Newsletter of the European Working Group "Multicriteria Aid for Decisions"
Bulletin du Groupe de Travail Européen "Aide Multicritère à la Décision"

Groupe de Travail Européen "Aide Multicritère à la Décision"
Série 3, nº18, automne 2008.

European Working Group "Multiple Criteria Decision Aiding"
Series 3, nº 18, Fall 2008.

Software

GRIP: an MCDA method using a set of additive value functions representing a reference preorder and intensities of preference

by

Piotr Zielniewicz[1], José Rui Figueira[2,5], Salvatore Greco[3], and Roman Słowiński [1,4]

[1] Poznan University of Technology, Poland
[2] Technical University of Lisbon, Portugal
[3] University of Catania, Italy
[4] Systems Research Institute, Polish Academy of Sciences, Poland
[5]Associate Research to LAMSADE Univesrty Paris-Dauphine

1. Introduction

In this short paper, we provide a very short presentation of a method called GRIP (Generalized Regression with Intensities of Preference) for ranking a finite set of actions evaluated on multiple criteria (Figueira et al., 2009), along with its software implementation. GRIP builds a set of additive value functions compatible with preference information composed of a partial preorder and required intensities of preference on a subset of actions, called reference actions.

It constructs not only some specific preference relations in the considered set of actions, but it also gives information about intensities of preference for pairs of actions from this set for a given decision maker (DM). The basic concepts of GRIP are the *necessary preference* and *possible preference* (Greco et al., 2008). An alternative *a* is necessarily preferred to another alternative *b*, if *a* is preferred to *b* for all additive value functions compatible with the preferences expressed by the DM on a set of referencs actions. An alternative *a* is possibly preferred to another alternative *b*, if *a* is preferred to *b* for at least one additive value function compatible with the preferences expressed by the DM on a set of referencs actions. The necessary preference relation is a partial pre-order (reflexive and transitive binary relation), and the possible preference relation is a strongly complete and negatively transitive binary relation. Necessary and possible relations between intensities of preference are analogously determined with respect to the whole set of criteria or specific single criteria. Distinguishing necessary and possible consequences of preference information on the considered set of actions, GRIP answers questions of robustness analysis (Greco et al., 2008).

The proposed methodology can be seen as an extension of the UTA method based on ordinal regression (Jacquet-Lagrèze and Siskos, 1982, Siskos et al., 2005). GRIP can also be compared to the AHP method (Saaty, 2005), which requires pairwise comparison of all actions and criteria, and yields a priority ranking of actions. As for the preference information being used, GRIP can be compared, moreover, to the MACBETH method (Bana et Costa et al., 2005) which also takes into account a preference order of actions and intensity of preference for pairs of actions.

The preference information used in GRIP does not need, however, to be complete, i.e. the DM is not required to give a complete order, from the best to the worst, of the reference actions. Instead the DM is asked to provide comparisons of only those pairs of reference actions on particular criteria for which his/her judgment is sufficiently certain. This is an important advantage comparing to methods, which, instead, require comparison of all possible pairs of actions on all the considered criteria. Moreover, GRIP works with a set of general additive value functions compatible with the preference information, while other methods use a single and less general value function, such as the weighted sum.

2. GRIP Decision Support Process

GRIP decision support process is composed of five main levels shown in Fig. 1 (see also Figueira et al., 2009):

- Level 1 concerns the input data, i.e., the consistent family of criteria F, and the set of actions *A*. In addition to the actions to be ranked by GRIP, set *A* can also contain some fictitious, past or other auxiliary actions, which will enter the set of reference actions A^R in order to facilitate elicitation of preference information by the DM.

- Level 2 is related to the preference information provided by the DM. The set of reference or training actions A^R is defined with the help of the DM. The major piece of information provided by the DM is a partial preorder on A^R, which is composed of holistic pairwise comparisons of actions from A^R, and holistic and/or partial preference information on intensities of preferences for some pairs of actions from A^R. It is worth noting that GRIP can easily handle other kinds of preference information, like local tradeoffs.

- In Level 3, the preference information provided by the DM is formally represented by a set of linear constraints.

- Level 4 concerns the computation phase, where the procedure should check for the existence of at least one value function compatible with the preference information provided by the DM. If there is no such a value function, then the DM is supported to revise his/her preference information.

