Elicitation of Weighting Parameters in Rank-Dependent Preference Models

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CNRS - LIP6 - UPMC

EURO 2013 - Preference Learning Stream

July, 1th, 2013

Problem Domain

Preference Aggregation with Rank-Dependent Models

- Aggregation of preference information (for instance, from different conflicting critera) is an essential component of a decision-support tool
- Growing interest in rank-dependent aggregation functions to represent the preferences of a decision maker (OWA, WOWA, Choquet integrals, etc)
- Such models are expressive as they are able to represent synergies between criteria, such as compensatory effects

Key question: how to assess the parameters required in order to use such models ?

$\rightarrow$ We consider parameter assessment and adaptive elicitation with **minimax regret**

Preference Parameters in Aggregation Functions

Aggregation function:

$\mathbf{x} = (x_1, \ldots, x_n), \mathbf{y} = (y_1, \ldots, y_n) \quad \rightarrow \quad \mathbf{x} \succsim \mathbf{y} \Leftrightarrow u(\mathbf{x}; \omega) \geq u(\mathbf{y}; \omega)$

Weighting parameters provide a control on:

- the type of compromise sought in MCDM
- the attitude towards equity in Social Choice

Standard weighting parameters (weights attached to criteria)

- **Weighted sum** : $u(\mathbf{x}; \omega) = \sum_{i=1}^{n} \omega_i x_i$
- **Weighted Tchebycheff** : $u(\mathbf{x}; \omega) = \max_{i \in [\![1;n]\!]} \{\omega_i \frac{x_i^* - x_i}{x_i^* - x_{*i}}\}$

Rank-dependent weighting parameters (weights attached to ranks)

- **OWA** : $u(\mathbf{x}; w) = \sum_{i=1}^{n} w_i x_{(i)}$
- **Choquet** : $u(\mathbf{x}; v) = \sum_{i=1}^{n} \left[x_{(i)} - x_{(i-1)}\right] v(X_{(i)})$

Utility-based Intereactive Recommender System:

- *Bel*: belief about the user's utility function u
- *Opt*(*Bel*): optimal decision given incomplete beliefs about u

Algorithm: Adaptive Utility Elicitation

1. **Repeat** until *Bel* meets some termination condition
   1. Ask user some query
   2. Observe user response r
   3. Update *Bel* given r
2. Recommend *Opt*(*Bel*)

Types of Beliefs

• *Probabilistic Uncertainty:* probability distribution of parameters, updated using Bayes

• *Strict Uncertainty:* feasible region of utility parameters (if linear constraints: a convex polytope)

Minimax Regret

Intuition

Adversarial game; the recommender selects the item reducing the "regret" wrt the "best" item when ω is chosen by the adversary

- Robust criterion for decision making under uncertainty [Savage; Kouvelis]
- Showed to be effective when used for decision making under utility uncertainty [Boutilier et al., 2006] and as driver for elicitation

Advantages

- No heavy Bayesian updates
- No prior assumption required
- MMR computation suggests queries to ask to the user

Limitations

- No account for noisy responses
- Formulation of the optimization heavily depends on the assumption about the utility model

Minimax Regret

Assumption: a set of feasible utility functions W is given

The *pairwise max regret*

$$PMR(\mathbf{x}, \mathbf{y}; W) = \max_{w \in W} u(\mathbf{y}; w) - u(\mathbf{x}; w)$$

