Robust Ordinal Regression and SMAA in Multiple Criteria Hierarchy Process for the Choquet Integral

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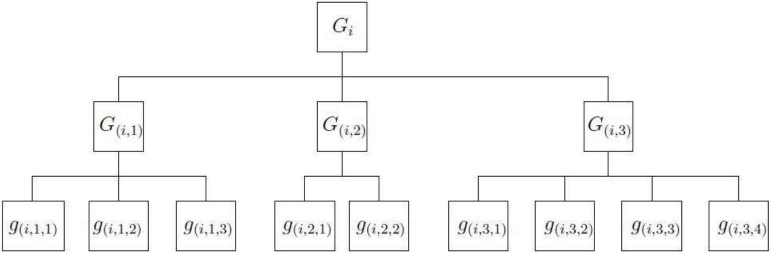
- Multiple Criteria Hierarchy Process (MCHP)



- Choquet integral preference model

  Let us suppose to consider of buying a car and of evaluating the different alternatives with respect to several criteria: price, acceleration, maximum speed, etc.,

  - Maximum speed and acceleration are redundant criteria,
  - Price and maximum speed are synergetic criteria.

Plan of the presentation

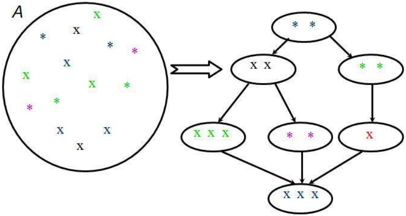
- Problem statement,
- Choquet integral,
- MCHP basic concepts,
- MCHP and Choquet integral,
- ROR and SMAA in MCHP applied to the Choquet integral preference model,
- Didactic example,
- Conclusions

Problem statement

Consider a finite set A of alternatives (actions, solutions, objects) evaluated by m criteria from a consistent family $G = \{g_1, \ldots, g_m\}$, $\mathcal{J} = \{1, \ldots, m\}$ and $\mathcal{I}_j = \{g_j(a), a \in A\}$ for all $j \in \mathcal{J}$.

Taking into account preferences of a Decision Maker (DM), we can deal with three main problems:

- Choice
- Sorting
- Ranking



Independence between Criteria

- Aggregation using utility functions:

$$U(a) = \sum_{j=1}^{m} u_j(g_j(a)),$$

that, in the simplest case, becomes:

$$U(a) = \sum_{j=1}^{m} w_j g_j(a)$$

- Aggregation using binary outranking relations S:

$$aSb \Leftrightarrow a \text{ is at least as good as } b$$

Interaction between Criteria

"Maximum speed and Acceleration so as Maximum speed and Price are interacting criteria",

Given $x \in \mathbb{R}^m$, and a capacity $\mu : 2^{\mathcal{J}} \to [0,1]$ such that:

- $\mu(\emptyset) = 0$, $\mu(\mathcal{J}) = 1$,
- $\mu(A) \leq \mu(B)$, if $A \subseteq B \subseteq \mathcal{J}$

$$Ch(x,\mu) = \sum_{j=1}^{m} x_{(j)} \left[\mu(A_{(j)}) - A_{(j+1)}\right] = \sum_{j=1}^{m} \left[x_{(j)} - x_{(j-1)}\right] \mu(A_{(j)})$$

where

- $0 = x_{(0)} \leq x_{(1)} \leq \ldots \leq x_{(m)}$,
- $A_{(j)} = \{i \in \mathcal{J} : x_i \geq x_{(j)}\}$,
- $A_{(m+1)} = \emptyset$.

Shapley value and interaction index

- The Shapley value expressing the importance of criterion $g_i \in G$, is given by:

$$\varphi(\{i\}) = \sum_{T \subseteq G: i \notin T} \frac{(|G \setminus T| - 1)!|T|!}{|G|!} \cdot [\mu(T \cup \{i\}) - \mu(T)], \qquad (1)$$

- The *interaction index* expressing the sign and the magnitude of the synergy in a couple of criteria $\{g_i, g_j\} \in G$, is given by:

$$\varphi(\{i,j\}) = \sum_{T \subseteq G: i,j \notin T} \frac{(|G \setminus T| - 2)!|T|!}{(|G| - 1)!} \cdot \tau(T, i, j), \qquad (2)$$

where $\tau(T, i, j) = [\mu(T \cup \{i, j\}) - \mu(T \cup \{i\}) - \mu(T \cup \{j\}) + \mu(T)]$.

