg H$ | |
|---|---|---|---|
| $E$ | $a$ | $c$ | $a+c$ |
| $\neg E$ | $b$ | $d$ | $b+d$ |
| | $a+b$ | $c+d$ | $n$ |

# Confirmation measures

- Measures that satisfy

$$c(H,E)\begin{cases} >0 & \text{if } P(H|E) > P(H), & \textbf{Good} \\ =0 & \text{if } P(H|E) = P(H), & \textbf{Neutral} \\ <0 & \text{if } P(H|E) < P(H). & \textbf{Bad} \end{cases}$$

- Confirmation measures say what is a "value of information" that *E* <u>adds</u> to the credibility of *H*
- **Intuition**: evidence should support the hypothesis

Ex: talk=*long* and slides=*boring* ⟶ audience=*happy*

*confidence > 0, confirmation definitely < 0 …*

# Confirmation measures

$$D(H,E) = P(H|E) - P(H) = \frac{a}{a+c} - \frac{a+b}{n} = \frac{ad-bc}{n(a+c)}$$

$$M(H,E) = P(E|H) - P(E) = \frac{a}{a+b} - \frac{a+c}{n} = \frac{ad-bc}{n(a+b)}$$

$$S(H,E) = P(H|E) - P(H|\neg E) = \frac{a}{a+c} - \frac{b}{b+d} = \frac{ad-bc}{(a+c)(b+d)}$$

$$N(H,E) = P(E|H) - P(E|\neg H) = \frac{a}{a+b} - \frac{c}{c+d} = \frac{ad-bc}{(a+b)(c+d)}$$

$$C(H,E) = P(E \wedge H) - P(E)P(H) = \frac{a}{n} - \frac{(a+c)(a+b)}{n^2} = \frac{ad-bc}{n^2}$$

$$F(H,E) = \frac{P(E|H) - P(E|\neg H)}{P(E|H) + P(E|\neg H)} = \frac{\frac{a}{a+b} - \frac{c}{c+d}}{\frac{a}{a+b} + \frac{c}{c+d}} = \frac{ad-bc}{ad+bc+2ac}$$

$$Z(H,E) = \begin{cases} 1 - \dfrac{P(\neg H|E)}{P(\neg H)} = \dfrac{ad-bc}{(a+c)(c+d)} & \text{in case of confirmation} \\ \dfrac{P(H|E)}{P(H)} - 1 = \dfrac{ad-bc}{(a+c)(a+b)} & \text{in case of disconfirmation} \end{cases}$$

$$A(H,E) = \begin{cases} \dfrac{P(E|H) - P(E)}{1 - P(E)} = \dfrac{ad-bc}{(a+b)(b+d)} & \text{in case of confirmation} \\ \dfrac{P(H) - P(H|\neg E)}{1 - P(H)} = \dfrac{ad-bc}{(b+d)(c+d)} & \text{in case of disconfirmation} \end{cases}$$

$$c_1(H,E) = \begin{cases} \alpha + \beta A(H,E) & \text{in case of confirmation when } c = 0 \\ \alpha Z(H,E) & \text{in case of confirmation when } c > 0 \\ \alpha Z(H,E) & \text{in case of disconfirmation when } a > 0 \\ -\alpha + \beta A(H,E) & \text{in case of disconfirmation when } a = 0 \end{cases}$$

$$c_2(H,E) = \begin{cases} \alpha + \beta Z(H,E) & \text{in case of confirmation when } b = 0 \\ \alpha A(H,E) & \text{in case of confirmation when } b > 0 \\ \alpha A(H,E) & \text{in case of disconfirmation when } d > 0 \\ -\alpha + \beta Z(H,E) & \text{in case of disconfirmation when } d = 0 \end{cases}$$

$$c_3(H,E) = \begin{cases} A(H,E)Z(H,E) & \text{in case of confirmation} \\ -A(H,E)Z(H,E) & \text{in case of disconfirmation} \end{cases}$$

$$c_4(H,E) = \begin{cases} \min(A(H,E), Z(H,E)) & \text{in case of confirmation} \\ \max(A(H,E), Z(H,E)) & \text{in case of disconfirmation} \end{cases}$$

| | $H$ | $\neg H$ | |
|---|---|---|---|
| $E$ | $a$ | $c$ | $a+c$ |
| $\neg E$ | $b$ | $d$ | $b+d$ |
| | $a+b$ | $c+d$ | $n$ |

