Multiple criteria ranking with additive value models and holistic pair-wise preference statements

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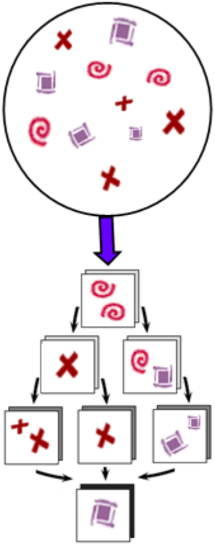
Context: multiple criteria ranking

- Ranking (or choosing a small subset from) a set of decision alternatives A
- Alternatives **deterministically** evaluated on multiple criteria
- Possibly multiple decision makers, though we concentrate on a single one



Additive multi-attribute value functions

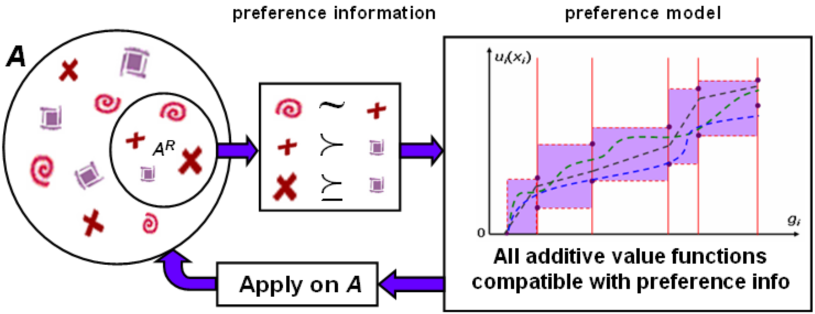
- Value functions $U(\cdot) : A \to \mathbb{R}$ satisfying

$$x \succsim y \Leftrightarrow U(x) \geq U(y)$$

- Additive model (assuming mutual preferential independence)

$$U(x) = \sum_{i=1}^{n} u_i(x_i)$$

Robust Ordinal Regression (UTA^{GMS})



- Take into account all value functions ($\mathcal{U}^{A^R}$) compatible with the preference information
- Identify necessary and possible consequences of using all value functions:

  $x \succsim^N y \Leftrightarrow \forall U \in \mathcal{U}^{A^R} : U(x) \geq U(y)$

  $x \succsim^P y \Leftrightarrow \exists U \in \mathcal{U}^{A^R} : U(x) \geq U(y)$

Motivation: application

Ranking of European countries based on their universities' quality (according to Webometrics)

- g_1 (system), the number of universities in the Top 500 in the given country, divided by the mean position of these institutions
- g_2 (access), a score built according to ranks (5 points for a university in the top 100, 4 points for $101 - 200$, etc., divided by the population size of the country)
- g_3 (flagship), a normalized score based on the leading university rank
- g_4 (economy), same score as for g_3 but divided by GDP per capita.

Criteria evaluations

Country	Short name	g_1	g_2	g_3	g_4
Germany	GER	82	94	80	91
United Kingdom	UK	74	91	96	82
Spain	SPA	59	73	72	67
Sweden	SWE	47	77	90	46
Netherlands	NET	50	73	88	47
Italy	ITA	51	50	84	55
Finland	FIN	42	59	88	39
Belgium	BEL	44	57	84	41
Austria	AUS	42	53	88	38
Denmark	DEN	42	61	68	39
France	FRA	45	37	80	44
Czech Rep.	CZE	41	43	80	40
Portugal	POR	41	41	60	40
Slovenia	SIO	38	37	72	34
Ireland	IRE	40	40	60	34
Hungary	HUN	39	34	48	38
Estonia	EST	38	36	44	34
Greece	GRE	39	28	40	34
Poland	POL	39	26	36	37
Slovakia	SVK	37	21	8	37