The *max regret*

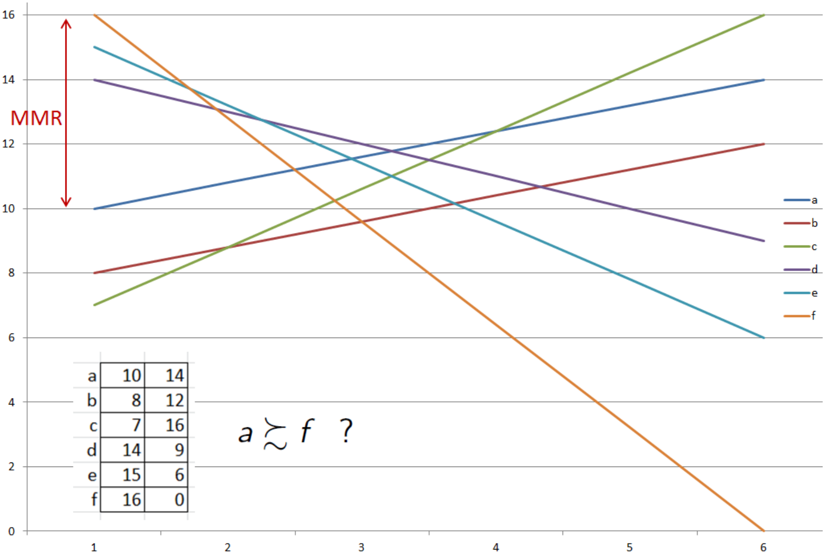
$$MR(\mathbf{x}; W) = \max_{\mathbf{y} \in \mathbf{X}} PMR(\mathbf{x}, \mathbf{y}; W)$$

The *minimax regret*

$MMR(W)$ of W and the *minimax optimal item* $\mathbf{x}^*_W$:

$$\begin{aligned} MMR(W) &= \min_{\mathbf{x} \in \mathbf{X}} MR(\mathbf{x}, W) \\ \mathbf{x}^*_W &= \arg\min_{\mathbf{x} \in \mathbf{X}} MR(\mathbf{x}, W) \end{aligned}$$

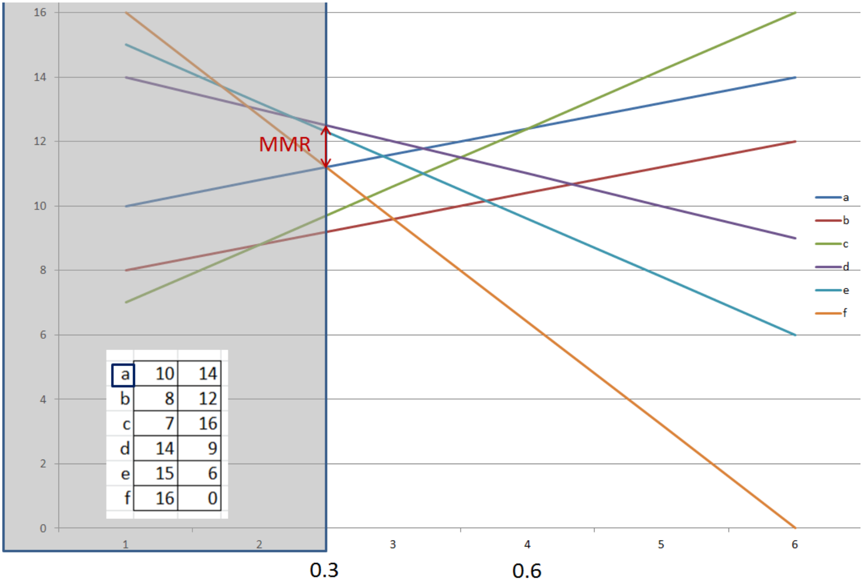
A graphical illustration for the weighted sum (1/3)



a	10	14
b	8	12
c	7	16
d	14	9
e	15	6
f	16	0

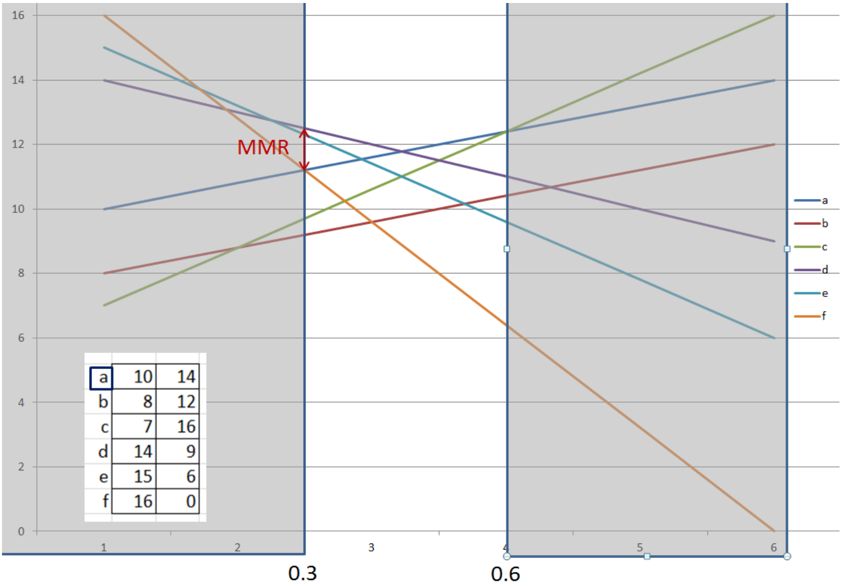
$a \succsim f$?

