The Property of χ^2_{01}-Concordance for Bayesian Confirmation Measures

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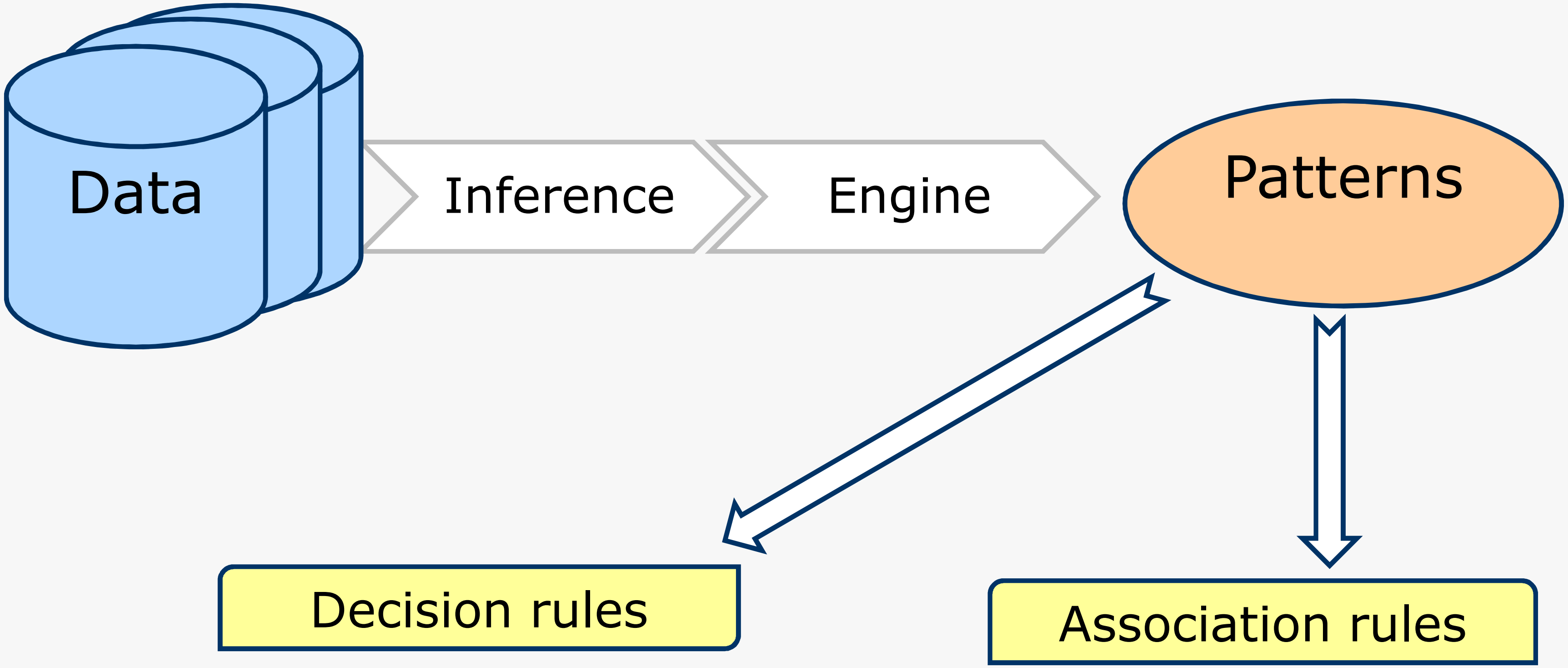
Presentation plan

- Rule induction and interestingness measures
- Property of confirmation and popular confirmation measures
- Using confirmation measures in error-prone situations
- Property of concordance
- Experimental setup
- Experimental results and risk related interpretations
- Conclusions

Rule induction



If symptom s_1 is present
and symptoms s_2
and s_3 are absent
then disease d_1

If symptom s_1 is present
then symptoms s_2
and s_3 are absent

Rule induction

- Patterns in form of rules are induced from a data table
- $S=\langle U, A\rangle$ – *data table*, where U and A are finite, non-empty sets
  U – *universe of objects*; A – set of *attributes*
- $S=\langle U, C, D\rangle$ – *decision table*, where C – set of *condition attributes*,
  D – set of *decision attributes*, $C \cap D = \emptyset$
- *Rule* induced from S is a *consequence relation*:
  E* → *H *read as* ***if E then H***
  where

  E is condition (evidence or premise) and
  H is conclusion (hypothesis or decision)
  formula built from attribute-value pairs (q,v)

Rule induction

Characterization of nationalities

U	*Height*	*Hair*	*Eyes*	*Nationality*	Support
1	*tall*	*blond*	*blue*	*Swede*	270
2	*medium*	*dark*	*hazel*	*German*	90
3	*medium*	*blond*	*blue*	*Swede*	90
4	*tall*	*blond*	*blue*	*German*	360
5	*short*	*red*	*blue*	*German*	45
6	*medium*	*dark*	*hazel*	*Swede*	45

C *D*

- E.g. decision rules induced from „characterization of nationalities”:

1) If (*Height=tall*) then (*Nationality=Swede*)

2) If (*Height=medium*) & (*Hair=dark*) then (*Nationality=German*)

If *Evidence* then *Hypothesis*

Interestingness measures

The number of rules
induced from datasets is usually quite large

- overwhelming for human comprehension,
- many rules are irrelevant or obvious (low practical value)

rule evaluation – interestingness (attractiveness) measures
(e.g. support, confidence, gain, rule interest, lift, measures of Bayesian confirmation)

- each measure was proposed to capture different characteristics of rules
- the number of proposed measures is very large

In this work we focus on a group of **measures of confirmation**

Notation

- Used notation corresponding to a 2x2 contingency table of rule's premise and conclusion

  $a=sup(H,E)$ is the number of objects in U satisfying both the premise E and the conclusion H of a rule $\boldsymbol{E \rightarrow H}$**,**

  $b=sup(H, \neg E)$,

  $c=sup(\neg H, E)$,

  $d=sup(\neg H, \neg E)$,

  $a+c=sup(E)$,

  $a+b=sup(H)$,…

	H	$\neg H$	Σ
E	a	c	$a+c$
$\neg E$	b	d	$b+d$
Σ	$a+b$	$c+d$	$a+b+c+d=n$

- a, b, c and d can also be regarded as frequencies that can be used to estimate probabilities:
  e.g., $P(E)=(a+c)/n$, $P(H)=(a+b)/n$, $P(H|E) = a/(a+c)$.

Property of confirmation

- An attractiveness measure *c*(*H*,*E*), has the property of confirmation (i.e. is a confirmation measure) if is satisfies the following condition:

$$c(H,E)\begin{cases} >0 & \text{if } P(H|E)>P(H) \\ =0 & \text{if } P(H|E)=P(H) \\ <0 & \text{if } P(H|E)<P(H) \end{cases} \Rightarrow c(H,E)\begin{cases} >0 & \textit{if } \dfrac{a}{a+c}>\dfrac{a+b}{n} \\ =0 & \textit{if } \dfrac{a}{a+c}=\dfrac{a+b}{n} \\ <0 & \textit{if } \dfrac{a}{a+c}<\dfrac{a+b}{n} \end{cases}$$

- Measures of confirmation quantify the strength of confirmation that premise *E* gives to conclusion *H*
- „*H* is verified more often, when *E* is verified, rather than when *E* is not verified"

Property of confirmation

$$
c(H, E)\begin{cases} > 0 & if \ \dfrac{a}{a+c} > \dfrac{a+b}{n} \\ = 0 & if \ \dfrac{a}{a+c} = \dfrac{a+b}{n} \\ < 0 & if \ \dfrac{a}{a+c} < \dfrac{a+b}{n} \end{cases}
$$

- The condition does not put any constraint on the value to be assigned to confirmatory arguments (as long as they are positive) or disconfirmatory arguments (as long as they are negative)
- There are many alternative, non-equivalent measures of confirmation

Popular confirmation measures

There are many alternative, non-equivalent measures of confirmation

$$D(H,E)=\frac{a}{a+c}-\frac{a+b}{|U|}$$ (Carnap 1950/1962)

$$S(H,E)=\frac{a}{a+c}-\frac{b}{b+d}$$ (Christensen 1999)

$$M(H,E)=\frac{a}{a+b}-\frac{(a+c)}{|U|}$$ (Mortimer 1988)

$$N(H,E)=\frac{a}{a+b}-\frac{c}{c+d}$$ (Nozick 1981)

$$C(H,E)=\frac{a}{|U|}-\frac{(a+c)(a+b)}{|U|^2}$$ (Carnap 1950/1962)

$$R(H,E)=\frac{a|U|}{(a+c)(a+b)}-1$$ (Finch 1960)

$$G(H,E)=1-\frac{c|U|}{(a+c)(c+d)}$$ (Rips 2001)

$$F(H,E)=\frac{ad-bc}{ad+bc+2ac}$$ (Kemeny and Oppenheim 1952)

Popular confirmation measures

$$Z(H,E)=\begin{cases}\dfrac{ad-bc}{(a+c)(c+d)} & \textit{in case of confirmation}\\[2ex] \dfrac{ad-bc}{(a+c)(a+b)} & \textit{in case of disconfirmation}\end{cases}$$