Möbius representation and k-additive measure

$$m : 2^G \rightarrow [0,1]$$

$$\mu(R) = \sum_{T \subseteq R} m(T), \quad m(R) = \sum_{T \subseteq R} (-1)^{|R \setminus T|} \mu(T)$$

1b) $m(\emptyset) = 0, \ \sum_{T \subseteq G} m(T) = 1,$

2b) $\forall\, i \in G$ and $\forall R \subseteq G \setminus \{i\}, \ \sum_{T \subseteq R} m(T \cup \{i\}) \geq 0.$

A *capacity* is called k-additive if $m(T) = 0$ for $T \subseteq G$ such that $|T| > k$ and there exists at least one $T \subseteq G$, with $|T| = k$, such that $m(T) > 0$.

Möbius representation and 2-additive Choquet integral

$$\mu(R) = \sum_{i \in R} m(\{i\}) + \sum_{\{i,j\} \subseteq R} m(\{i,j\}), \; \forall R \subseteq G.$$

1c) $m(\emptyset) = 0, \; \sum_{i \in G} m(\{i\}) + \sum_{\{i,j\} \subseteq G} m(\{i,j\}) = 1,$

2c)

$$\begin{cases} m(\{i\}) \geq 0, \; \forall i \in G, \\ m(\{i\}) + \sum_{j \in T} m(\{i,j\}) \geq 0, \; \forall i \in G \text{ and } \forall \; T \subseteq G \setminus \{i\}, \; T \neq \emptyset. \end{cases}$$

$$C_\mu(a) \;=\; \sum_{\{i\} \subseteq G} m(\{i\}) (g_i(a)) + \sum_{\{i,j\} \subseteq G} m(\{i,j\}) \min\{g_i(a), g_j(a)\}.$$

Shapley value and interaction index for 2-additive capacities

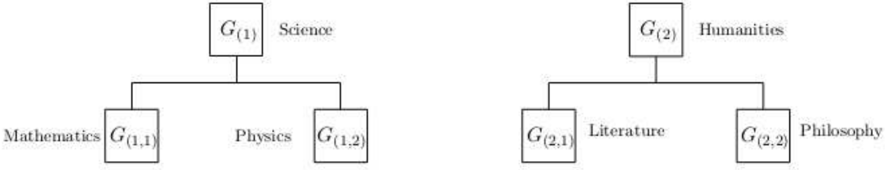
- The Shapley value can be expressed as:

$$\varphi(\{i\}) = m(\{i\}) + \sum_{j \in G \setminus \{i\}} \frac{m(\{i,j\})}{2}, \;\; i \in G, \qquad (3)$$

- The interaction index can be expressed as:

$$\varphi(\{i,j\}) = m(\{i,j\}). \qquad (4)$$

Multiple Criteria Hierarchy Process (MCHP)



Basic concepts

- $H = \{1, \ldots, l\}$ set of levels in the Hierarchy,
- $\mathcal{G}$ set of criteria of all level of hierarchy,
- $\mathcal{I}_{\mathcal{G}}$ set of indices of all criteria in the different levels,
- $G_{\mathbf{r}} \in \mathcal{G}$ with $\mathbf{r} = (i_1, \ldots, i_h) \in \mathcal{I}_{\mathcal{G}}$, criterion of the level h in the hierarchy,
- $n(\mathbf{r})$: number of subcriteria of $G_{\mathbf{r}}$ in the subsequent level,
- $\left\{G_{(\mathbf{r},j)},\ \jmath = 1, \ldots, n(\mathbf{r})\right\}$ set of subcriteria of $G_{\mathbf{r}}$ in the subsequent level,
- EL set of indices of elementary subcriteria (i.e. set of criteria in the leaves of the tree),
- $E(G_{\mathbf{r}})$ set of indices of elementary subcriteria descending from criterion $G_{\mathbf{r}}$.