The values of the presented measures range from -1 to 1

# Research questions

1. Can confirmation measures be applied to predictive classification problems?

2. How to discover  and prune decision rules with high confirmation?

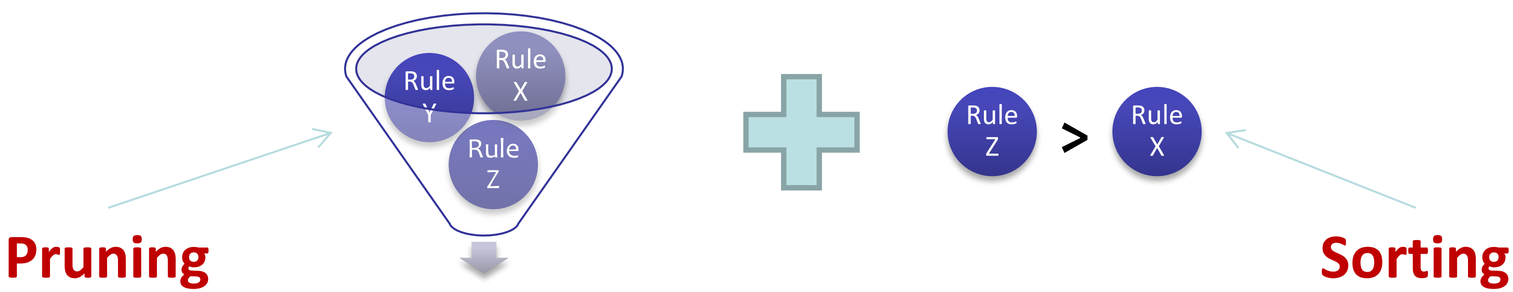
3. Which confirmation measures are best suited for classification?

# CM-CAR

- Algorithm for creating classification association rules
- Generalization of CBA algorithm
- Tries to create predictive and descriptive rule lists