(Crupi, Tentori, Gonzalez 2007)

$$A(H,E)=\begin{cases}\dfrac{ad-bc}{(a+b)(b+d)} & \textit{in case of confirmation}\\[2ex] \dfrac{ad-bc}{(b+d)(c+d)} & \textit{in case of disconfirmation}\end{cases}$$

(Greco, Słowiński, Szczęch 2012)

Derived confirmation measures

$$c_1(H,E)=\begin{cases}\alpha+\beta A(H,E)\ \textit{in case of confirmation when } c=0\\ \alpha Z(H,E)\ \textit{in case of confirmation when } c>0\\ \alpha Z(H,E)\ \textit{in case of disconfirmation when } a>0\\ -\alpha+\beta A(H,E)\ \textit{in case of disconfirmation when } a=0\end{cases}$$

$$c_2(H,E)=\begin{cases}\alpha+\beta Z(H,E)\ \textit{in case of confirmation when } b=0\\ \alpha A(H,E)\ \textit{in case of confirmation when } b>0\\ \alpha A(H,E)\ \textit{in case of disconfirmation when } d>0\\ -\alpha+\beta Z(H,E)\ \textit{in case of disconfirmation when } d=0\end{cases}$$

$$c_3(H,E)=\begin{cases}A(H,E)Z(H,E)\ \textit{in case of confirmation}\\ -A(H,E)Z(H,E)\ \textit{in case of disconfirmation}\end{cases}$$

$$c_4(H,E)=\begin{cases}\min(A(H,E),Z(H,E))\ \textit{in case of confirmation}\\ \max(A(H,E),Z(H,E))\ \textit{in case of disconfirmation}\end{cases}$$

Properties of confirmation measures

The choice of a confirmation measure for a certain application is a difficult problem

- there is no evidence which measure(s) is the best
- the users' expectations vary,
- the number of proposed measures is overwhelming

properties of confirmation measures, which reflect users' expectations towards the behaviour of measures in particular situations

- property of monotonicity M (Greco, Pawlak & Słowiński 2004)
- Ex_1 property and its generalization to weak Ex_1
- property of logicality L and its generalization to weak L (Fitelson 2006; Crupi, Tentori & Gonzalez 2007 Greco, Słowiński & Szczęch 2012)
- …

need to analyze measures with respect to their properties

Motivation for this work: **Do confirmation measures reflect the statistically significant dependencies in data (between *E* and *H*)?**

Statistically significant dependency between *E* and *H*

Height	**Hair**	**Eyes**	**Nationality**
tall	*blond*	*blue*	*Swede*
medium	*dark*	*hazel*	*German*
medium	*blond*	*blue*	*Swede*
tall	*blond*	*blue*	*German*
short	*red*	*blue*	*German*
medium	*dark*	*hazel*	*Swede*

VE	**VH**
$\neg E$	$\neg H$
$\neg E$	H
$\neg E$	$\neg H$
$\neg E$	H
E	H
$\neg E$	$\neg H$

	H	$\neg H$
E	a	c
$\neg E$	b	d

$a = sup(E,H)$
$b = sup(\neg E,H)$
$c = sup(E,\neg H)$
$d = sup(\neg E,\neg H)$

if (Hair = red) & (Eyes = blue) then (Nationality = German)

if Evidence then Hypothesis

if (VE = E) then (VH = H)

- The contingency table constitutes a form of information about *VE* and *VH* that is to be used in inferring whether these variables are independent or not
- The contingency table is also the form used to calculate the value of confirmation measures

Statistically significant dependency between *E* and *H*

- In real-life situations the existence of possible measurement errors (finally reflected in contingency tables) must be taken into account

- Thus, we should look for a statistically significant dependency between *E* and *H*

- This may be quantified and measured with e.g. two dimensional χ^2 test, often used to test for the independence of two discrete-valued variables

Testing for independency of *E* and *H* - χ^2_{01} coefficient

- For 2 x 2-sized contingency tables, as used in defining confirmation measures, a coefficient χ^2_0 is defined:

	H	¬ *H*
E	*a*	*c*
¬ *E*	*b*	*d*

$$\chi_0^2 = \frac{n(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

- This coefficient is approximately χ^2-distributed and ranges from 0 to *n*. To make it *n*-independent, it is scaled down (divided) by *n*, producing a value belonging to the interval [0, 1] and denoted as χ^2_{01}

Using confirmation measures in error-prone situations

- In practice, two potentially unfavourable situations can concern the confirmation measure applied to a contingency table created from error-prone data:
  - the value of $c(H,E)$ indicates either strong confirmation or strong disconfirmation, while there is only a **weak dependency between *E* and *H***
  - the value of $c(H,E)$ indicates either weak confirmation or weak disconfirmation, while there is a strong dependency E and H

Property of concordance

- To counteract those situations, there arises a need to evaluate the concordance between confirmation measures and statistical significance of the evidence-hypothesis dependency

- For such an evaluation to be useful, it should provide continuous measurements, the higher the more the measure $c(H,E)$ 'agrees' with the level of dependency between the evidence and the hypothesis

- This evaluation may be performed using different statistical tools.
  In this study we use linear Pearson correlation r
  between $|c(H,E)|$ and χ^2_{01}

The experimental dataset

- Given $n > 0$ (the total number of observations), a synthetic dataset is generated as the set of all possible contingency tables satisfying $a + b + c + d = n$
- The set is thus exhaustive and non-redundant (i.e. it contains exactly one copy of each contingency table satisfying the above condition)

Property of concordance – example for n=8

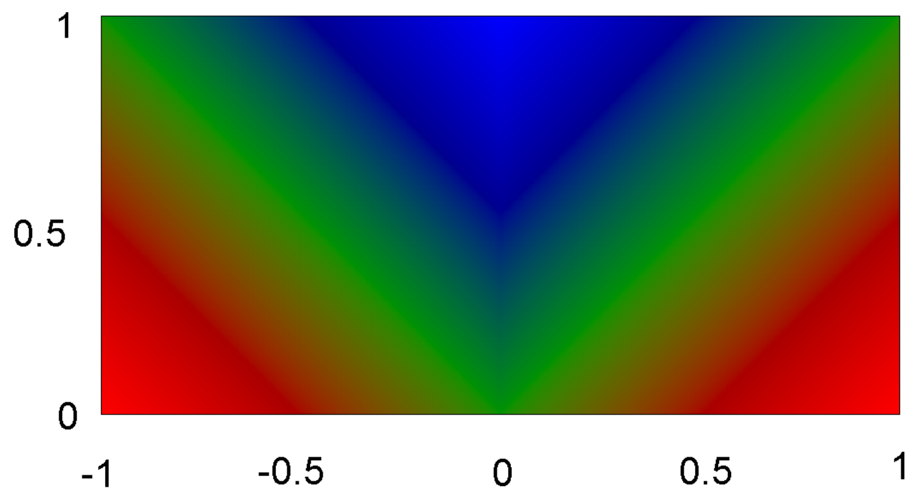
	H	$\neg H$
E	a	c
$\neg E$	b	d

$r(|c(H,E)|, \chi^2_{01})$

a	**b**	**C**	**d**	**c(H,E)**	χ^2_{01}
0	0	0	8	0.00	0.01
0	0	1	7	-0.13	0.13
0	0	2	6	-0.25	0.15
0	0	3	5	0.00	0.00
0	0	4	4	-0.13	0.43
0	0	5	3	0.00	0.00
0	0	6	2	0.88	0.80
0	0	7	1	0.75	0.91
0	0	8	0	0.63	0.63
0	1	0	7	0.50	0.17
0	1	1	6	0.38	0.72
0	1	2	5	0.25	0.25
0	1	3	4	0.13	0.19
…	…	…	…	…	…
8	0	0	0		

Property of concordance

- The relation between χ^2_{01} coefficient and a given confirmation measure *c*(*H*,*E*) may be additionally visualized with a scatter-plot of *c*(*H*,*E*) (horizontal axis [-1;1]) against χ^2_{01} (vertical axis [0;1])
- Given a measure *c*(*H*,*E*), the points of the *c*(*H*,*E*) versus- χ^2_{01} scatter-plot should possibly occupy the green regions of the figure, while possibly avoiding any of the red or blue ones



- Risk related interpretations: red region – risk-prone, blue region – risk-averse