MCHP and the Choquet integral

Given $G_{\mathbf{r}}$, $\mathbf{r} \in \mathcal{I}_{\mathcal{G}} \setminus EL$, we define:

- Capacity on $\mathcal{G}_{\mathbf{r}}^k = \left\{ G_{(\mathbf{r},w)} \in \mathcal{G} : (\mathbf{r}, w) \in \mathcal{I}_{\mathcal{G}} \cap \mathbb{N}^k \right\}$

$$\mu_{\mathbf{r}}^k : 2^{\mathcal{G}_{\mathbf{r}}^k} \to [0,1],$$

  Given $\mathcal{F} \subseteq \mathcal{G}_{\mathbf{r}}^k$,

$$\mu_{\mathbf{r}}^k(\mathcal{F}) = \frac{\mu(E(\mathcal{F}))}{\mu(E(G_{\mathbf{r}}))}$$

- Choquet integral of a on criterion $G_{\mathbf{r}}$, $\mathbf{r} \notin EL$,

$$C_{\mu_{\mathbf{r}}}(a) = \frac{C_{\mu}(a_{\mathbf{r}})}{\mu(E(G_{\mathbf{r}}))}$$

  where $g_{\mathbf{t}}(a_{\mathbf{r}}) = \begin{cases} g_{\mathbf{t}}(a), & \text{if } \mathbf{t} \in E(G_{\mathbf{r}}), \\ 0 & \text{if } \mathbf{t} \notin E(G_{\mathbf{r}}). \end{cases}$

Shapley value and Interaction index w.r.t a 2-additive capacity

$$\varphi^k_{\mathbf{r}}(G_{(\mathbf{r},w)}) =$$

$$= \left\{ \sum_{\mathbf{t}\in E(G_{(\mathbf{r},w)})} m(g_{\mathbf{t}}) + \sum_{\mathbf{t}_1,\mathbf{t}_2\in E(G_{(\mathbf{r},w)})} m(g_{\mathbf{t}_1}, g_{\mathbf{t}_2}) + \sum_{\substack{\mathbf{t}_1\in E(G_{(\mathbf{r},w)}) \\ \mathbf{t}_2\in E(\mathcal{G}^k_{\mathbf{r}}\setminus\{G_{(\mathbf{r},w)}\})}} \frac{m(g_{\mathbf{t}_1}, g_{\mathbf{t}_2})}{2} \right\} \cdot \frac{1}{\mu(E(G_{\mathbf{r}}))}$$

$$\varphi^k_{\mathbf{r}}(G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)}) = \left\{ \sum_{\substack{\mathbf{t}_1\in E(G_{(\mathbf{r},w_1)}), \\ \mathbf{t}_2\in E(G_{(\mathbf{r},w_2)})}} m(g_{\mathbf{t}_1}, g_{\mathbf{t}_2}) \right\} \cdot \frac{1}{\mu(E(G_{\mathbf{r}}))}.$$

Direct and indirect preference information

- Direct: The Decision Maker provides all the parameters necessary to apply the considered method (in our case, the Möbius coefficients),
- Indirect: The Decision Maker provides some preference information on alternatives or criteria in order to induce parameters compatible with these preferences:
  - Robust Ordinal Regression (ROR),
  - Stochastic Multiobjective Acceptability Analysys (SMAA).

Evaluations of the students

	Science		Humanities	
Student	Mathematics	Physics	Literature	Philosophy
a	18	18	12	12
b	16	16	16	16
c	14	14	18	18
d	18	12	16	16
e	15	15	18	14
f	18	14	14	18
g	15	17	18	16
h	10	20	10	20
k	14	14	14	14

Möbius measures

$m(G_{(\mathbf{1},1)})$	0.29
$m(G_{(\mathbf{1},2)})$	0.19
$m(G_{(\mathbf{2},1)})$	0.29
$m(G_{(\mathbf{2},2)})$	0.19
$m(G_{(\mathbf{1},1)}, G_{(\mathbf{1},2)})$	-0.1
$m(G_{(\mathbf{1},1)}, G_{(\mathbf{2},1)})$	0
$m(G_{(\mathbf{1},1)}, G_{(\mathbf{2},2)})$	0
$m(G_{(\mathbf{1},2)}, G_{(\mathbf{2},1)})$	0
$m(G_{(\mathbf{1},2)}, G_{(\mathbf{2},2)})$	0.24
$m(G_{(\mathbf{2},1)}, G_{(\mathbf{2},2)})$	-0.1