- When, the preference information is consistent, i.e., there exists at least one value function compatible with such information, in Level 5, the method is producing the following output results:
  1. The necessary preference relation on the set of all the actions in A.
  2. The possible preference relation on the set of all the actions in A.
  3. The necessary relation related to comparison of comprehensive intensities of preferences between pairs of actions in $A \times A$.
  4. The possible relation related to comparison of comprehensive intensities of preferences between pairs of actions in $A \times A$.
  5. The necessary relation related to comparison of intensities of preferences with respect to partial (on each criterion) between pairs of actions in $A \times A$.
  6. The possible relation related to comparison of intensities of preferences with respect to partial (on each criterion) between pairs of actions in $A \times A$.

Of course in practice, there is no need to compute all the above results. Indeed, the most useful output information is provided by the necessary and possible preferences. Other results can be computed on request concerning particular couples of pairs of actions. If the DM feels comfortable and agrees on the conclusions, GRIP stops; otherwise, preference information should be augmented or revised, or the input data should be revised.

Recently, a methodology to identify the “most representative” value function in GRIP has been proposed by Figueira et al. (2008d), without loosing the advantage of taking into account all compatible value functions. This function is also implemented in the GRIP software. The idea is to select among all compatible value functions that one value function which better highlights the necessary preference, by maximizing the difference of evaluations between actions for which there is a necessary preference. As secondary objective, one can consider minimizing the difference of evaluations between actions for which there is not necessary preference.

GRIP is based on the robust ordinal regression paradigm (Greco et al., 2008b) and has also been applied within interactive multiobjective optimization procedure (Figueira et al., 2009)

The GRIP interaction scheme generalizes the UTA method (Siskos et al., 2005), the UTA^{GMS} method (Greco et al., 2008a) and, in a certain sense, the MACBETH method (Bana et Costa et al., 2005). Indeed, in case of using only the information on the intensities of preferences and checking if there exists a compatible additive value function, we obtain similar results to MACBETH. We do not need, however, to determine the weights, as MACBETH does, and the DM does not need, moreover, to define “good” and “neutral” levels on each criterion, as it it the case in MACBETH.

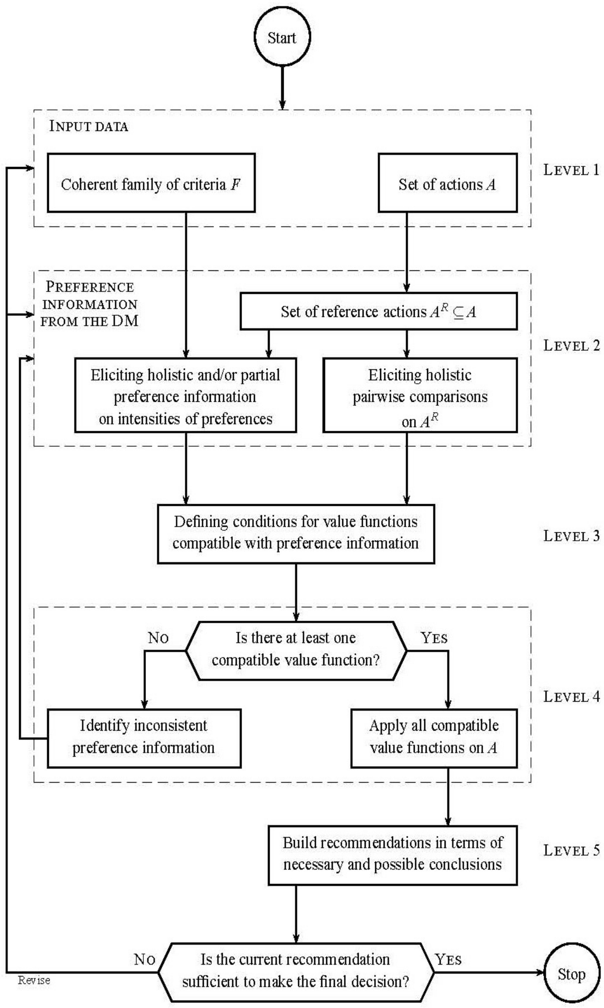

Figure 1: GRIP decision support process

4. Software

Short description of the D2 GRIP plugin.

The GRIP software is written in the Java language as a plugin to the Decision-Deck (D2) platform. It uses GLPK (*GNU Linear Programming Kit*) solver to conclude the truth or falsity of preference relations, the JGraph (*Java

Graph visualization library) to visualize ranking of actions and JFreeChart (*Java Chart library*) to visualize representative utility function and marginal utilities.

Illustrative example.

In the following didactic example, we shall simulate an interaction with a fictitious DM to illustrate the type of interaction proposed in the D2 GRIP plugin.