The experimental set-up

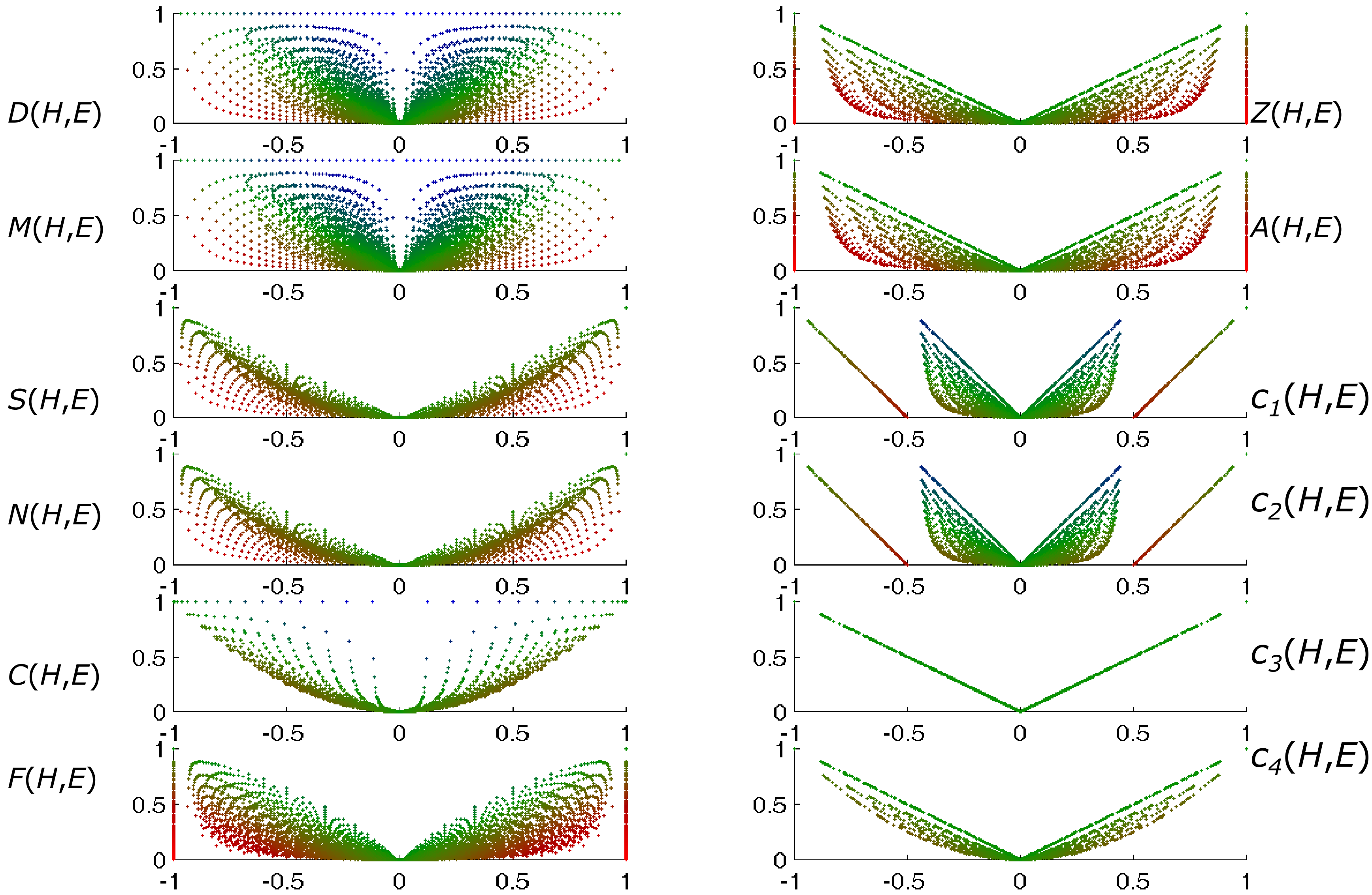
- After having set the total number of observations n to 128, the following operations were performed:
  - the exhaustive and non-redundant set of contingency tables satisfying $a + b + c + d = n$ was generated (there are 366 145 such tables)
  - the values of the 12 selected confirmation measures for all the generated tables were calculated (with $c_1(H,E)$ and $c_2(H,E)$ for $\alpha = \beta = 0.5$)
  - the values of the χ^2_{01} coefficient for all the generated tables were computed
  - the correlations between the absolute values of each of 12 selected confirmation measures and the χ^2_{01} coefficient (i.e. concordances) were established
  - scatter-plots of the 12 selected confirmation measures against χ^2_{01} and triple-region histograms were drawn

The experimental results

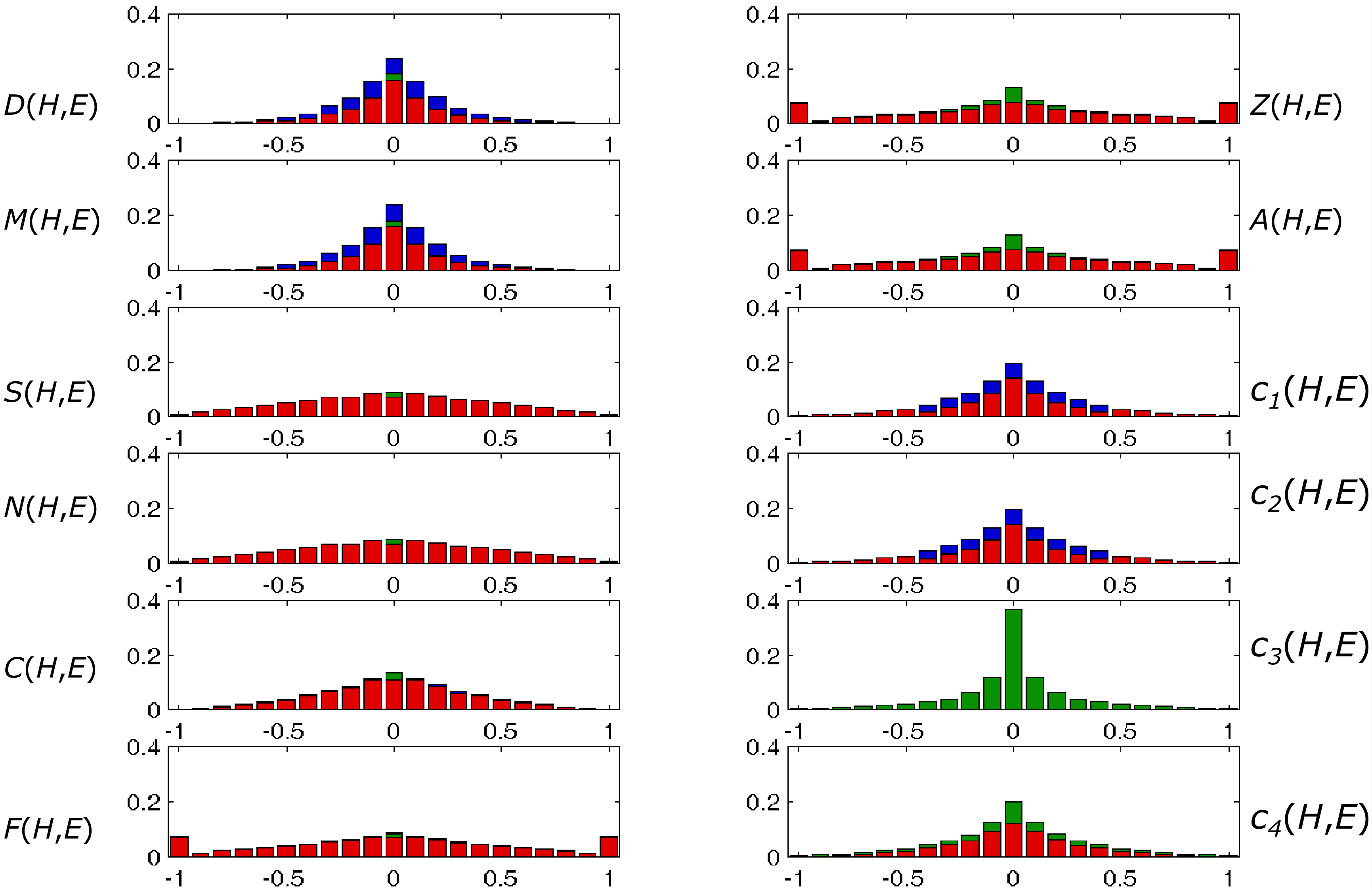
- Some interesting results:
  - measure $c_3(H,E)$ enjoys an ideal χ^2_{01} -concordance, which is due to the fact that | $c_3(H,E)$ | = χ^2_{01}
  - the concordances of the other measures range from 0.957 ($c_4(H,E)$) down to 0.694 ($Z(H,E)$ and $A(H,E)$), in result of which all of the 12 selected measures can be referred to as approximately concordant
  - the less concordant measures should thus be used with some care, especially when applied to real-life, error-prone data, as the may express either strong confirmation or strong disconfirmation in statistically insignificant situations

***c*(*H*,*E*)**	***r***
$D(H,E)$	0.713
$M(H,E)$	0.713
$S(H,E)$	0.912
$N(H,E)$	0.912
$C(H,E)$	0.908
$F(H,E)$	0.711
$Z(H,E)$	0.694
$A(H,E)$	0.694
$c_1(H,E)$	0.697
$c_2(H,E)$	0.697
$c_3(H,E)$	1.000
$c_4(H,E)$	0.957