Choquet integrals w.r.t. different criteria

	Science		Humanities			
	Mathematics	Physics	Literature	Philosophy		Choquet integrals
a_1	18	18	0	0	$C_{\mu_1}(a)$	18
a_2	0	0	12	12	$C_{\mu_2}(a)$	12
a	18	18	12	12	$C_{\mu}(a)$	14.28
b_1	16	16	0	0	$C_{\mu_1}(b)$	16
b_2	0	0	16	16	$C_{\mu_2}(b)$	16
b	16	16	16	16	$C_{\mu}(b)$	16
c_1	14	14	0	0	$C_{\mu_1}(c)$	14
c_2	0	0	18	18	$C_{\mu_2}(c)$	18
c	14	14	18	18	$C_{\mu}(c)$	15.52
d_1	18	12	0	0	$C_{\mu_1}(d)$	16.57
d_2	0	0	16	16	$C_{\mu_2}(d)$	16
d	18	12	16	16	$C_{\mu}(d)$	15.26
e_1	15	15	0	0	$C_{\mu_1}(e)$	15
e_2	0	0	18	14	$C_{\mu_2}(e)$	17.05
e	15	15	18	14	$C_{\mu}(e)$	15.54
f_1	18	14	0	0	$C_{\mu_1}(f)$	17.05
f_2	0	0	14	18	$C_{\mu_2}(f)$	16
f	18	14	14	18	$C_{\mu}(f)$	15.92
g_1	15	17	0	0	$C_{\mu_1}(g)$	16
g_2	0	0	18	16	$C_{\mu_2}(g)$	17.52
g	15	17	18	16	$C_{\mu}(g)$	16.58
h_1	10	20	0	0	$C_{\mu_1}(h)$	13.5
h_2	0	0	10	20	$C_{\mu_2}(h)$	13.5
h	10	20	10	20	$C_{\mu}(h)$	15.06
k_1	14	14	0	0	$C_{\mu_1}(k)$	14
k_2	0	0	14	14	$C_{\mu_2}(k)$	14
k	14	14	14	14	$C_{\mu}(k)$	14

Shapley values and Interaction indices

(a) Shapley values of every elementary criterion with respect to every macro-subject $G_{\mathbf{r}}$

	Science		Humanities	
	Mathematics	Physics	Literature	Philosophy
$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)})$	0.63	0.37	0.63	0.37

(b) Shapley values of the elementary criteria

	$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)})$
Mathematics	0.24
Physics	0.26
Literature	0.24
Philosophy	0.26

Table : The Shapley values and interaction index of Science (G_1) and Humanities (G_2)

	$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)})$
Science	0.5
Humanities	0.5
	$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)})$
Science and **Humanities**	0.24

Direct and indirect preference information

- Direct: The Decision Maker provides all the parameters necessary to apply the considered method (in our case, the Möbius coefficients),
- Indirect: The Decision Maker provides some preference information on alternatives or criteria in order to induce parameters compatible with these preferences:
  - Robust Ordinal Regression (ROR),
  - Stochastic Multiobjective Acceptability Analysys (SMAA).

Robust Ordinal Regression (ROR)

Considering the indirect technique, there could exists more than one model compatible with the preference information provided by the DM

$\Downarrow$

Robust Ordinal Regression

- a is necessarily preferred to b ($a \succsim^N b$) $\Leftrightarrow$ a is at least as good as b for all compatible models,
- a is possibly preferred to b ($a \succsim^P b$) $\Leftrightarrow$ a is at least as good as b for at least one compatible model.

SMAA methods

Basic assumptions:

- Imprecision or lack of data (weights and evaluations)
- density functions $f_W(w)$ and $f_\chi(\xi)$ over the weight space $W \subseteq \mathbb{R}^{n+}$ and the evaluation space $\chi \subseteq \mathbb{R}^{m \times n}$,

Computations for each alternative of:

- Rank acceptability index: $b_j^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_j^r(\xi)} f_W(w)\, dw\, d\xi,$
- Central weight vector: $w_j^c = \frac{1}{b_j^1} \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_j^1(\xi)} f_W(w) w\, dw\, d\xi,$

  where $W_j^r(\xi) = \{w \in W : rank(j, \xi, w) = r\}.$
- Besides we can compute the frequency of the preference:

  $p(a_h, a_k) = \int_{w \in W} f_W(w) \int_{\xi \in \chi : u(\xi_h, w) \geq u(\xi_k, w)} f_\chi(\xi) d\xi\, dw.$

