On the estimation of the parameters of Electre Tri model in multi criteria ordinal sorting problem: a proposal of new approach in two phases

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The grouping problems may be divided into:

-- groups defined \emph{a posteriori} (clusters)

-- groups defined \emph{a priori} without preference info (classes)

-- groups defined \emph{a priori} with preference info (categories)

\begin{itemize}
  \item \textbf{Multiple Criteria Sorting Problem (MCSP)}
  
  Real world case studies of MCSP have been reported in the literature in various domains:
  
  \begin{itemize}
    \item evaluation of applicants for loans or grants ([Groleau \textit{et al.} 95], [Veilleux \textit{et al.} 96] and [Greco \textit{et al.} 98b])
    \item business failure risk assessment ([Dimitras \textit{et al.} 95], et al.)
    \item screening methods prior to project selection ([Anandalingam \textit{et al.} 89]),
    \item satellite shot planning ([Gabrel 94]),
    \item medical diagnosis ([Slowinski 92], [Tanaka \textit{et al.} 92]),
    \item transportation system
  \end{itemize}

  \begin{itemize}
    \item Marie Sawadogo, Didier Anciaux: Intermodal transportation within the green supply chain: an approach based on the Electre method - Laboratoire de Genie Industriel et de Production de Metz Universite Paul Verlaine, Ile du Saulcy, Metz Cedex 1, France
  \end{itemize}
\end{itemize}

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\begin{itemize}
  \item \textbf{ELECTRE TRI method} (implemented in R 2.13)
\end{itemize}
Given a finite set of alternatives $A = \{a_1, a_2, \ldots, a_n\}$ the ordinal sorting problem consists in assigning each alternative to one of the $p$ \textit{a priori} known categories, defined in ordinal way, such that $C_{h+1} > C_h$. They are limited by category tipical elements ('profiles') which play the role of reference central points (\textit{reference points}) between categories.

The assignment of an alternative $a$ to one category results from the comparison of $a$ with the profiles $b_1, b_2, \ldots, b_{p-1}$ defining the limits of the categories.

\textbf{Aim}: construct the relation $S$: “$a$ outranks $b_h$” $\iff$ “$a$ is at least as good as $b_h$” $\iff$ $aSb_h$
How to construct the outranking relation $S \ [2]$. 

Electre Tri is pseudo-criteria based model.

Partial concordance index expresses how many a criterion $g_i$ is agreed with the statement ' $a$ is at least as good as $b_h$ ' ($aSb_h$ ).

$$c_i(a, b_h) = \begin{cases} 
0 & \frac{a - (b_h - p_h)}{p_h - q_h} \\
1 & 1 \\
0 & \frac{b_h + p_h - a}{p_h - q_h} 
\end{cases} = c_i(b_h, a)$$

$$c(a, b_h) = \frac{\sum_{j=1}^{m} w_j \cdot c_j(a, b_h)}{\sum_{j=1}^{m} w_j}$$

global concordance index
Credibility Index $\sigma(a, b_h)$ provides the degree of credibility of the outranking relation $S$.

It is the synthesis of concordance and discordance indices, i.e. it corresponds to the global concordance index weakened by eventual veto effects.

W.A.L.G we assume no veto situations and then $\sigma(a, b_h) = c(a, b_h)$

from $\sigma(a, b_h)$ - fuzzy - [exploitation procedure] defuzzied using cutting level $\lambda \in [0,1]$ $\rightarrow aSb_h$ - crisp -

\[
\sigma(a, b_h) \begin{cases} 
\geq \lambda & \rightarrow aSb_h \\
< \lambda & \rightarrow \neg aSb_h
\end{cases}
\]

\[
\sigma(b_h, a) \begin{cases} 
\geq \lambda & \rightarrow b_hSa \\
< \lambda & \rightarrow \neg b_hSa
\end{cases}
\]

Assignment procedures:

**pessimistic procedure**: compare an alternative successively to profile $b_h$, $h = p - 1, p - 2, ..., 1$

$b_h$ being the first profile such that $aSb_h$, then stop: $a \rightarrow C_{h+1}$

\[
\begin{cases}
\sigma(a, b_h) < \lambda & \Leftrightarrow \neg aSb_h \\
\sigma(a, b_{h-1}) \geq \lambda & \Leftrightarrow a \rightarrow C_h
\end{cases}
\]

**optimistic procedure**: compare an alternative successively to profile $b_h$, $h = 1, 2, ..., p - 1$

$b_h$ being the first profile such that $b_h \succ a$, then stop: $a \rightarrow C_h$

\[
\begin{cases}
\sigma(a, b_h) < \lambda & \Leftrightarrow \neg aSb_h \\
\sigma(b_h, a) \geq \lambda & \Leftrightarrow a \rightarrow C_h
\end{cases}
\]
The Electre Tri model parameters are:

-- profiles \(b_h, \ h = 1, \ldots, p - 1\)

-- preference, indifference, veto thresholds \((p_h, q_h, v_h)\)

-- weights \(w_j, \ j = 1, \ldots, m\)

-- cutting level \(\lambda \in [0,1]\)

Mousseau & Slowinski [5] proposed a methodology in order to infer indirectly (rather than directly) the model's parameters through a certain form of regression on Assignment Examples (A.E.) which represent *olistic information* provided by DM. Inferring a form of knowledge from examples of DM's decisions is a typical approach of *artificial intelligence*.

**Aim**: to find model's parameters as compatible as possible with A.E. provided; in order to minimize the difference between Electre assignments and those given by DM, an optimization procedure is used (without veto situations)

\[ a \rightarrow C_h \Rightarrow \begin{cases} 
\sigma(a, b_h) < \lambda \\
\sigma(a, b_{h-1}) \geq \lambda 
\end{cases} \]

It's realistic to assume that the DM prefers to provide some A.E. rather than to fix directly the used parameters values.
NLP problem to estimate profiles, thresholds (no veto), weights, $\lambda$-cut

$$\text{max } z = \alpha + \varepsilon \sum_{a_k \in A^*} (x_k + y_k) \quad \text{where } \alpha = \min_{a_k \in A^*} \{x_k, y_k\} \text{ and } \varepsilon > 0 \text{ small}$$

$$g_j (b_{h+1}) \geq g_j (b_h) + p_j (b_h) + p_j (b_{h+1}) \quad j = 1, \ldots, m \quad h = 1, \ldots, p - 1$$

$$p_j (b_h) \geq q_j (b_h) \quad j = 1, \ldots, m \quad h = 1, \ldots, p$$

$$\lambda \in [0.5, 1]$$

$$\alpha \leq x_k \quad \forall a_k \in A^*$$

$$\alpha \leq y_k \quad \forall a_k \in A^*$$

$$w_j \geq 0 \quad j = 1, \ldots, m$$

$$q_j (b_h) \geq 0 \quad j = 1, \ldots, m \quad h = 1, \ldots, p$$

$$\sum_{j=1}^{m} w_j \cdot c_j (a_k, b_{h-1})$$

\[
\frac{\sum_{j=1}^{m} w_j \cdot c_j (a_k, b_{h-1})}{\sum_{j=1}^{m} w_j} - x_k = \lambda \quad \forall a_k \in A^* \]

\[
\frac{\sum_{j=1}^{m} w_j \cdot c_j (a_k, b_{h-1})}{\sum_{j=1}^{m} w_j} + y_k = \lambda \quad \forall a_k \in A^* \]

Variables $= 2n + 1 + m + 3mp$

Constraints $= 4n + 3mp + 2$


Suggestions [5]: in the case of large problems, the use of metaheuristics in particular genetic algorithms are recommended

Not differentiable

Not convex

Variables $n = \# \text{ASS.E.}$

$\# \text{criteria} m$

$\# \text{profiles} p$
Illustrative example [5]

<table>
<thead>
<tr>
<th></th>
<th>(g_1)</th>
<th>(g_2)</th>
<th>(g_3)</th>
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<tbody>
<tr>
<td>(a_1)</td>
<td>70</td>
<td>64.75</td>
<td>46.25</td>
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<tr>
<td>(a_2)</td>
<td>61</td>
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<tr>
<td>(a_6)</td>
<td>15</td>
<td>15</td>
<td>30</td>
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</table>