Scatter-plots of the 12 selected measures against χ^2_{01}

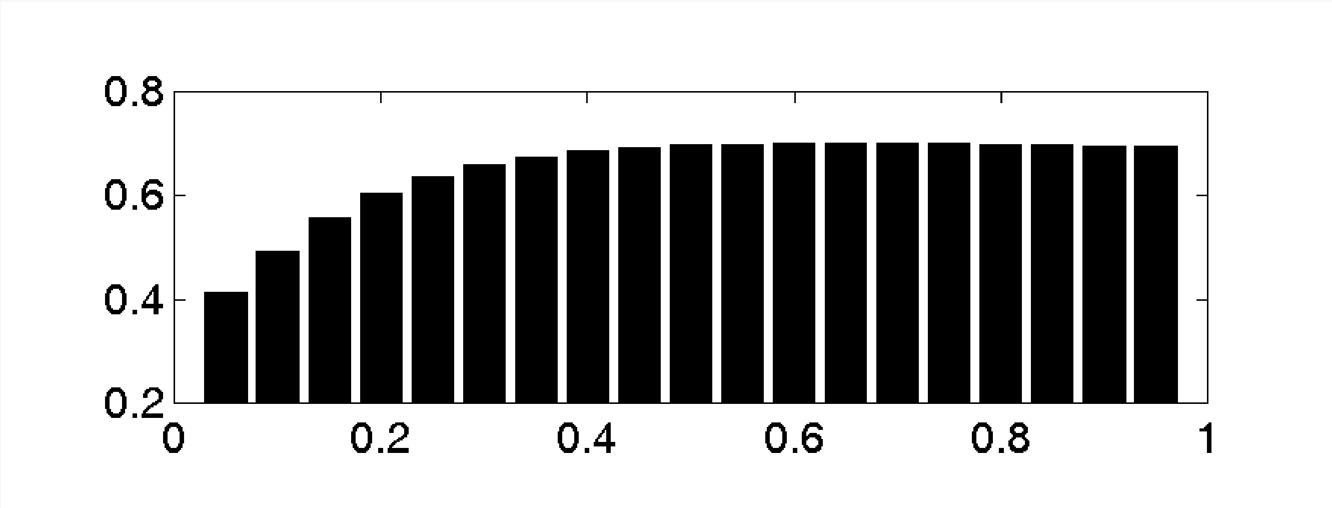


Triple-region histograms of the 12 selected measures



Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)

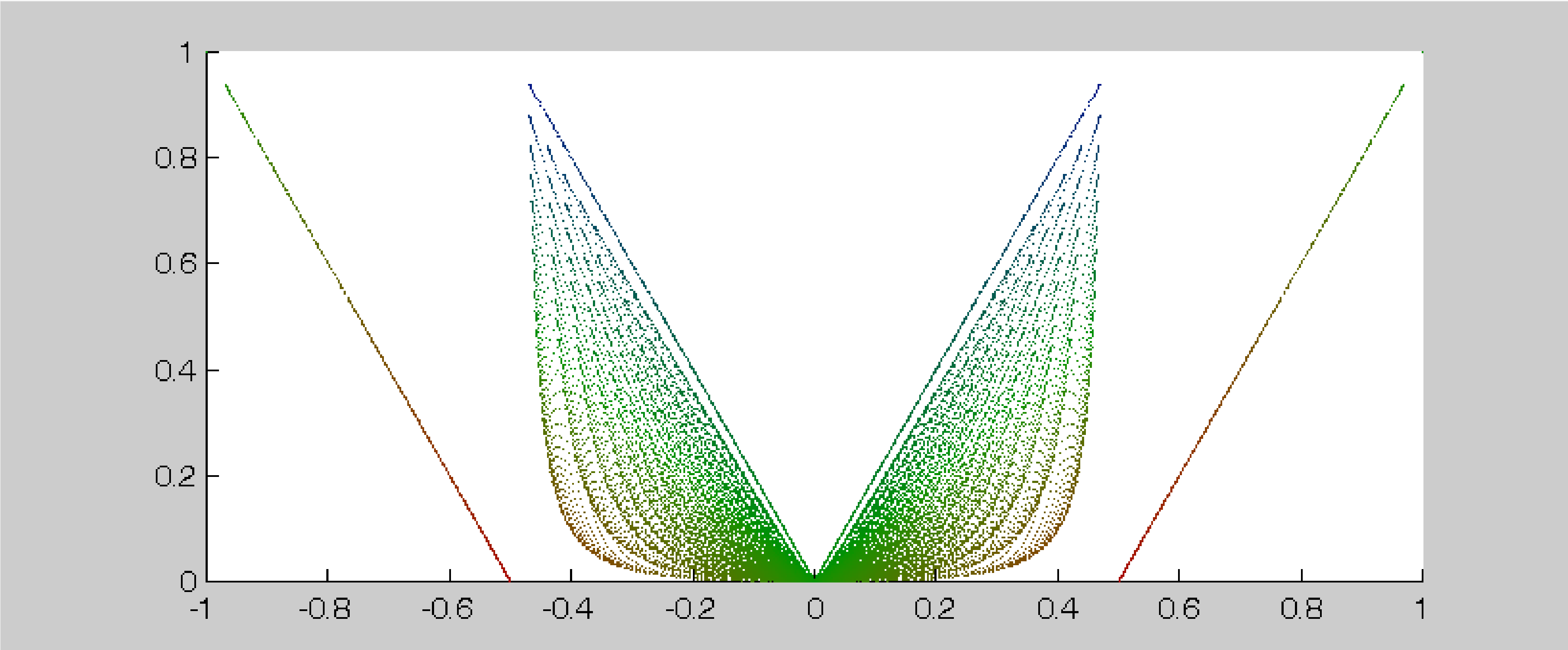
- Measures $c_1(H,E)$ and $c_2(H,E)$ depend on α parameter, i.e. the free parameter that is used to define them (β = 1 - α)
- They manifest varying shapes of their corresponding scatter-plots
- α influences their χ^2_{01}-concordance