## Main idea

**Use two seperate sets of (confirmation) measures to select and sort classification association rules**

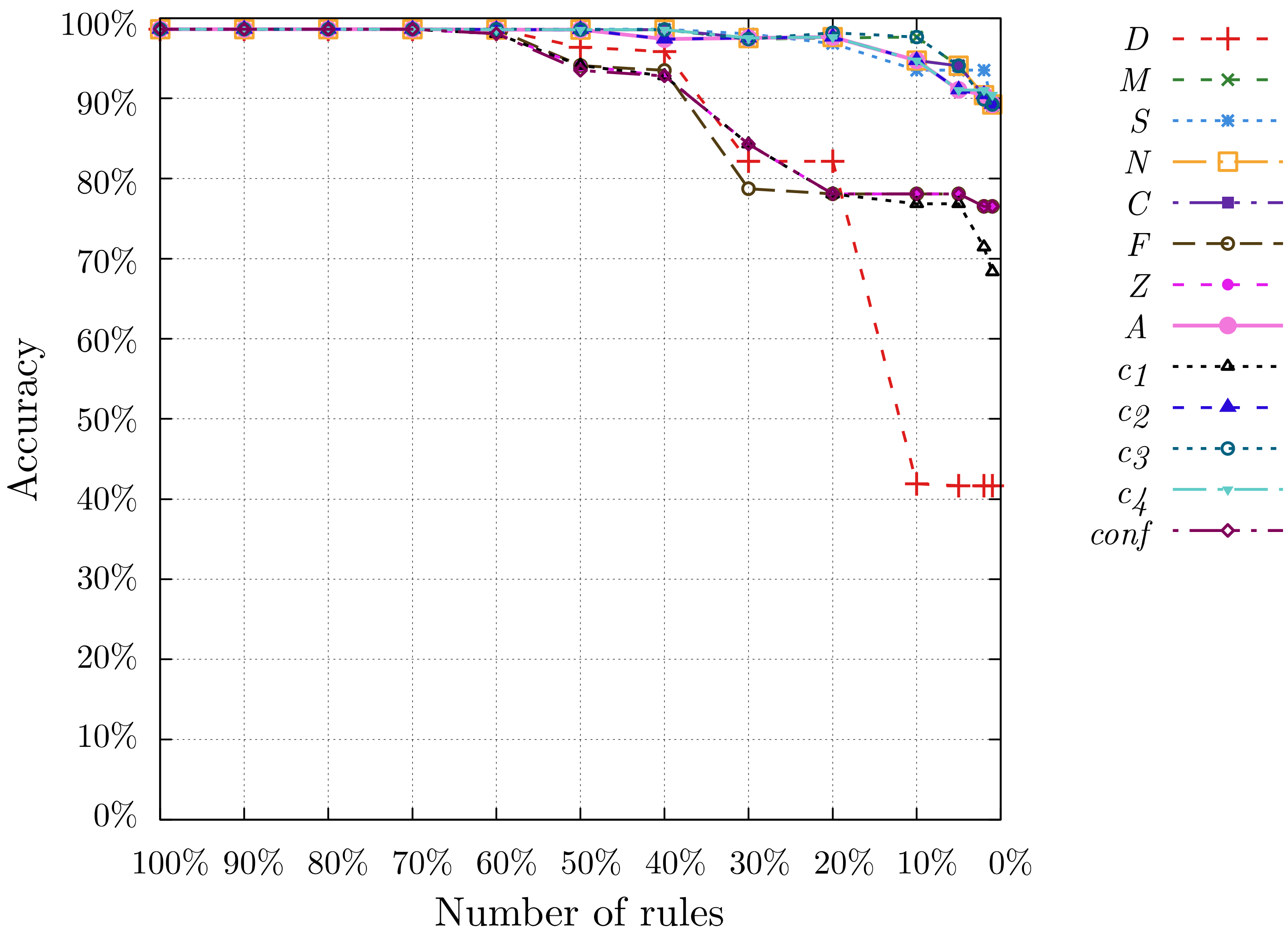


# Experimental setup

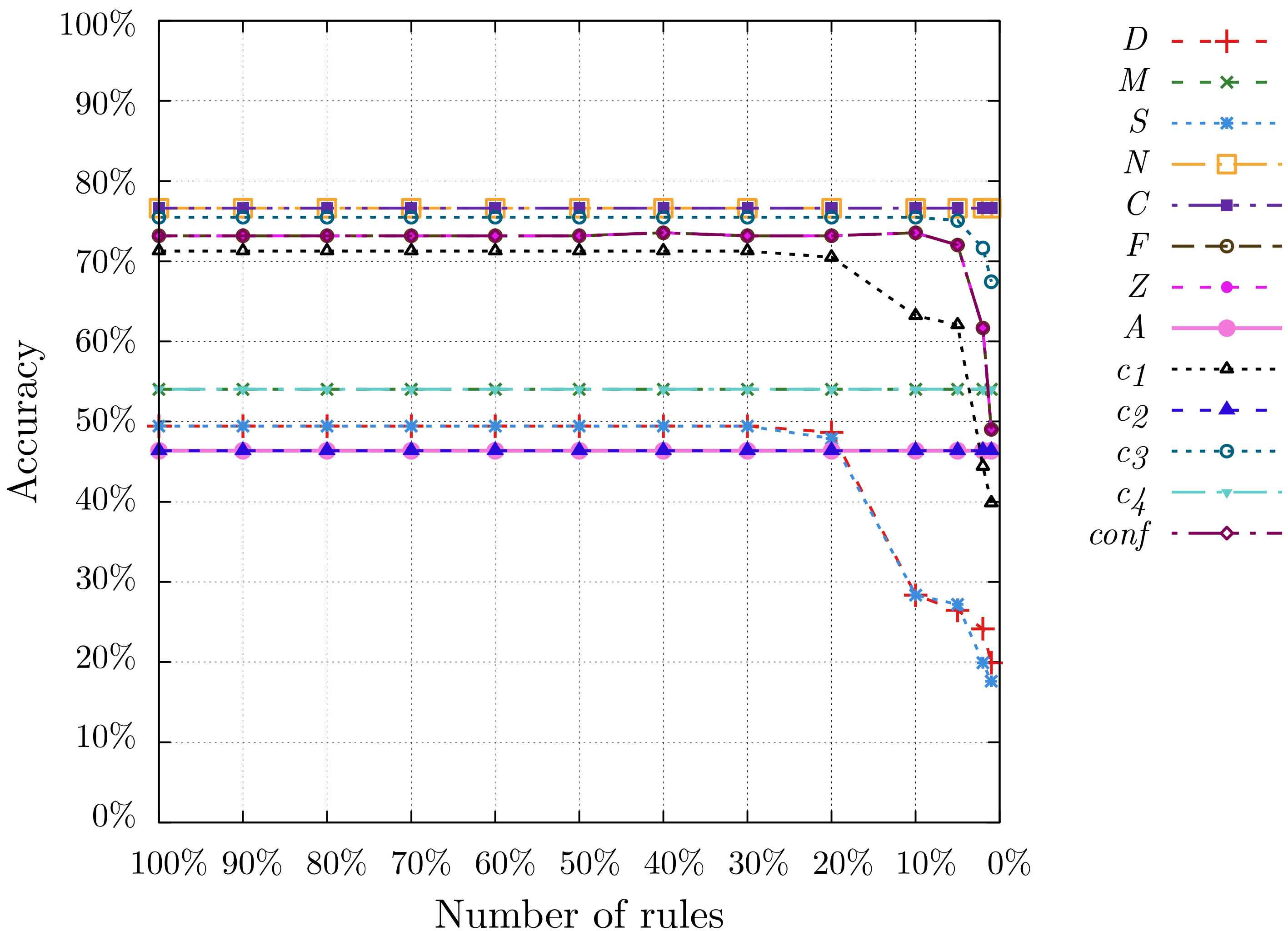
- 12 confirmation measures
- 20 datasets: 10 balanced, 10 imbalanced