A graphical illustration for the weighted sum (2/3)



a	10	14
b	8	12
c	7	16
d	14	9
e	15	6
f	16	0

A graphical illustration for the weighted sum (3/3)



a	10	14
b	8	12
c	7	16
d	14	9
e	15	6
f	16	0

Minimax Regret Computation

Computation of Pairwise Max Regret as Linear Program

- Objective function: $\max_{w \in W} w \cdot (\mathbf{y} - \mathbf{x})$
- Usually W expressed by linear constraints such as $w \cdot \mathbf{z}^+ \geq w \cdot \mathbf{z}^-$ for $(\mathbf{z}^+, \mathbf{z}^-) \in \mathcal{D}_{pref}$ (a set of comparisons)

Computation of Minimax Regret

- Naive approach: test all $n^2 - n$ combinations of choices
- Better idea: implement a *search* problem [Braziunas, 2012]
  - i=choice of recommender, j=choice of adversary
  - UB: upper bound on minimax regret (max regret of best solution found)
  - LB_i: lower bound on the max regret of option i
  - After testing i against j: $LB_i \leftarrow \max(LB_i, PMR(i,j))$
  - Whenever $LB_i \geq UB$: prune option i
- Empirically, a small number of PMR checks is needed

Example of Minimax Regret Computation

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (0 *PMR* checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	?	?	?	0
i=2	?	0	?	?	0
i=3	?	?	0	?	0
i=4	?	?	?	0	0

UB
+Inf

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (1 *PMR* checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	1	?	?	1
i=2	?	0	?	?	0
i=3	?	?	0	?	0
i=4	?	?	?	0	0

UB
+Inf

Example of Minimax Regret Computation

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (3 *PMR* checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	1	2	3	3
i=2	?	0	?	?	0
i=3	?	?	0	?	0
i=4	?	?	?	0	0

UB
3

Example of Minimax Regret Computation

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (6 PMR checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	?	?	0	?	0
i=4	?	?	?	0	0

UB
2

Example of Minimax Regret Computation

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (7 *PMR* checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	?	0	?	4
i=4	?	?	?	0	0

UB
2

Example of Minimax Regret Computation

Example

Complete "pairwise max regret" table

$PMR(i,j)$	j=1	j=2	j=3	j=4	$MR(i)$
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	1	0	1	4
i=4	3	2	3	0	3

Evaluation (8 *PMR* checks)

$PMR(i,j)$	j=1	j=2	j=3	j=4	LB_i
i=1	0	1	2	3	3
i=2	2	0	2	2	2
i=3	4	?	0	?	4
i=4	3	?	?	0	3

UB
2

Application to OWA

Positive normalized weights (general case)

$u(\mathbf{x}; w) = \sum_{i=1}^{n} w_i x_{(i)}$ non-linear in x but linear in w !

$$u(x, w) \geq u(y, w) \iff w.x^{\uparrow} \geq w.y^{\uparrow}$$

where $x^{\uparrow}$ is the vector x sorted by increasing order
$\rightarrow$ preference of type $x \succsim y$ is equivalent to a linear inequality

Positive normalized decreasing weights (fair optimization)

OWA with decreasing weights: $w_i > w_j$ whenever $i < j$
ensures the compatibility with *Pigou-Dalton transfers*, i.e:
$\forall i, j : x_j > x_i, \forall \varepsilon \in (0, x_j - x_i)$,
$(x_1, x_i + \varepsilon, \ldots, x_j - \varepsilon, x_n) \succ (x_1, \ldots, x_n)$
$\rightarrow$ add inequalities of type $w_i - w_{i+1} \geq \delta$ with $\delta > 0$