α - horizontal axis

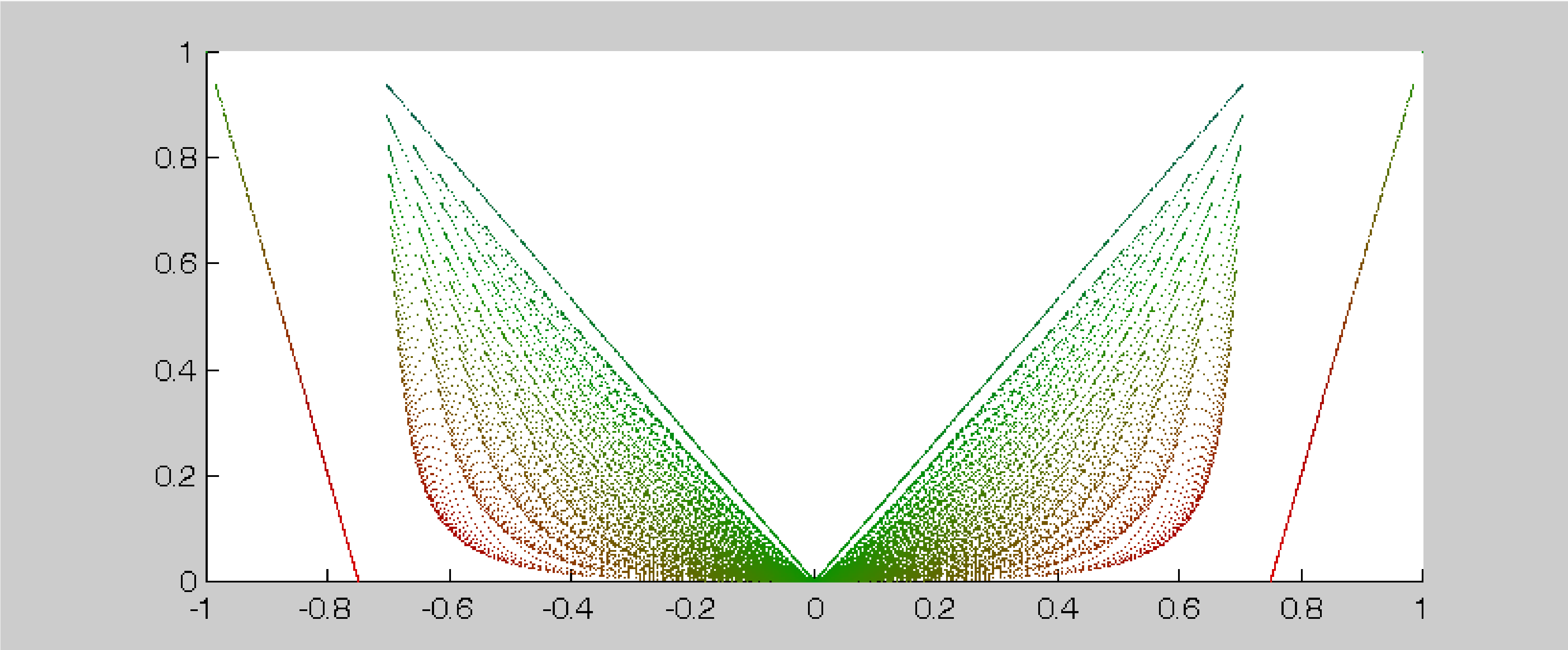
r – vertical axis

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



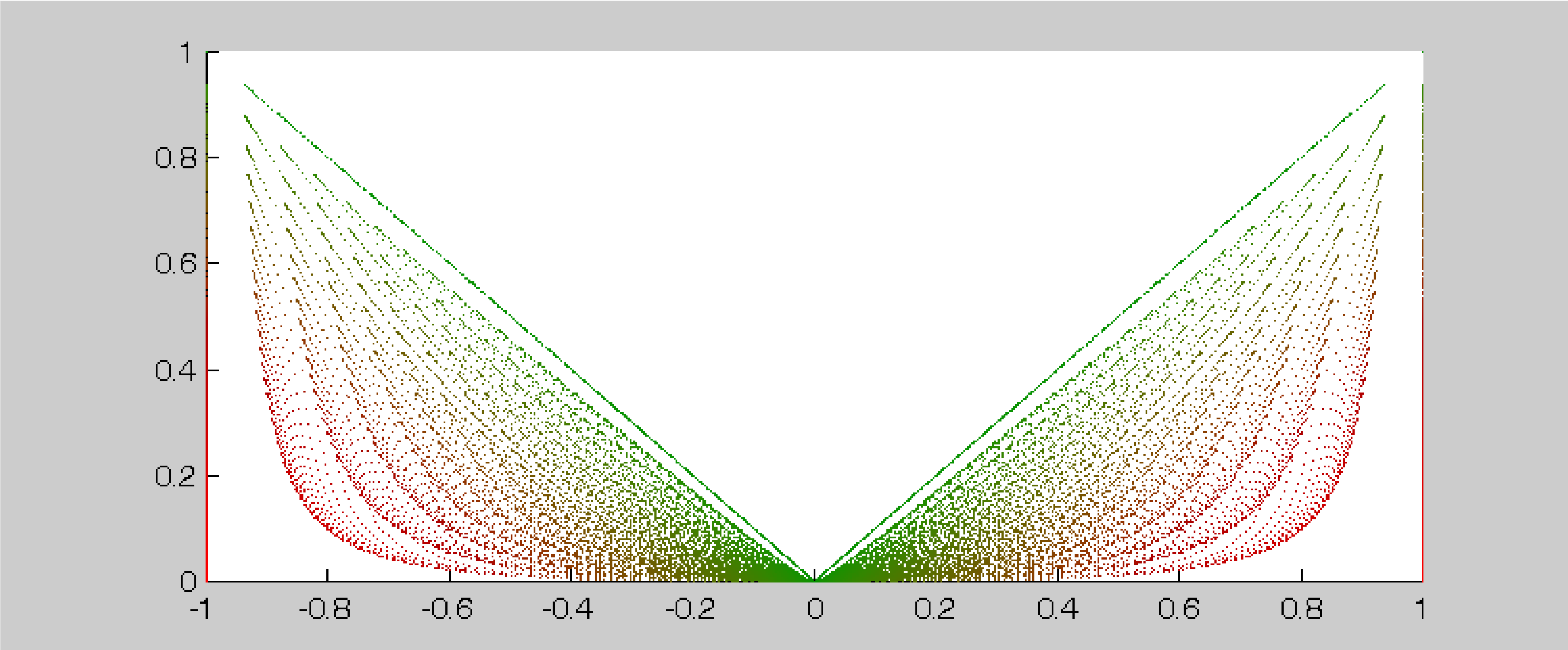
- $\alpha = 0.5$ and $\beta = 0.5$
  - r = 0.697

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



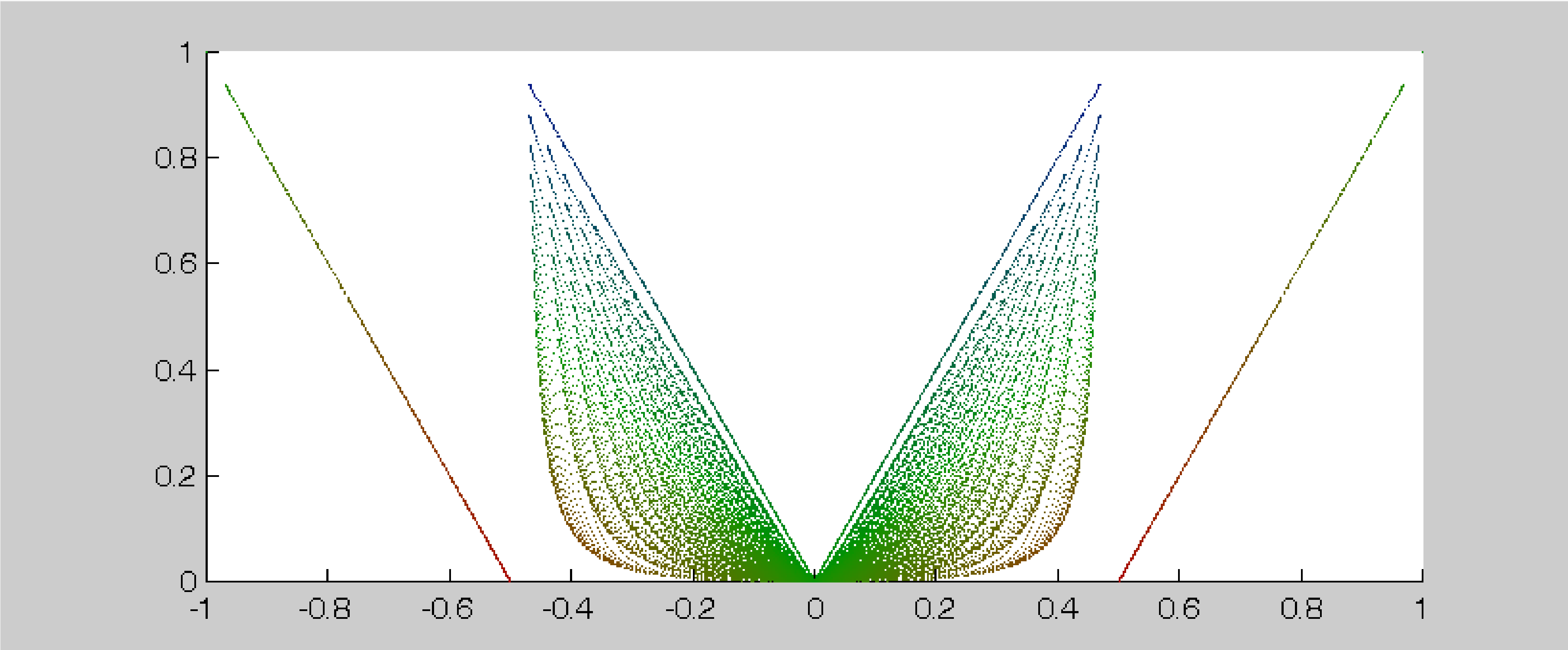
- α = 0.75 and β = 0.25
  - r = 0.700

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



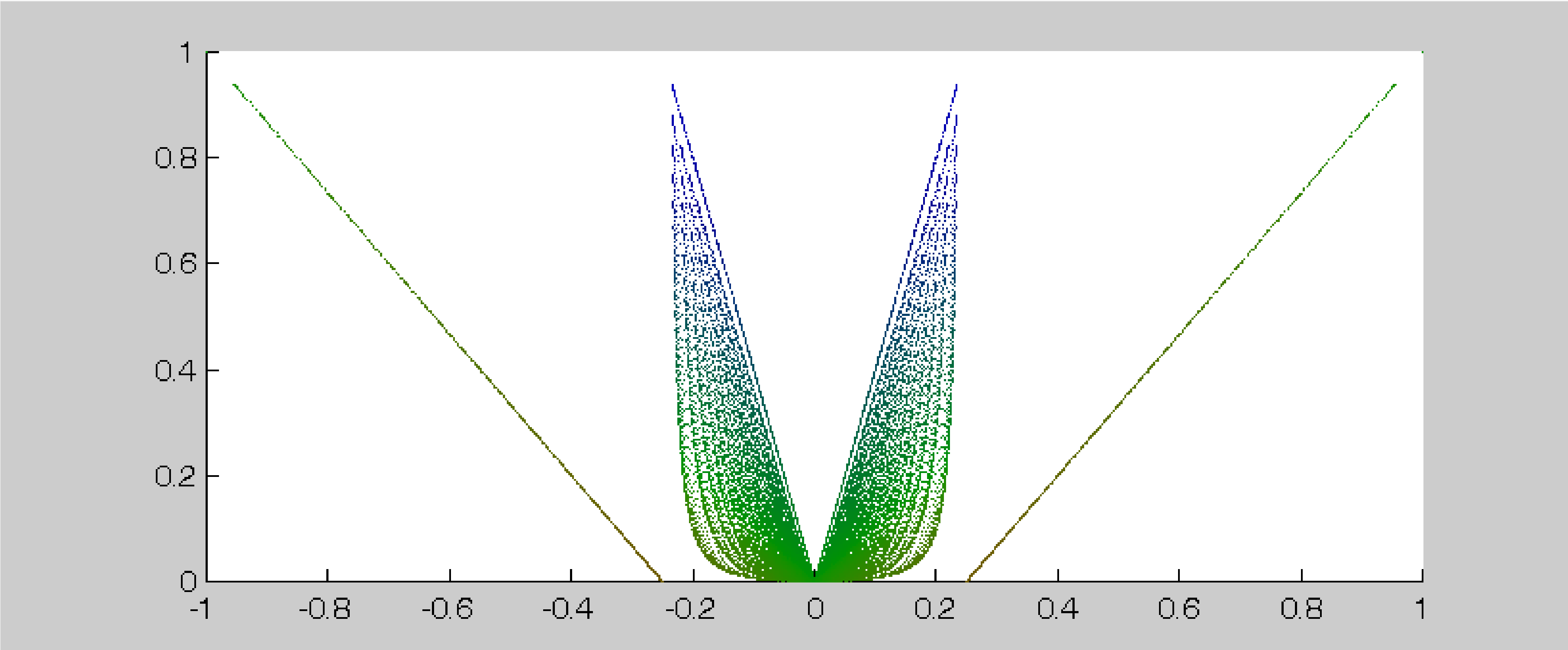
- α = 0.999 and β = 0.001
  (approaching $Z(H,E)$ and $A(H,E)$ respectively)