DM's preference information

- Comparisons between importance of criteria:
  - $G_{(\mathbf{r},w)}$ is more important than $G_{(\mathbf{r},k)}$ iff $\varphi^k_{\mathbf{r}}(G_{(\mathbf{r},w)}) > \varphi^k_{\mathbf{r}}(G_{(\mathbf{r},k)})$,
  - $G_{(\mathbf{r},w)}$ and $G_{(\mathbf{r},k)}$ are equally important iff $\varphi^k_{\mathbf{r}}(G_{(\mathbf{r},w)}) = \varphi^k_{\mathbf{r}}(G_{(\mathbf{r},k)})$,
- Comparisons between alternatives:
  - a is preferred to b on criterion $G_{\mathbf{r}}$ iff $C_{\mu_{\mathbf{r}}}(a) > C_{\mu_{\mathbf{r}}}(b)$,
  - a and b are indifferent on criterion $G_{\mathbf{r}}$ $C_{\mu_{\mathbf{r}}}(a) = C_{\mu_{\mathbf{r}}}(b)$,
  - a is preferred to b more than c is preferred to d on criterion $G_{\mathbf{r}}$ iff $C_{\mu_{\mathbf{r}}}(a) - C_{\mu_{\mathbf{r}}}(b) > C_{\mu_{\mathbf{r}}}(c) - C_{\mu_{\mathbf{r}}}(d)$.

Set of constraints translating DM's preferences

$$
\begin{cases}
\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)}) > \varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},k)}), & \text{if } G_{(\mathbf{r},w)} \text{ is more important than } G_{(\mathbf{r},k)}, \\
\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)}) = \varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},k)}), & \text{if } G_{(\mathbf{r},w)} \text{ and } G_{(\mathbf{r},k)} \text{ are indifferent,} \\
\\
C_{\mu_{\mathbf{r}}}(a) > C_{\mu_{\mathbf{r}}}(b), & \text{if } a \text{ is preferred to } b \text{ on } G_{\mathbf{r}}, \\
C_{\mu_{\mathbf{r}}}(a) = C_{\mu_{\mathbf{r}}}(b), & \text{if } a \text{ and } b \text{ are indifferenton } G_{\mathbf{r}}, \\
C_{\mu_{\mathbf{r}}}(a) - C_{\mu_{\mathbf{r}}}(b) > C_{\mu_{\mathbf{r}}}(c) - C_{\mu_{\mathbf{r}}}(d), & \text{if} \\
a \text{ is preferred to } b \text{ more than } c \text{ is preferred to } d \text{ on } G_{\mathbf{r}}, \\
\\
m(\{\emptyset\}) = 0, \ \sum_{\mathbf{t} \in EL} m(\{g_{\mathbf{t}}\}) + \sum_{\{\mathbf{t}_1,\mathbf{t}_2\} \subseteq EL} m(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}) = 1 \\
m(\{g_{\mathbf{t}}\}) \geq 0, \ \forall \mathbf{t} \in EL \\
m(\{g_{\mathbf{t}_1}\}) + \sum_{\mathbf{t}_2 \in T} m(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}) \geq 0, \forall \mathbf{t}_1 \in EL \text{ and } \forall\ T \subseteq EL \setminus \{\mathbf{t}_1\}
\end{cases}
$$

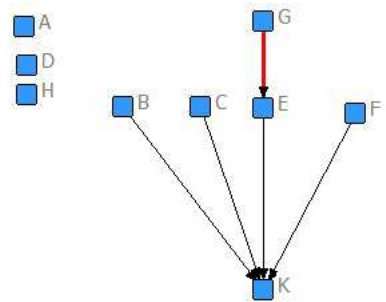
Evaluations of the students

	Science		Humanities	
Student	Mathematics	Physics	Literature	Philosophy
A	18	18	12	12
B	16	16	16	16
C	14	14	18	18
D	18	12	16	16
E	15	15	18	14
F	18	14	14	18
G	15	17	18	16
H	10	20	10	20
K	14	14	14	14

ROR: First preference information

- Physic is more important than Mathematics with respect to the totality of criteria,

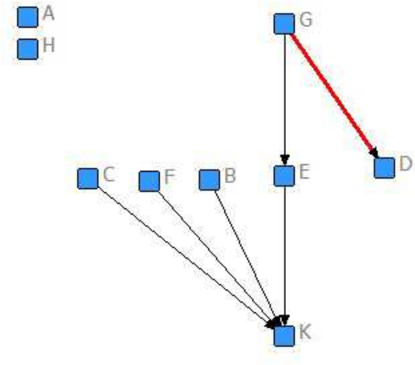
$$\varphi_{\mathbf{0}}(G_{(\mathbf{1},\mathbf{1})}) > \varphi_{\mathbf{0}}(G_{(\mathbf{1},\mathbf{2})})$$