ASS.E.

\[ a_1 \rightarrow C_3 \]
\[ a_2 \rightarrow C_3 \]
\[ a_3 \rightarrow C_2 \]
\[ a_4 \rightarrow C_2 \]
\[ a_5 \rightarrow C_1 \]
\[ a_6 \rightarrow C_1 \]

Assignment Examples set (Training set)

Starting Point:

\[ g_j (b_h) = \frac{1}{2} \left( \sum_{a_i \rightarrow C_h} g_j(a_i) \frac{n_h}{n_{h+1}} + \sum_{a_i \rightarrow C_{h+1}} g_j(a_i) \frac{n_{h+1}}{n_h} \right) \]

Heuristic Rule

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</tr>
<tr>
<td>(q_1)</td>
<td>1.762</td>
<td>1.563</td>
<td>1.376</td>
</tr>
<tr>
<td>(q_2)</td>
<td>2.960</td>
<td>2.710</td>
<td>2.080</td>
</tr>
<tr>
<td>(p_1)</td>
<td>3.525</td>
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</tr>
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<td>(p_2)</td>
<td>5.925</td>
<td>5.419</td>
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\(w_j\)

\[ w_j = 0.517 \]
\[ 1.000 \]
\[ 0.483 \]

\[ \lambda = 0.629 \]
\[ \alpha = 0.37 \]

NLP

\[ b_1 \]
\[ 35.25 \]
\[ 31.25 \]
\[ 23.65 \]

\[ b_2 \]
\[ 59.25 \]
\[ 62.75 \]
\[ 41.60 \]

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\[ w_j \]

\[ 0.517 \]
\[ 1.000 \]
\[ 0.483 \]

\[ \lambda = 0.629 \]
\[ \alpha = 0.37 \]

\[ q_j (b_h) = 0.05 \cdot g_j (b_h) \]
\[ p_j (b_h) = 0.10 \cdot g_j (b_h) \]

\[ w_j = 1 \quad j = 1, 2, 3 \]

\[ \lambda = 0.75 \]
\[ a_4 \rightarrow C_1 \]
\[
q_j(b_h) = 0.05 \cdot g_j(b_h) \quad W.A.L.G.: \text{veto thresholds are not considered}
\]
\[
p_j(b_h) = 0.10 \cdot g_j(b_h)
\]
\[
v_j(b_h) = 0.20 \cdot g_j(b_h) \quad \sigma(a, b_h) = c(a, b_h) = \frac{\sum_{j=1}^{m} w_j \cdot c_j(a, b_h)}{\sum_{j=1}^{m} w_j}
\]

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<td>(c_j(a_1, b_2))</td>
<td>1</td>
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\[
\left\{ \begin{array}{l}
\frac{w_1}{w_1 + w_2 + w_3} < \lambda \\
\frac{w_3}{w_1 + w_2 + w_3} < \lambda
\end{array} \right\} \Rightarrow \lambda > \max \left\{ \begin{array}{l}
\frac{w_1}{w_1 + w_2 + w_3}, \frac{w_3}{w_1 + w_2 + w_3}
\end{array} \right\}
\]

If \(w_3 < w_1\) \quad \frac{w_1}{w_1 + w_2 + w_3} < \lambda < 1; \quad \text{if instead} \quad w_3 > w_1 \quad \frac{w_3}{w_1 + w_2 + w_3} < \lambda < 1
Illustrative example [5]

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**ASS.E.**

- $a_1 \rightarrow C_3$
- $a_2 \rightarrow C_3$
- $a_3 \rightarrow C_2$
- $a_4 \rightarrow C_2$
- $a_5 \rightarrow C_1$
- $a_6 \rightarrow C_1$

Assignment Examples set (Training set)