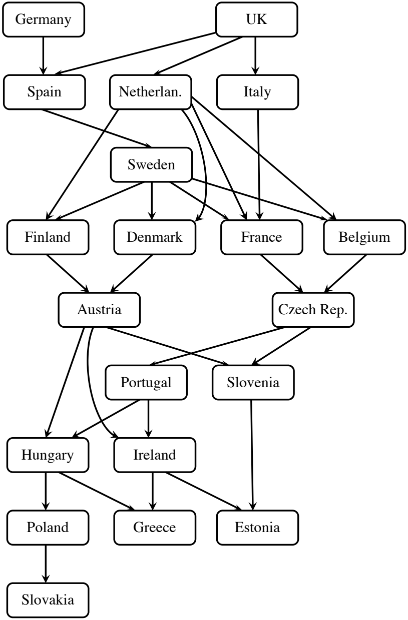
Preference information

$$\text{Denmark} \succ \text{Austria}$$

$$\text{Spain} \succ \text{Sweden}$$

$$\text{France} \succ \text{Czech Rep.}$$

Necessary preference relations



- Most preference relations are also dominance-relations (e.g. Sweden $\succ$ Denmark)
- Some part of the preference information (Denmark $\succ$ Austria, Spain $\succ$ Sweden, France $\succ$ Czech Rep.)
- Others almost non-existent due to generality of the preference model! (c.f. Spliet and Tervonen, 2013)

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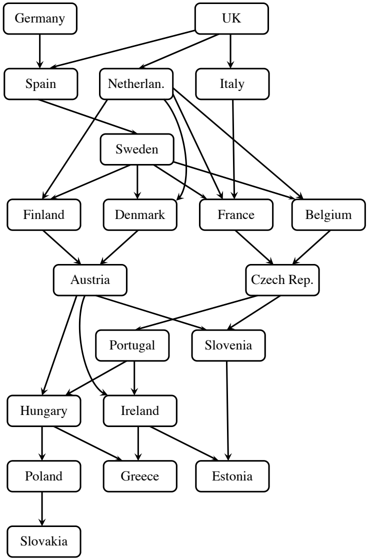
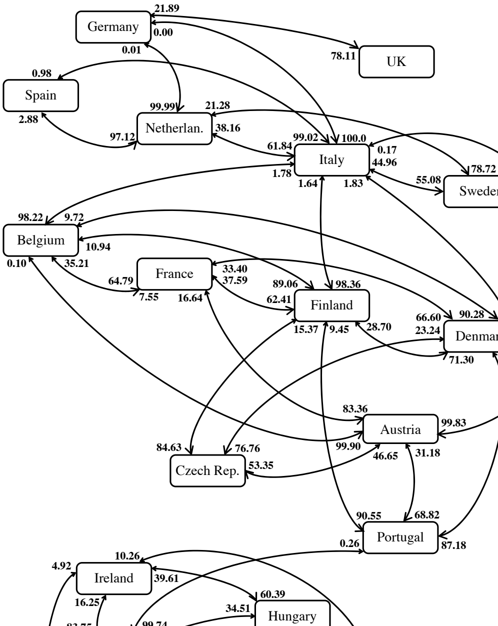
Stochastic pair-wise indices

- Pair-wise Outranking Index POI(a, b): share of $\mathcal{U}^{A^R}$ so that $U(a) \geq U(b)$
- Relates to $\succsim^P$
- Exact computation #P hard; estimated through Monte Carlo integration
- Pair-wise Winning Index PWI(a, b) $=$ 1-POI(b,a) (relates to $\succsim^N$)

Necessary and Possible preference relations and POIs





Possible relations and extreme ranking analysis

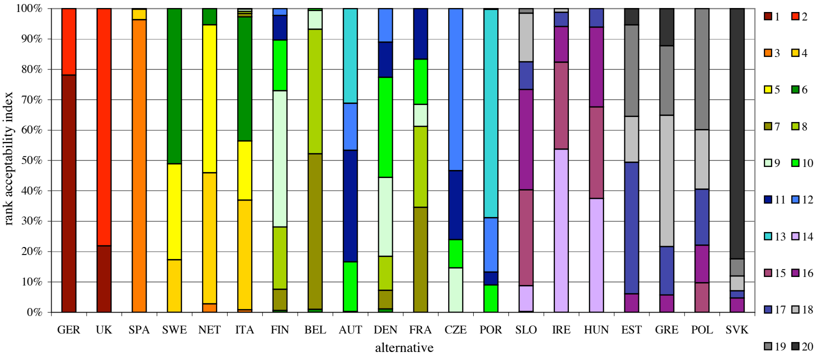
- $\forall a, b \in A, \neg(a \succ^N b) \Rightarrow b \succsim^P a$
- Often the necessary relation (apart from dominance, transitivity and preference statements) is empty
- Extreme ranking analysis tells ranges of ranks an alternative can obtain
- Assuming no shared ranks, all ranks between the extremes can be obtained