We consider a problem of ranking 7 students evaluated by a set of 3 criteria (to be maximized). The performances of the students are presented in Table 1.

student	mathematics	physics	literature
s_1	medium	medium	good
s_2	good	good	medium
s_3	medium	good	medium
s_4	medium	medium	medium
s_5	good	good	bad
s_6	medium	bad	good
s_7	bad	medium	good

Table 1. Performance matrix of the set of students

Let as suppose that the DM has chosen the following set of reference actions A^R (see Figure 2):

$$A^R = \{s_1, s_2, s_4, s_5, s_6, s_7\}.$$

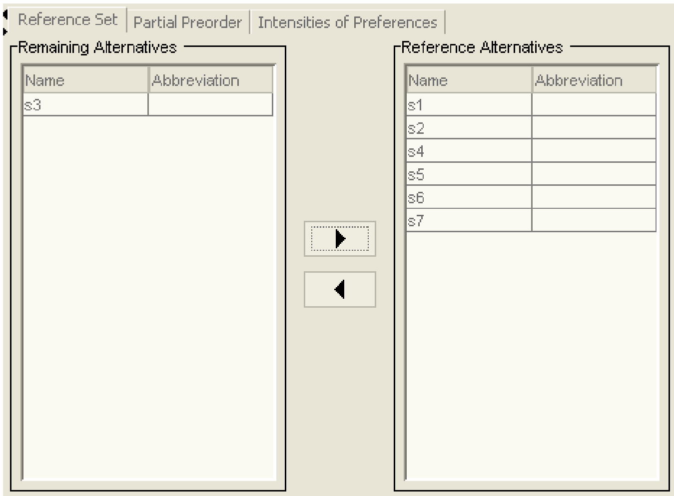

Figure 2: The set of reference actions tab window

Then, we suppose that the DM expresses preference information in terms of pairwise comparisons of actions from A^R (Figure 3) and intensities of preferences (Figures 4 and 5). Each of those windows used to define preference information is composed of two parts: the right panel presents preference information already defined, the left panel presents additional information (i.e. evaluation of performances, comparison of selected actions) helpful to the DM.

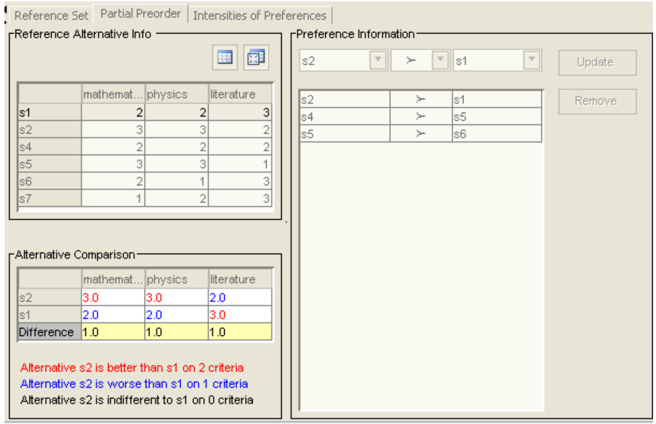

Figure 3. Partial pre-order tab window

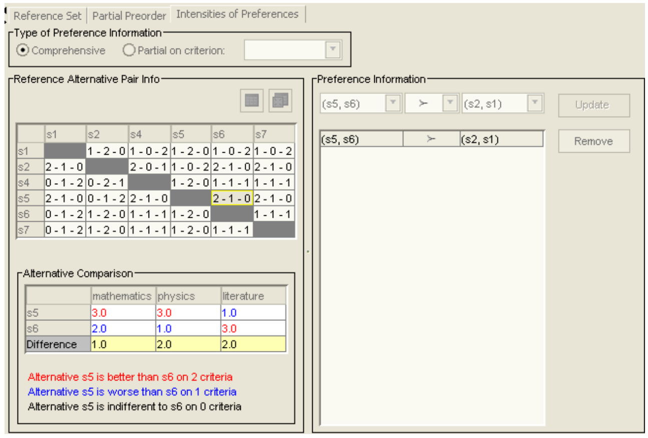

Figure 4. Comprehensive Intensities of Preferences tab window

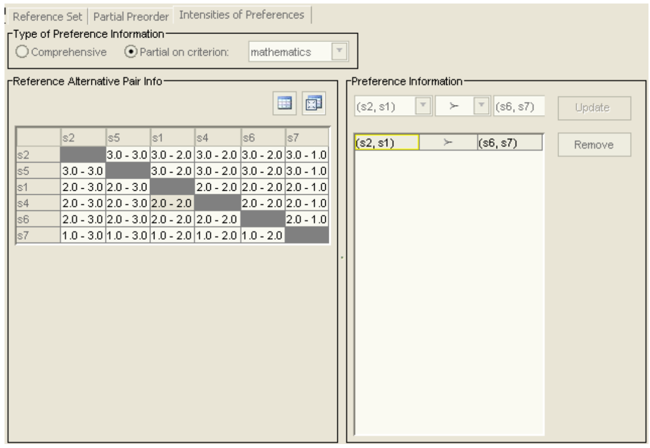

Figure 5. Partial Intensities of Preferences tab window

The preference information shown in Figures 3, 4 and 5 corresponds to the following GRIP constraints:

$U(s_2) > U(s_1)$ $U(s_4) > U(s_5)$ $U(s_5) > U(s_6)$
$U(s_5) - U(s_6) > U(s_2) - U(s_1)$ $U_{mathematics}(s_2) - U_{mathematics}(s_1) > U_{mathematic}(s_6) - U_{mathematics}(s_7)$

Considering the provided preference information, we can compute the necessary and possible preference and the necessary and possible relations with respect to comparison of intensities of preference in the whole set of actions. Moreover, we can compute the ranking (being a complete pre-order) given by the most representative value function.

The obtained necessary preference relations can be presented in two forms: as in table of Figure 6, or as in the graph of Figure 7. In this graph, blue nodes correspond to reference actions, actions aggregated in light gray boxes are indifferent, blue edges correspond to pairwise comparisons of reference actions, and black edges mark necessary preference relations.

Necessary Ranking Graph | Representative Ranking | Marginal Utilities
Dominance Relation | Possible Preference Relation | Necessary Preference Relation

	s1	s2	s3	s4	s5	s6	s7
s1	True	False	False	True	False	True	True
s2	True	True	True	True	True	True	True
s3	False	False	True	True	True	True	False
s4	False	False	False	True	True	True	False
s5	False	False	False	False	True	True	False
s6	False	False	False	False	False	True	False
s7	False	False	False	False	False	True	True

Figure 6. Necessary Preference Relation tab window

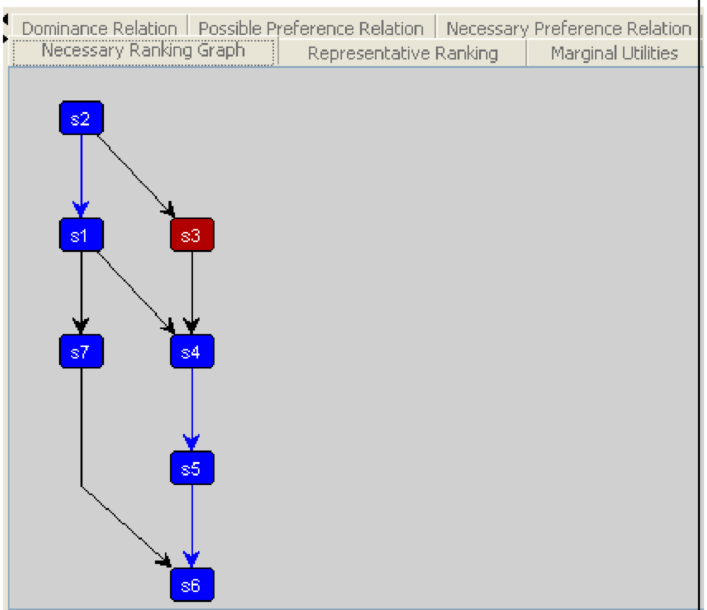

Figure 7. Necessary Relation Graph tab window after the first iteration

Figure 8 presents the complete ranking of actions by a “representative value function”, and Figure 9 shows its marginal components.