- ~10,000 rules generated per dataset
- 1%-100% rules left after pruning
- Comparison of accuracy, AUC, F1-score, and G-mean

- CM-CAR:
  - Confirmation measure used only for rule list pruning
  - Confirmation measure used for rule sorting and pruning

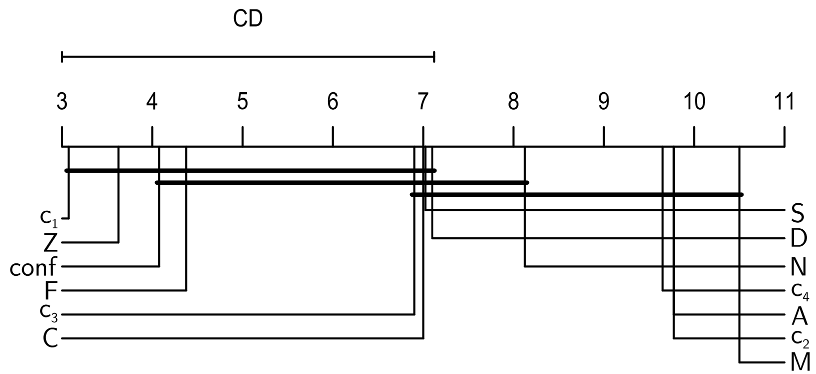
# Results – rule sorting (mushroom)