Country	P^*	P_*
GER	1	4
UK	1	2
SPA	3	5
SWE	4	6
NET	2	6
ITA	2	10
FIN	6	12
BEL	6	11
AUS	8	13
DEN	6	12
FRA	7	11
CZE	9	12
POR	10	14
SLO	13	19
IRE	14	18
HUN	14	17
EST	16	20
GRE	16	20
POL	15	19
SVK	16	20

Rank Acceptability Index

- Rank Acceptability Index RAI(a, r): share of $\mathcal{U}^{A^R}$ so that a has rank r
- The main decision aiding metric in SMAA-2
- Also #P hard to compute: estimate with MC integration

Extreme ranks and RAIs



Country	P^*	P_*
GER	1	4
UK	1	2
SPA	3	5
SWE	4	6

So what's the ranking?

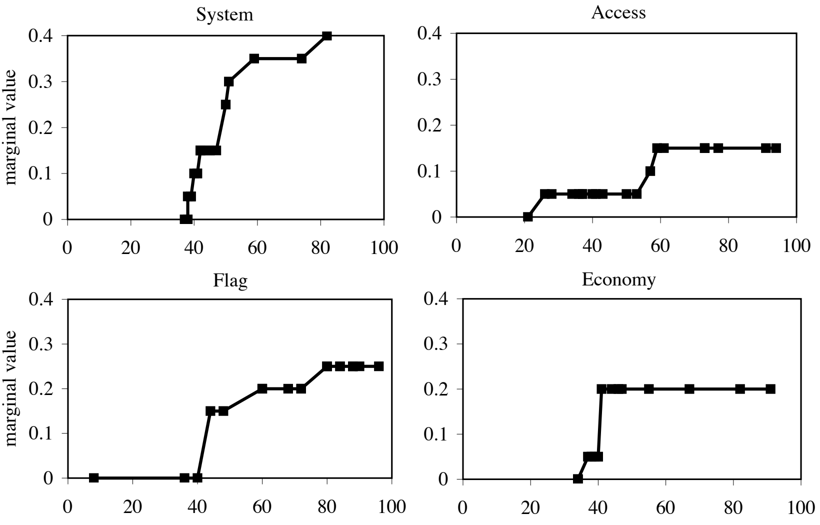
Selection of a representative value function

1. Require that for the representative value function U^R, $U^R(a) > U^R(b)$ if $PWI(a, b) > PWI(b, a)$

2. Maximize differences between comprehensive values of pairs of alternatives $(a, b) \in A \times A$ for which $PWI(a, b) > PWI(b, a)$

The inequality can also be defined w.r.t. a threshold (e.g. 0.02) to take into account imprecision caused by the estimation procedure

Representative value function



Extreme ranks and ranking with the representative VF

Country	U^R	P^*	P_*
GER	1.00 (1)	1	4
UK	0.95 (2)	1	2
SPA	0.90 (3)	3	5
SWE	0.75 (6)	4	6
NET	0.85 (4)	2	6
ITA	0.80 (5)	2	10
FIN	0.60 (9)	6	12
BEL	0.70 (7)	6	11
AUS	0.50 (11)	8	13
DEN	0.55 (10)	6	12
FRA	0.65 (8)	7	11
CZE	0.45 (12)	9	12
POR	0.40 (13)	10	14
SLO	0.25 (16)	13	19
IRE	0.35 (14)	14	18
HUN	0.30 (15)	14	17
EST	0.20 (17)	16	20
GRE	0.10 (19)	16	20
POL	0.15 (18)	15	19
SVK	0.05 (20)	16	20

- PWI(GER, a) $>$ PWI(a, GER) $\forall a \in A \setminus \{\text{GER}\}$
- PWI(SVK, b) $<$ PWI(b, SVK) $\forall b \in A \setminus \{\text{SVK}\}$

Decision aiding with stochastic ordinal regression

- What is always and sometimes true: necessary and possible preference relations
- Enrich the necessary relation with the help of pair-wise outranking/winning indices (new preference statements)
- Final ranking: know what is possible through extreme ranking analysis
- Rank distributions with rank acceptability indices
- Representative value function for the DM to better understand the results (and to provide a complete pre-order)