**Starting Point:**

$$g_j(b_h) = \frac{1}{2} \left\{ \frac{\sum_{a_i \rightarrow C_h} g_j(a_i)}{n_h} + \frac{\sum_{a_i \rightarrow C_{h+1}} g_j(a_i)}{n_{h+1}} \right\}$$

Heuristic Rule

NLP

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$\lambda = 0.629$

$\alpha = 0.37$

$q_j(b_h) = 0.05 \cdot g_j(b_h)$  
$p_j(b_h) = 0.10 \cdot g_j(b_h)$

$w_j \equiv 1  \quad j = 1, 2, 3 \quad \Rightarrow 0.33 < \lambda \leq 0.67  \quad a_4 \rightarrow C_2'$

$\lambda \approx 0.75$
How to estimate profiles? 2 ideas ...

--- Heuristic Rule ---

Centroid of category $C_1$ on criterion $g_2$

Centroid of category $C_3$ on criterion $g_3$

--- Equal Size Intervals ---

\[ g_i^j = g_{i*} + \frac{j-1}{\alpha_i-1} (g_{i*}^j - g_{i*}) \]
Linear Programming (LP) problem to estimate profiles [10]

\[ \min \ z = \sum_{j=1}^{m} \sum_{a_k \to C_h} \mathcal{G}_j(a_k) \quad \text{where} \quad \mathcal{G}_j(a_k) = \begin{cases} 0 & \text{se } g_j(a_k) \in [g_j(b_{h-1}); g_j(b_h)] \\ \text{diff}_j(a_k) & \text{se } g_j(a_k) \notin [g_j(b_{h-1}); g_j(b_h)] \end{cases} \]

\[ g_j(b_h) > g_j(b_{h-1}) \quad \forall j = 1, ..., m, \forall h = 2, ..., p \]
\[ \mathcal{G}_j(a_k) \geq 0 \quad \forall j = 1, ..., m, \forall a_k \to C_h \]

\[ \min \ z = \sum_{j=1}^{m} \sum_{a_k \to C_h} \mathcal{G}_j(a_k) \quad \text{subject to} \]

\[ \mathcal{G}_j(a_k) \geq g_j(a_k) - g_j(b_h) \quad \forall j = 1, ..., m, \forall a_k \to C_h \quad (1) \]
\[ \mathcal{G}_j(a_k) \geq g_j(b_{h-1}) - g_j(a_k) \quad \forall j = 1, ..., m, \forall a_k \to C_h \quad (2) \]
\[ g_j(b_h) > g_j(b_{h-1}) \quad \forall j = 1, ..., m, \forall h = 2, ..., p - 1 \quad (3) \]
\[ \mathcal{G}_j(a_k) \geq 0 \quad \forall j = 1, ..., m, \forall a_k \to C_h \quad (4) \]

Note that \( z^* = \min (z) = 0 \) iff
\[ \mathcal{G}_j(a_k) = 0 \quad \forall j = 1, ..., m, \forall a_k \to C_h \quad \text{iff} \]
\[ g_j(a_k) \in [g_j(b_{h-1}); g_j(b_h)] \quad \forall j = 1, ..., m, \forall a_k \to C_h \]

We assume that \( b_0 \to -\infty, b_p \to +\infty \)
Application of the LP problem to the illustrative example by [5]

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<td>( a_2 \rightarrow C_3 )</td>
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<tr>
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<td>( a_6 )</td>
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weights

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<td>( b_1 ) = (20 40 30)'</td>
<td>( b_2 ) = (66 50 37)'</td>
<td></td>
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<td></td>
</tr>
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<td>( b_1 ) = (20 20 23.125)'</td>
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SAS

\( b_1 = (20 40 30)' \)
\( b_2 = (66 50 37)' \)

R

\( b_1 = (20 20 30)' \)
\( b_2 = (66 50 37)' \)

MPL

\( b_1 = (20 20 23.125)' \)
\( b_2 = (61 50 37.000)' \)

\( z^* = 11.875 \)