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



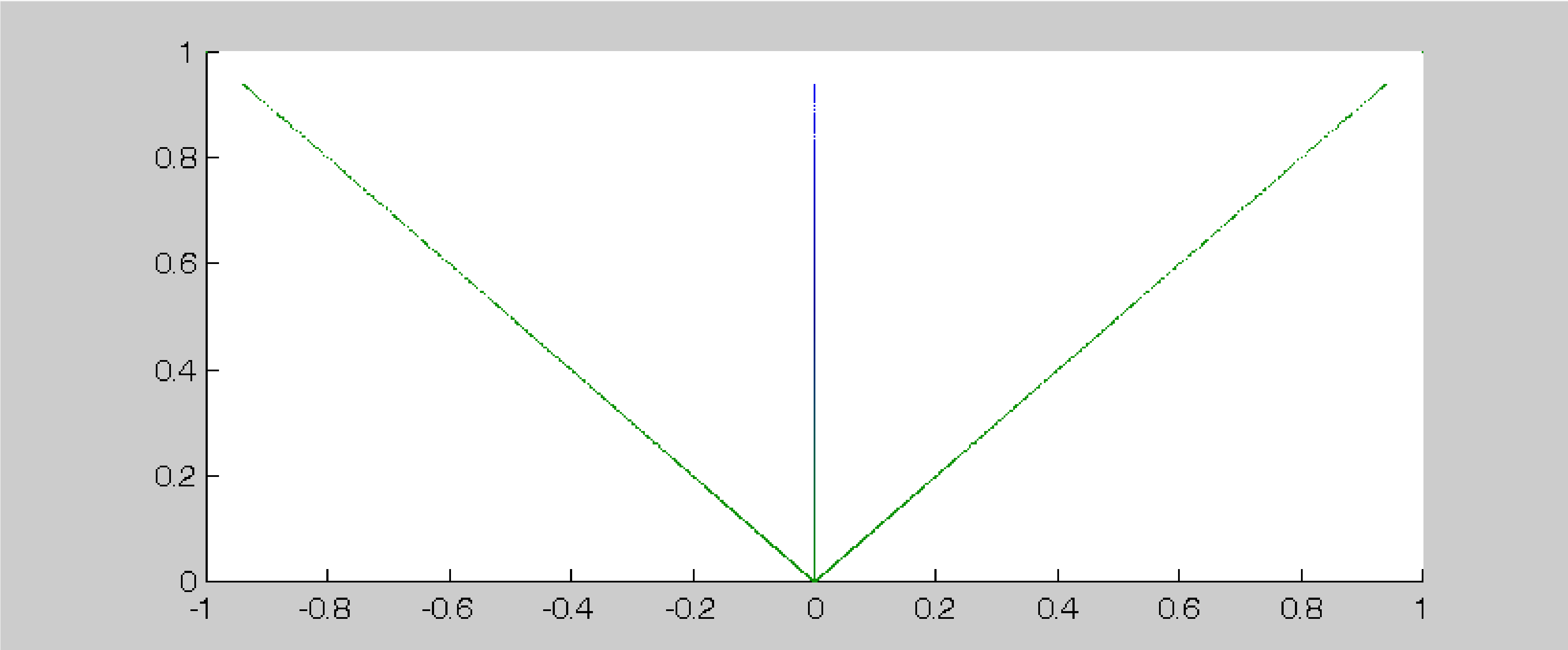
- α = 0.5 and β = 0.5
  - r = 0.697

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



- $\alpha = 0.25$ and $\beta = 0.75$
  - r = 0.637

Influence of α and β on the profile of $c_1(H,E)$ (same for $c_2(H,E)$)



- α = 0.001 and β = 0.999 (approaching $c_3(H,E)$)

Conclusions

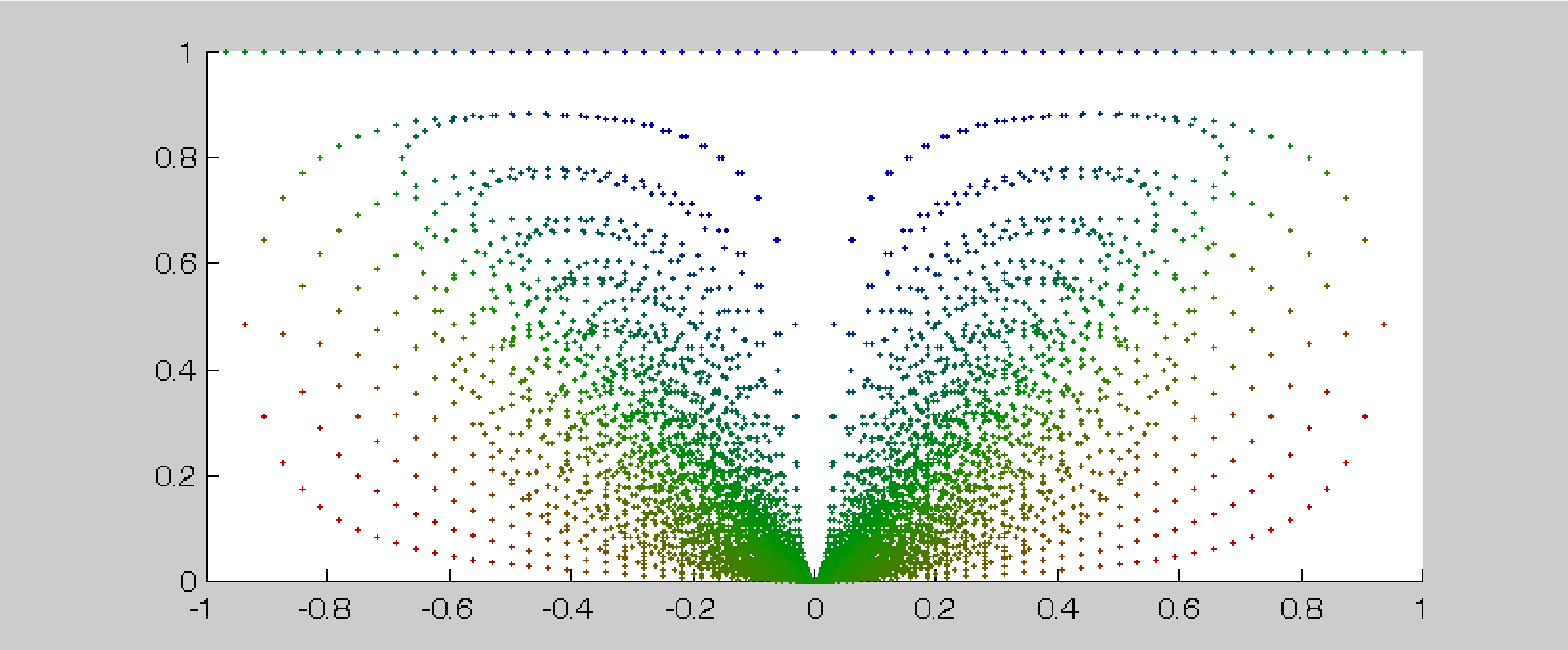
- Confirmation measures are popular tools for evaluation of rules induced from data
- Previous research concerning confirmation measures was confined to environments that had been explicitly or implicitly assumed to be free from observational errors
- In real-life situations, however, the existence of such errors must be taken into account and properly approached
- We incorporate the χ^2 test to examine for the dependence between the evidence and the hypothesis of an induced rule
- Using the Pearson correlation coefficient between the measure and an introduced χ^2_{01} coefficient we quantify how concordant the measure is with the level of dependency between *E* and *H*

Conclusions

- The general conclusion is that most measures possess rather high, although not ideal, concordance
- The scatter-plots and the triple-region histograms of these measures reveal particular situations in which they express either strong confirmation or strong disconfirmation in statistically insignificant situations
- This means that they should be used with special care in error-prone environments