Genericity of the model

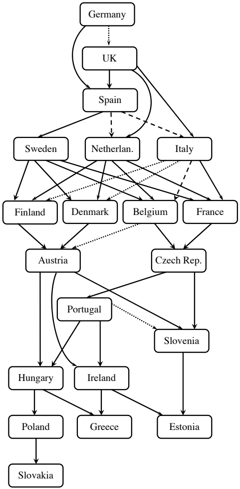
- We applied general monotone partial value functions
- Traditional UTA applies one- or two-piece linear ones

$$\mathcal{U}_1^{A^R} \subseteq \mathcal{U}_2^{A^R} \subseteq \mathcal{U}_G^{A^R}$$

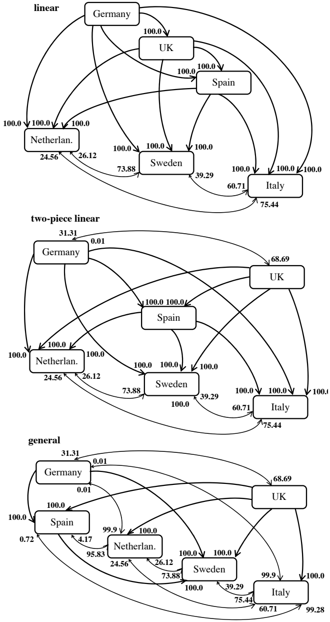
- The most flexible model gives the most general results, and the smallest necessary relation

$$\mathcal{U}_2^{A^R} \subseteq \mathcal{U}_G^{A^R} \Rightarrow \succ_2^N \supseteq \succ_G^N$$

Necessary relation
for different types of marginal value functions



Pair-wise outranking indices for a subset of alternatives
for different types of marginal value functions

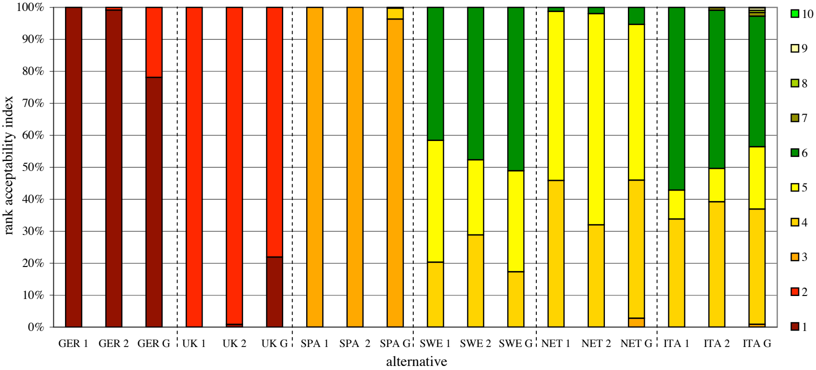


- Solid = General, 2-piece, linear
- Dashed = 2-piece, linear
- Dots = linear
- Italy vs Sweden

Extreme ranking analysis with different type VFs

Country	P_1^*	$P_{*,1}$	P_2^*	$P_{*,2}$	P_G^*	$P_{*,G}$
GER	1	1	1	2	1	4
UK	2	2	1	2	1	2
SPA	3	3	3	3	3	5
SWE	4	6	4	6	4	6
NET	4	6	4	6	2	6
ITA	4	6	4	9	2	10
FIN	7	12	6	12	6	12
BEL	7	8	7	10	6	11
AUS	10	13	8	13	8	13
DEN	7	12	6	12	6	12
FRA	7	11	7	11	7	11
CZE	9	12	9	12	9	12
POR	10	13	10	14	10	14
SLO	14	19	13	19	13	19
IRE	14	18	14	18	14	18
HUN	14	17	14	17	14	17
EST	16	20	16	20	16	20
GRE	16	20	16	20	16	20
POL	15	19	15	19	15	19
SVK	16	20	16	20	16	20

Rank acceptabilities with different type VFs



Conclusions

- We combined ROR, ERA, and SMAA under a unified decision support framework
- We looked at how the different approaches complement each other (without sacrificing model genericity)
- ROR approach to SMAA-2 enables practical use of general monotone value functions

Connection to other presentations in this stream

- We assumed preference model space shares to represent probabilities. This might not be a very good assumption (van Valkenhoef & Tervonen)

- As the general model is too flexible, how to choose another one? (Slowinski, Greco & Mousseau)

- How to work efficiently with sets of non-linear value functions? (Viappiani, Perny & Gonzales)

- What if the problem is larger? (many presentations)

Thanks for your attention!

M. Kadziński and T. Tervonen, Robust multi-criteria ranking with additive value models and holistic pair-wise preference statements, European Journal of Operational Research, 228(1):169–180, 2013