ROR: Second preference information

- K is preferred to H with respect to Science,

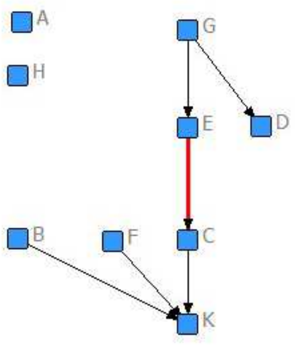
$$C_{\mu_1}(K) > C_{\mu_1}(H)$$



ROR: Second preference information

- E is comprehensively preferred to C,

$$C_{\mu_0}(E) > C_{\mu_0}(C)$$



SMAA Rank acceptability indices

(a) Comprehensive

	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9
A	12.51	4.18	9.46	6.24	7.76	15.44	12.92	17.00	14.48
B	14.57	56.74	17.20	7.35	2.82	1.32	0	0	0
C	0	0	6.11	26.09	43.09	22.20	2.52	0	0
D	0	0	0	0	1.36	18.75	34.98	30.84	14.07
E	0	11.34	37.25	38.82	11.59	0.99	0	0	0
F	0.05	4.5	17.77	18.88	30.70	25.04	3.05	0	0
G	70.51	21.45	8.02	0.02	0	0	0	0	0
H	2.36	1.78	4.18	2.59	2.67	8.58	11.47	12.5	53.86
K	0	0	0	0	0	7.68	35.07	39.66	17.58

(b) Science

	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9
A	100	0	0	0	0	0	0	0	0
B	0	77.62	9.61	12.77	0	0	0	0	0
C	0	0	0	0	0	57.90	42.1	0	0
D	0	0	12.77	4.43	5.17	19.72	33.94	23.96	0
E	0	0	0	43.57	34.05	22.38	0	0	0
F	0	22.38	13.91	20.14	43.57	0	0	0	0
G	0	0	63.71	19.08	17.20	0	0	0	0
H	0	0	0	0	0	0	23.96	76.04	0
K	0	0	0	0	0	57.90	42.1	0	0

(c) Humanities

	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9
A	0	0	0	0	0	0	19.89	80.11	0
B	0	0	6.93	50.79	18.39	23.89	0	0	0
C	84.58	15.42	0	0	0	0	0	0	0
D	0	0	6.93	50.79	18.39	23.89	0	0	0
E	0	0	63.3	9.05	15.82	11.82	0	0	0
F	0	0.86	19.03	20.75	59.36	0	0	0	0
G	0	77.86	7.79	14.34	0	0	0	0	0
H	15.42	5.86	2.95	5.06	6.43	20.38	24.01	19.89	0
K	0	0	0	0	0	43.91	56.1	0	0

Preferences between pairs of alternatives

(d) Comprehensive

	A	B	C	D	E	F	G	H	K
A	0	15.73	35.38	63.85	28.89	36.24	13.81	73.76	69.87
B	84.27	0	91.11	100	84.82	92.55	21.03	95.18	100
C	64.63	8.89	0	99.92	0	50.48	0	87.14	100
D	36.15	0	0.08	0	0	1.75	0	65.29	59.24
E	71.11	15.18	100	100	0	70.33	0	89.75	100
F	63.76	7.45	49.52	98.25	29.67	0	0.23	88.15	100
G	86.19	78.97	100	100	100	99.78	0	97.52	100
H	26.24	4.82	12.86	34.71	10.25	11.85	2.48	0	38.04
K	30.13	0	0	40.76	0	0	0	61.96	0

(e) Science

	A	B	C	D	E	F	G	H	K
A	0	100	100	100	100	100	100	100	100
B	0	0	100	87.23	100	77.62	100	100	100
C	0	0	0	57.90	0	0	0	100	0
D	0	12.77	42.1	0	22.38	0	17.20	76.04	42.1
E	0	0	100	77.62	0	43.57	0	100	100
F	0	22.38	100	100	56.43	0	36.29	100	100
G	0	0	100	82.79	100	63.71	0	100	100
H	0	0	0	23.96	0	0	0	0	0
K	0	0	0	57.90	0	0	0	100	0