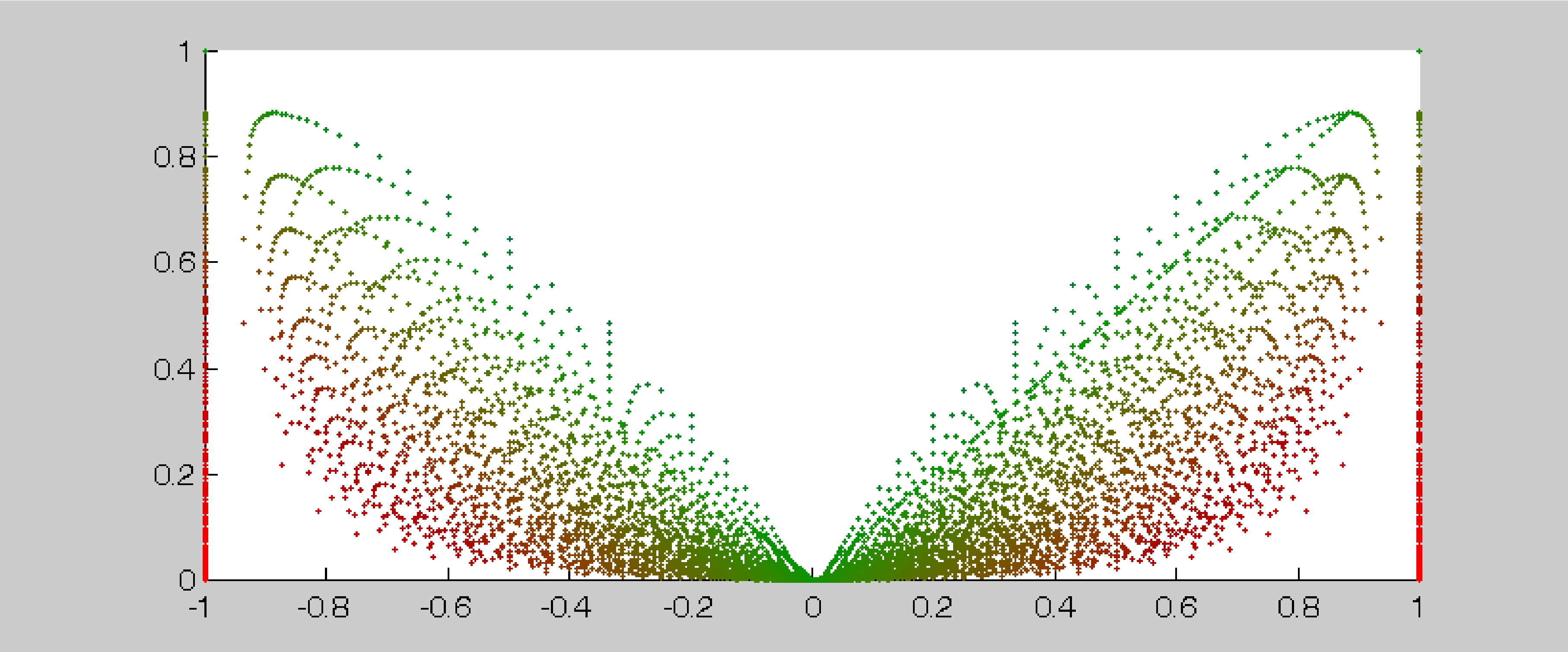
Thank you!

$D(H,E)$-versus-χ^2_{01} (same for $M(H,E)$)



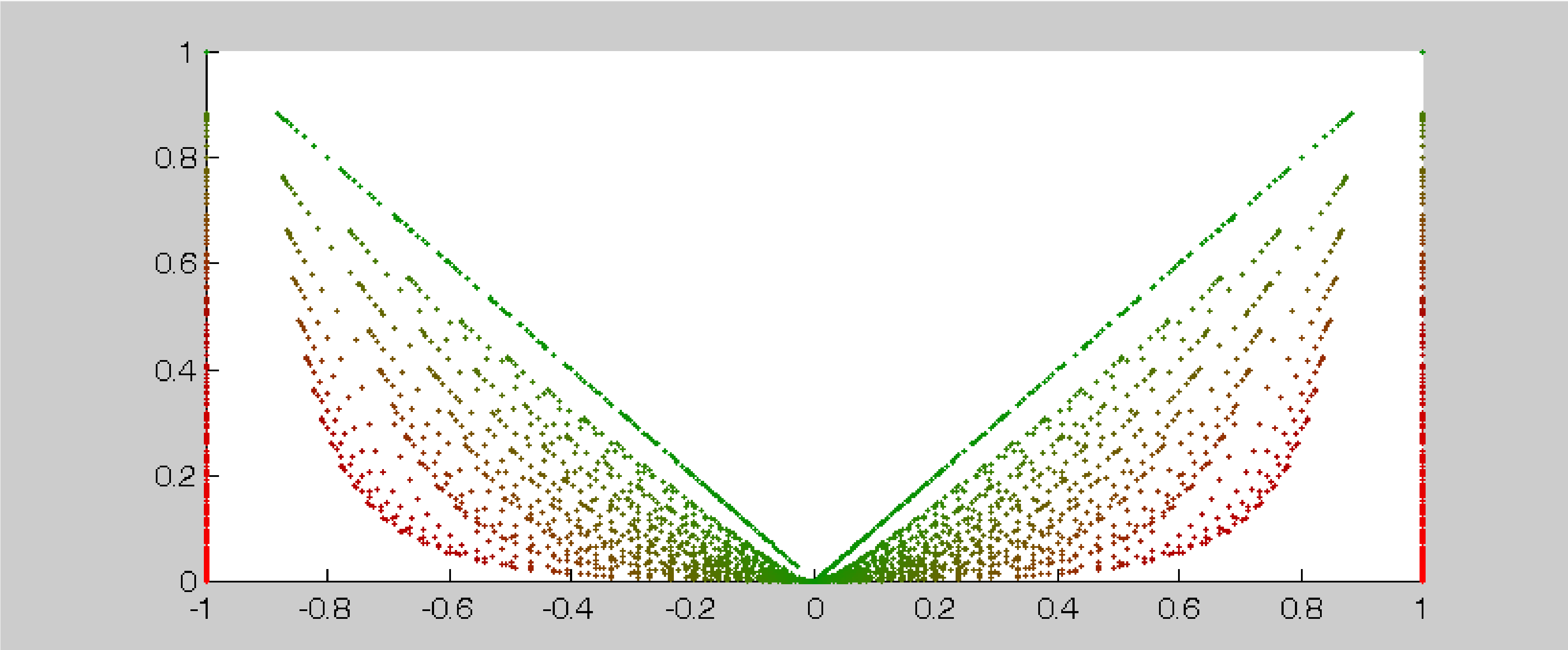
- r = 0.713

$F(H,E)$-versus-χ^2_{01}



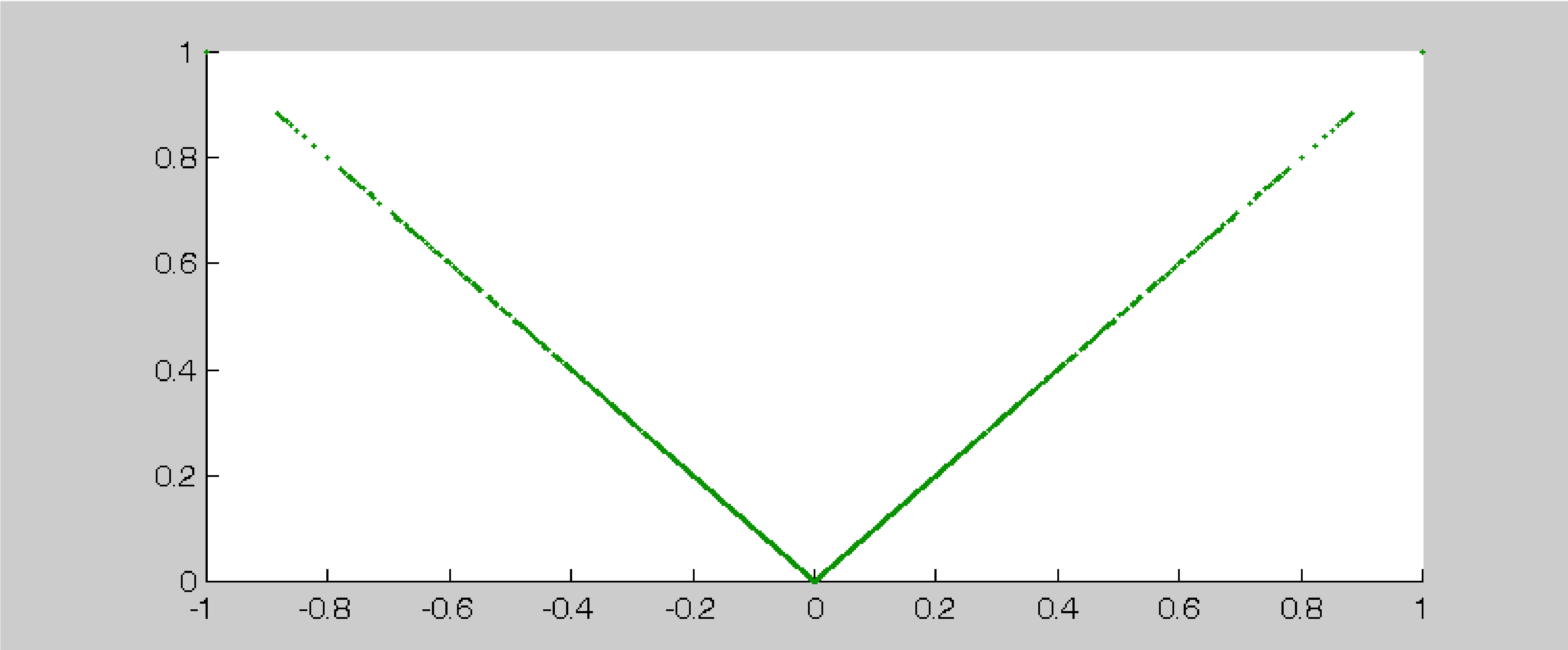
- r = 0.711

$Z(H,E)$-versus-χ^2_{01} (same for $A(H,E)$)