\( (5,4,3) \)
\( 0.583 < \lambda \leq 0.75 \)
0 misclass

\( (4,5,3) \)
\( 0.667 < \lambda \leq 0.75 \)
0 misclass

\( (3,4,5) \)
\( 0.75 < \lambda \leq 0.8712 \)
1 misclass

\( (0.5 0.3 0.2) \)
\( 0.5 < \lambda \leq 0.7424 \)
0 misclass

\( 0.75 < \lambda \leq 0.7854 \)
1 misclass

\( 0.75 < \lambda \leq 0.8283 \)
1 misclass

\( 0.75 < \lambda \leq 0.8712 \)
1 misclass

\( 0.75 < \lambda \leq 0.7424 \)
1 misclass

\( \lambda > 0.75 \)
0 misclass

\( \lambda > 0.75 \)
0 misclass

\( \lambda > 0.80 \)
0 misclass

\( 0.75 < \lambda \leq 0.7854 \)
1 misclass

\( \lambda > 0.75 \)
0 misclass

\( \lambda > 0.75 \)
0 misclass

\( \lambda > 0.80 \)
0 misclass
Remark that \( g_j(a_k) = 0 \iff g_j(a_k) \in [g_j(b_{h-1}); g_j(b_h)] \quad \forall j = 1, \ldots, m, \forall a_k \rightarrow C_h \)

on \( g_1 \)
\[
\begin{align*}
g_1(b_2) &\leq 70 \\
g_1(b_2) &\leq 61 \\
g_1(b_1) &\leq 40 \leq g_1(b_2) \\
g_1(b_1) &\leq 66 \leq g_1(b_2) \\
g_1(b_1) &\geq 20 \\
g_1(b_1) &\geq 15
\end{align*}
\]

on \( g_2 \)
\[
\begin{align*}
g_2(b_2) &\leq 50 \\
g_2(b_1) &\leq 64.75
\end{align*}
\]

on \( g_3 \)
\[
\begin{align*}
g_3(b_2) &\leq 60 \\
g_3(b_1) &\leq 37
\end{align*}
\]
Heuristic Rule

Non Linear Programming

\[ b_1 = (35.25 \ 31.25 \ 23.65) \]
\[ b_2 = (59.25 \ 62.75 \ 41.6) \]

\[ z(b_1, b_2) = 13.625 \]

Linear Programming

\[ z^* = \min (z) = 11.875 \]

\[ b_1 = (20 \ 40 \ 30)' \]
\[ b_2 = (66 \ 50 \ 37)' \]
(by SAS)

\[ b_1 = (20 \ 20 \ 30)' \]
\[ b_2 = (66 \ 50 \ 37)' \]
(by R)

\[ b_1 = (20 \ 20 \ 23.125)' \]
\[ b_2 = (61 \ 50 \ 37.000)' \]
(by MPL)

Equal Size Intervals

\[ b_1 = (33.33 \ 31.58 \ 33.33)' \]
\[ b_2 = (51.67 \ 48.167 \ 46.67)' \]

\[ z(b_1, b_2) = 26.79166 \]
I phase: inference on profiles (LP problem / other procedures)
II phase: inference on weights and cutting level (system of non linear inequalities)

\[ b_1 = (35.25, 31.25, 27.50)' \]

\[ b_2 = (59.25, 54.19, 41.60)' \]

**Heuristic Rule**

plot of lambda as a function of misclassified

if \( w = (3 \ 2 \ 1) \)

plot of lambda as a function of misclassified

if \( w = (1 \ 1 \ 1) \)

\[ 0.2 \ 0.4 \ 0.6 \ 0.8 \ 1.0 \]

\[ 0.0 \ 0.5 \ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \]

misclassified

\[ 0.5 \ 0.83 \]

\[ 0.33 \ 0.67 \]

\[ 0.75 \]
• choose in case of multiple optimal solutions: use of evolutionary algorithms
• profiles as stochastic process of random variables (probabilistic approach)
• inconsistent system of non-linear/linear inequalities
• additional constraints to be added

Work in progress

Thanks for your attention!!
References


[9] Vincent Mousseau, Luís C. Dias, José Figueira : Dealing with inconsistent judgments in multiple criteria sorting models - August 31, 2004