Regret Computation for OWA

Dataset Manipulation

$\mathbf{x}^{\uparrow} = (x_{(1)}, ..., x_{(n)})$ permutation of features from worst to best

PMR for OWA

$$\max_{w} \quad w \cdot (\mathbf{y}^{\uparrow} - \mathbf{x}^{\uparrow}) \tag{1}$$

$$s.t. \quad 0 \leq w_i \leq 1 \quad \forall i \tag{2}$$

$$\sum_i w_i = 1 \tag{3}$$

$$w_i - w_{i+1} \geq \delta \tag{4}$$

$$w \cdot \mathbf{z}_+^{\uparrow} \geq w \cdot \mathbf{z}_-^{\uparrow} \quad \text{for } (\mathbf{z}_+, \mathbf{z}_-) \in \mathcal{D}_{pref} \tag{5}$$

Application to Choquet integrals

$$C_v(x) = x_{(1)}v(X_{(1)}) + \sum_{i=2}^{n} \left[x_{(i)} - x_{(i-1)}\right] v(X_{(i)})$$

$x_{(i)} \leq x_{(i+1)}$ for all $i = 1, \ldots, n-1$ and $X_{(i)} = \{j \in N, x_j \geq x_{(i)}\}$,

Example

	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
v	0	0.1	0.2	0.3	0.5	0.6	0.7	1

$x = (10, 6, 14)$ and $y = (10, 12, 8)$

$$C_v(x) = 6 + (10-6)v(\{1,3\}) + (14-10)v(\{3\}) = \quad 9.6$$
$$C_v(y) = 8 + (10-8)v(\{1,2\}) + (12-10)v(\{2\}) = \quad 9.4$$

With C_v we observe that x is preferred to y.

Fairness and convex capacities in the Choquet integral

v is said to be *convex* or *supermodular* when
$v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$ for all $A, B \subseteq N$,

Proposition (Chateauneuf and Tallon, 1999)

When preferences are represented by a Choquet integral, then choosing v convex is equivalent to the following property:
$\forall x^1, x^2, \ldots x^p \in \mathbb{R}^n$, $\forall k \in \{1, 2, \ldots, p\}$ and
$\forall i \in \{1, 2, \ldots, p\}, \lambda_i \geq 0$ such that $\sum_{i=1}^{p} \lambda_i = 1$ we have:

$$C_v(x^1) = C_v(x^2) = \ldots = C_v(x^p) \Rightarrow C_v(\sum_{i=1}^{p} \lambda_i x^i) \geq C_v(x^k)$$

Example (convex v and fairness)

$x = (18, 18, 0), y = (0, 18, 18), z = (18, 0, 18)$
$t = (12, 12, 12) = (x + y + z)/3$
v convex $\Rightarrow$ $[C_v(x) = C_v(y) = C_v(z) \Rightarrow t \succsim x, t \succsim y, t \succsim z]$

Choquet integral and Möbius masses

an alternative representation in terms of the Möbius inverse:

Möbius inverse and Möbius masses

To any set-function $v : 2^N \to \mathbb{R}$ is associated $m : 2^N \to \mathbb{R}$ a mapping called *Möbius inverse*, defined by:

$$\forall A \in N, \quad m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} v(B), \quad v(A) = \sum_{B \subseteq A} m(B)$$

Coefficients $m(B)$ for $B \subseteq A$ are called Möbius masses.

Remark: v convex if Möbius masses are positive (*belief function*, Shafer, 1976)

Choquet integral as a function of Möbius masses

$$C_v(x) = \sum_{B \subseteq A} m(B) \bigwedge_{i \in B} x_i$$

2-additive capacity ($m(B) = 0$ iff $|B| > 2$) $\to$ capacity completely characterized by $(n^2 + n)/2$ coefficients.
Good compromise between compacity of the model and expressivity.