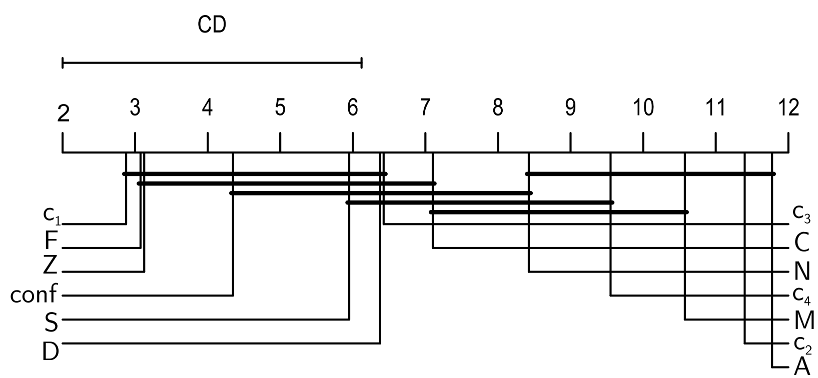
# Results – rule pruning (diabetes)



# Results (Accuracy)




Critical distance plot for **rule pruning**




Critical distance plot for **rule sorting**

# Results summary

- Confirmation measures influenced the predictive performance of decision rule lists
- Slightly different results for rule sorting and pruning
- To achieve good performance on imbalanced data *coverage* should be additionally controlled
- $F$, $Z$, $c_1$, $S$ performed better/comparable to the baseline

Full results for accuracy, AUC, F1-score, and G-mean:
http://www.cs.put.poznan.pl/dbrzezinski/software/CMCAR.html

# Conclusions

- **CM-CAR**: algorithm for sorting and pruning rule lists based on any interestingness measure
- The 12 analyzed **measures differed** in terms of resulting classifier performance
- Measures ***F*, *Z*, $c_1$, *S* comparable or better than *conf*** in terms of rule sorting and pruning
- **Future work**: algorithms using confirmation measures during **rule generation**

# Thank you!

# Property of monotonicity M

- Desirable property of $c(H,E) = f(a,b,c,d)$ : monotonicity (M)*

> $f$ should be non-decreasing with respect to $a$ and $d$
> and non-increasing with respect to $b$ and $c$

- Interpretation of (M): ($E \rightarrow H \equiv$ *if x is a raven, then x is black*)

  a) the more black ravens we observe, the more credible becomes $E \rightarrow H$

  b) the more black non-ravens we observe, the less credible becomes $E \rightarrow H$

  c) the more non-black ravens we observe, the less credible becomes $E \rightarrow H$

  d) the more non-black non-ravens we observe, the more credible becomes $E \rightarrow H$

*S.Greco, Z.Pawlak, R.Słowiński: Can Bayesian confirmation measures be useful for rough set decision rules? *Engineering Applications of Artificial Intelligence*, 17 (2004) no.4, 345-361

# Property of maximality/minimality

- Desirable property of $c(H,E)$: maximality/minimality*

$c(H,E)$ is maximal if and only if $P(E,\neg H) = P(\neg E,H) = 0$ and

$c(H,E)$ is minimal if and only if $P(E,H) = P(\neg E, \neg H) = 0$.

- Interpretation of maximality/minimality:

  a measure obtains its maximum iff $c=b=0$ and its minimum iff $a=d=0$.

*Glass, D.H.: Confirmation measures of association rule interestingness, Knowledge-Based Systems 44, (2013) 65–77

# Property of hypothesis symmetry HS

- Desirable property of $c(H,E)$: hypothesis symmetry (HS)*

$$\boxed{c(H,E) = -c(\neg H,E)}$$

- Interpretation of (HS): ($E \rightarrow H \equiv$ *if x is a square, then x is rectangle*)

  the strength with which

  the premise (*x is a square*) confirms the conclusion (*x is rectangle*)

  is the same as the strength with which

  the premise disconfirms the negated conclsuion (*x is not a rectangle*).

*Carnap, R.: Logical Foundations of Probability, second ed. University of Chicago Press, Chicago (1962)
Eells, E., Fitelson, B.: Symmetries and asymmetries in evidential support. Philosophical Studies, 107 (2) (2002), 129-142