- r = 0.694

$c_3(H,E)$-versus-χ^2_{01}



- r = 1.000

Using confirmation measures in error-prone situations

- Example

  $Z(H,E) = 1.000$

	H	***¬H***
E	100	0
¬E	99	1

$\chi^2_0 = 1.005$, we cannot reject that *VE* and *VH* are independent

Statistics-based Four-Step Procedure

- The four-step procedure of hypothesis testing
  1. Set up a null hypothesis H_0 and an alternative hypothesis H_1
  2. Assume α as the probability of a highly improbable event
  3. Carry out the experiment and get the resulting *DATA*.
     Use the *DATA* to compute the value of some statistics s_0.
     Use the value of the statistics s_0 to compute the probability p.
  4. Compare p to α:
     - if $p>\alpha$ -- the inconclusive result
     - if $p\leq\alpha$ -- the conclusive result (have right to reject H_0)

A 'Reversed Scheme' in the Four-Step Procedure

- Let's reproduce the computations 'in reverse'
  1. H_0, H_1, …
  2. α …
  3. Experiments/computing:

     The *DATA*
     $\Downarrow$
     the statistics s_0 — the statistics s_α
     $\Downarrow$ — $\Uparrow$
     the probability p — the probability α

  4. What is the relation between s_0 and s_α?
- In this reversed scheme the computation $s_0 \Rightarrow p$ is replaced with the computation $\alpha \Rightarrow s_\alpha$

A statistical approach

- Are two variables independent or are they not?
  - the problem is formulated as follows:
    - the two variables are concluded to be dependent when the degree of their dependency is unlikely to have occurred by chance
  - in practice:
    - assume that the two variables are independent and compute the probability that the degree of dependency between them is just like the observed one;
      if the probabilty is very low, then conclude that the two variables are not independent
    - (may be carried out using the χ^2-based statistical test)

A statistical approach

- Idea of the procedure
  - the discrete-valued variables V_1 and V_2
  - the contingency tables O_{ij} and E_{ij} (observed and expected), with i=1..m and j=1..n, where m is the cardinality of the domain of V_1 and n is the cardinality of the domain of V_2
  - the χ^2_0 coefficient and its (approximate) distribution (χ^2 with df = (m−1)(n−1))
  - the probabilities α and *p*

A statistical approach

- Detailed procedure (χ^2-based testing)
  - assume α (probability of what will be considered an unlikely event)
  - formulate H_0 and H_1
  - compute p assuming that H_0 holds
  - conclude (reject H_0 if $p \leq \alpha$)

A statistical approach

- Postulated/desired property of c(H,E):

  - variables *VE* and *VH* <u>should not be independent</u> if the values of c(H,E) are to determine how E confirms/disconfirms H

The Problem of 'Raw Material Supplier'

- The 'raw material supplier' example:
  - John Doe's home company produces three brands of batteries. The management is interested in examining the origin of a particular raw material on the battery's performance. The analyst, John Doe, described 300 tested batteries in terms of:
    - the raw material supplier (A, B, and C)
    - the performance (below 5 hours, about 5 hours, over 5 hours)

    After having analyzed the contingency table (rows: supplier, columns: performance) John Doe concluded that there is no significant dependency supplier<-->performance.

30	60	10
15	60	25
30	30	40

  - Is J. Doe's conclusion correct from the statistical point of view?

The Counts and the Contingency Tables

- The idea of occurrence counts can be applied to two qualitative variables at once
  - assume we have *N* observations described in terms of two qualitative variables, one *K*-elementary and one *L*-elementary
  - because each observation is then described by a combination of two qualitative values, the counts can be of form o_{ij}, where
    - *i*-denotes the *i*-th value of the first scale and *j* denotes the *j*-th value of the second scale
    - $o_{ij}=x$ indicates that there were exactly *x* observations described by the combination of the corresponding qualitative elements
  - of course: $\sum_{i=1}^{K}\sum_{j=1}^{L} o_{ij} = N$
  - the counts are usually stored in form of a $K \times L$ table, which is called the contingency table
  - the contingency table is also called the table of observed values

The Table of Expected Values

- Expectations about $K \times L$ contingency table is also expressed in form of a $K \times L$ table
  - it turns out that the expected e_{ij} are strongly influenced by the marginal distributions of the o_{ij} values
    - a contingency table can be viewed as a two-dimensional discrete distribution and as such it has two main characteristics (so-called marginal distributions)
      - the sums of rows r_i for i=1..K
      - the sums of columns c_j for j=1..L
  - under the condition that the two qualitative variables that delivered the observed contingency table are independent the expected values e_{ij} are given by:

$$e_{ij} = \frac{r_i c_j}{N}$$ where $$r_i = \sum_{j=1}^{L} o_{ij}$$ and $$c_j = \sum_{i=1}^{K} o_{ij}$$

Introducing the Two-Dimensional χ^2-test

- The differences between the observed and the expected tables can be used to characterize the dependency of the qualitative variables that delivered the tables
- The measure used is the following *u* statistics:

$$u_0 = \sum_{i=1}^{K} \sum_{j=1}^{L} \frac{(o_i - e_i)^2}{e_i}$$

- The statistics follows approximately the χ^2-distribution with $df=(K-1)*(L-1)$ degrees of freedom

Testing the Qualitative Dependency

- The test for verifying the dependency between two qualitative variables is constructed as follows:
  1. H_0: $o_{ij}=e_{ij}$ for all pairs *i,j* -- ‘the variables are independent’
     H_1: $o_{ij}\neq e_{ij}$ for at least one pair *i,j* -- ‘the variables are dependent’
  2. Assume α
  3. Compute u_0, establish *df* and compute *p*
  4. Reject H_0 if $p\leq\alpha$
- The ‘reversed scheme’ can be used similarly to the one-dimensional χ^2-test

Solution to 'Raw Material Supplier' #1

- The tables appearing in the solution of the problem
  - the table of the observed values:

30	60	10
15	60	25
30	30	40

  - the sum of rows:

100
100
100

  - the sum of columns:

75	150	75

  - the table of the expected values:

25	50	25
25	50	25
25	50	25

Solution to 'Raw Material Supplier' #2

- Solution to the problem of raw material supplier

1. H_0: 'the performance of the battery
   and the supplier of the raw material
   are independent'

   H_1: 'the variables are dependent'
2. α=0.01
3. $df=(K-1)*(L-1)=4$

   $u_0=36.000$

   $\chi^2_{0.01,4}=13.277$
4. Because $u_0>\chi^2_{0.01,4}$ H_0 is rejected

Is $|c_3(H,E)|$ equal to χ^2_{01} by a coincidence?

- By definition, for an mxn contingency table $[o_{ij}]$

$$\chi_0^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(o_{ij}-e_{ij})^2}{e_{ij}}$$

  with every $e_{ij} = f(o_{11}, o_{12}, \ldots, o_{21}, o_{22}, \ldots, o_{mn})$

- For 2x2 contingency tables, with $a = o_{11}$, $b = o_{21}$, $c = o_{12}$, and $d = o_{22}$, this resolves itself to

$$\chi_0^2 = \frac{n(ad-bc)^2}{(a+b)(b+d)(d+c)(c+a)}$$

- In result

$$\chi_{01}^2 = \frac{\chi_0^2}{n} = \frac{(ad-bc)^2}{(a+b)(b+d)(d+c)(c+a)}$$

Is $|c_3(H,E)|$ equal to χ^2_{01} by a coincidence?

- Now:

$$c_3(H,E) = \begin{cases} A(H,E)Z(H,E) & \textit{in case of confirmation} \\ -A(H,E)Z(H,E) & \textit{in case of disconfirmation} \end{cases}$$

$$Z(H,E) = \begin{cases} 1 - \dfrac{\mathrm{P}(\neg H \mid E)}{\mathrm{P}(\neg H)} = \dfrac{ad - bc}{(a+c)(c+d)} & \textit{in case of confirmation} \\[2ex] \dfrac{\mathrm{P}(H \mid E)}{\mathrm{P}(H)} - 1 = \dfrac{ad - bc}{(a+c)(a+b)} & \textit{in case of disconfirmation} \end{cases}$$

$$A(H,E) = \begin{cases} \dfrac{\mathrm{P}(E \mid H) - \mathrm{P}(E)}{1 - \mathrm{P}(E)} = \dfrac{ad - bc}{(a+b)(b+d)} & \textit{in case of confirmation} \\[2ex] \dfrac{\mathrm{P}(H) - \mathrm{P}(H \mid \neg E)}{1 - \mathrm{P}(H)} = \dfrac{ad - bc}{(b+d)(c+d)} & \textit{in case of disconfirmation} \end{cases}$$