PMR for Choquet (1/2)

m vector of variables:
$m = (m_1, \ldots, m_n, m_{11}, m_{12}, \ldots, m_{1n}, m_{23}, \ldots, m_{2n} \ldots, m_{n-1n})$
$\bar{x} = (x_1, \ldots, x_n, x_{11}, x_{12}, \ldots, x_{1n}, x_{23}, \ldots, x_{2n} \ldots, x_{n-1n})$, $x_{ij} = x_i \wedge x_j$

The case of a 2-additive capacity

$$\max_m \quad m.(\bar{\mathbf{x}} - \bar{\mathbf{y}}) \tag{6}$$

$$s.t. \sum_{j \in J} m_{ij} \geq 0, \quad i = 1, \ldots, n, \{i\} \subseteq J \subseteq N \tag{7}$$

$$\sum_{i=1}^{n} m_i + \sum_{i}^{n} \sum_{j=i+1}^{n} m_{ij} = 1 \tag{8}$$

$$m.\bar{\mathbf{z}}_+ \geq m.\bar{\mathbf{z}}_-, \ \ (\mathbf{z}_+, \ \mathbf{z}_-) \in \mathcal{D}_{pref} \tag{9}$$

Remark: monotonicity (7) requires an exponential number of constraints

PMR for Choquet (2/2)

m vector of variables:
$m = (m_1, \ldots, m_n, m_{11}, m_{12}, \ldots, m_{1n}, m_{23}, \ldots, m_{2n} \ldots, m_{n-1n})$
$\bar{x} = (x_1, \ldots, x_n, x_{11}, x_{12}, \ldots, x_{1n}, x_{23}, \ldots, x_{2n} \ldots, x_{n-1n})$, $x_{ij} = x_i \wedge x_j$

The case of (2-additive) belief functions

$$\max_m \quad m.(\bar{\mathbf{x}} - \bar{\mathbf{y}}) \tag{10}$$

$$s.t. \sum_{i=1}^{n} m_i + \sum_{i}^{n} \sum_{j=i+1}^{n} m_{ij} = 1 \tag{11}$$

$$m.\bar{\mathbf{z}}_+ \geq m.\bar{\mathbf{z}}_-, \quad (\mathbf{z}_+, \ \mathbf{z}_-) \in \mathcal{D}_{pref} \tag{12}$$

$$m_i \geq 0, \ i = 1, \ldots, n, \tag{13}$$

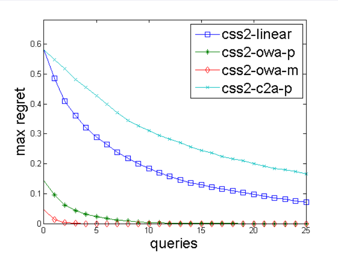
$$m_{ij} \geq 0, \ i = 1, \ldots, n, \ j = i+1, \ldots, n, \tag{14}$$

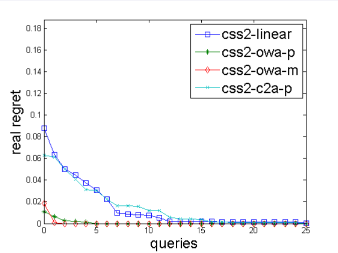
Interactive Elicitation

Strategies for Choosing the Next Query

- Different types of Queries: comparisons, bound queries (local/global),...
- Choice of Query: *Current Solution Strategy (CSS)* asking to compare $\mathbf{x}^*$ and $\mathbf{x}^a$ (the adversary's choice): $\mathbf{x}^a = \arg\max_{\mathbf{y}} PMR(\mathbf{x}, \mathbf{y})$
- It is possible to optimize queries wrt analogous of value of information [Viappiani and Boutilier, 2009]

Simulations





Random dataset: 100 data points, 10 features. Sampled random utility weights between 0 and 1; then normalized

The simulated user answers a comparison query at each step according to utility weights; 30 runs

Discussion

Observation

- Empirically, a user whose utility function is based on OWA requires less queries to identifying the true best item
- When assuming monotonicity for the weights (favouring well-balanced solutions) this is even more striking
- MMR seems to be lower when assuming OWA than linear aggregation

Conclusions

- The assessment of the parameters of models such as OWA and Choquet is usually problematic
- Minimax regret can be used to assess the parameters of non-linear utility models such as OWA and Choquet
- Regret can be used to select the next query to ask in Interactive elicitation
- Preliminary simulation results