(f) Humanities

	A	B	C	D	E	F	G	H	K
A	0	0	0	0	0	0	0	19.89	0
B	100	0	0	0	19.6	55.14	0	66.01	100
C	100	100	0	100	100	100	100	84.58	100
D	100	0	0	0	19.59	55.14	0	66.01	100
E	100	80.40	0	80.40	0	71.33	0	72.10	100
F	100	44.86	0	44.86	28.66	0	14.84	73.02	100
G	100	100	0	100	100	85.16	0	78.36	100
H	80.11	33.99	15.42	33.99	27.9	26.98	21.64	0	56.1
K	100	0	0	0	0	0	0	43.90	0

Central weight vectors

(g) Comprehensive

	m_1	m_2	m_3	m_4	m_{12}	m_{13}	m_{14}	m_{23}	m_{24}	m_{34}
A	0.148	0.224	0.178	0.106	0.402	-0.024	-0.009	-0.041	0.019	-0.004
B	0.085	0.107	0.084	0.085	0.273	0.089	0.045	0.063	0.162	0.002
F	0.141	0.187	0.132	0.169	0.186	-0.049	0.218	0.034	0.054	-0.074
G	0.145	0.103	0.270	0.156	0.148	-0.024	0.020	0.165	0.095	-0.082
H	0.126	0.094	0.246	0.241	0.102	0.025	-0.004	0.010	0.339	-0.182

(h) Science

	m_1	m_2	m_3	m_4	m_{12}	m_{13}	m_{14}	m_{23}	m_{24}	m_{34}
A	0.136	0.119	0.231	0.142	0.197	-0.006	0.020	0.121	0.101	-0.062

(i) Humanities

	m_1	m_2	m_3	m_4	m_{12}	m_{13}	m_{14}	m_{23}	m_{24}	m_{34}
C	0.141	0.123	0.239	0.122	0.199	-0.018	0.022	0.111	0.103	-0.044
H	0.108	0.097	0.188	0.252	0.185	0.055	0.005	0.173	0.094	-0.161

Conclusions

- We have proposed the application of the Multiple Criteria Hierarchy Process (MCHP) to a preference model expressed in terms of Choquet integral.
- Application of the MCHP to the Choquet integral permits the handling of importance and interactions of criteria with respect to any subcriterion of the hierarchy.
- The added value of the MCHP is that it permits the DM expressing the preference information related to any criterion of the hierarchy.
- We applied the Robust Ordinal Regression (ROR) and the Stochastic Multiobjective Acceptability Analysisf in order to take into account the whole set of parameters compatible with some preference information provided by the DM.

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Shapley value and Interaction index in MCHP

- The Shapley value of $G_{(\mathbf{r},w)} \in \mathcal{G}_{\mathbf{r}}^{k}$

$$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)}) = \sum_{T \subseteq \mathcal{G}_{\mathbf{r}}^{k} \setminus \left\{ G_{(\mathbf{r},w)} \right\}} \frac{(|\mathcal{G}_{\mathbf{r}}^{k} \setminus T| - 1)!|T|!}{|\mathcal{G}_{\mathbf{r}}^{k}|!} \cdot \left[\mu_{\mathbf{r}}^{k}(T \cup \left\{ G_{(\mathbf{r},w)} \right\}) - \mu_{\mathbf{r}}^{k}(T)\right]$$

- Interaction index of $\left\{ G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)} \right\} \subseteq \mathcal{G}_{\mathbf{r}}^{k}$ is given by:

$$\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)}) = \sum_{T \subseteq \mathcal{G}_{\mathbf{r}}^{k} \setminus \left\{ G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)} \right\}} \frac{(|\mathcal{G}_{\mathbf{r}}^{k} \setminus T| - 2)!|T|!}{(|\mathcal{G}_{\mathbf{r}}^{k}| - 1)!} \cdot \tau_{\mathbf{r}}^{k}(T, G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)})$$

where

$$\tau_{\mathbf{r}}^{k}(T, A, B) = \left[\mu_{\mathbf{r}}^{k}(T \cup \{A, B\}) - \mu_{\mathbf{r}}^{k}(T \cup \{A\}) - \mu_{\mathbf{r}}^{k}(T \cup \{B\}) + \mu_{\mathbf{r}}^{k}(T)\right].$$