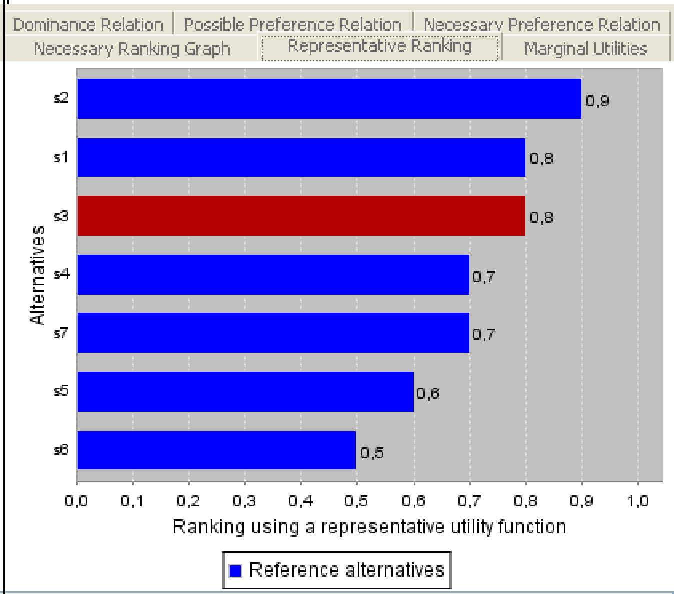

Figure 8. Representative Ranking tab window

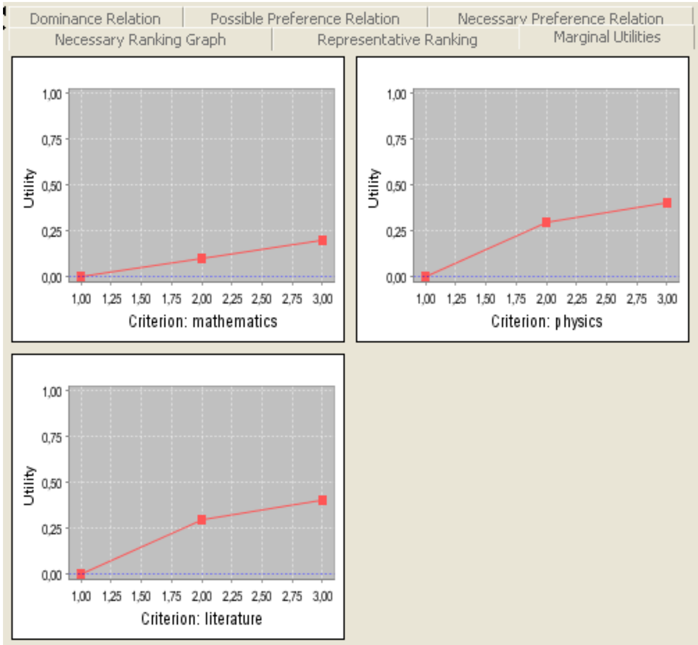

Figure 9. Representative Marginal Value Functions tab window

Let us suppose that in the next iteration the DM adds the following preference information: $s_5 \succ s_7$.

Figure 10 presents the obtained necessary preference relation in the graph form. In this graph, dashed edges mark the differences between the current relation graph and the previous one.

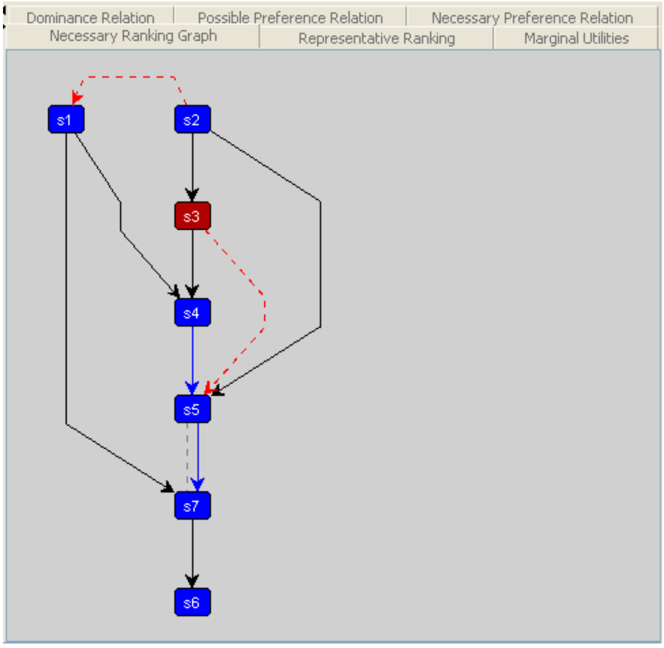

Figure 10. Necessary Relation Graph tab window after the second iteration

If the DM is not convinced by the obtained results (because, for instance, he matures the conclusion that a given alternative *a* is preferred to another alternative *b*, but *a* does not result necessarily preferred to *b*), the DM can introduce new preference information and/or can modify the previous preference information and proceed to a new application of GRIP.

References

Bana e Costa, C. A., J.M. De Corte, Vansnick, J. C., 2005. On the Mathematical Foundation of MACBETH, in Figueira, J., Greco, S. and Ehrgott, M. (eds) *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer Science + Business Media, Inc., New York, 409-443.

Figueira, J., Greco, S., Mousseau, V., Słowiński, R., 2008a. Interactive Multiobjective Optimization using a Set of Additive Value Functions, chapter 4 in J.Branke, K.Deb, K.Miettinen, R.Słowiński (eds.), *Multiobjective Optimization: Interactive and Evolutionary Approaches*, LNCS 5252, Springer, Berlin, 97-120.

Figueira, J., Greco, S., Mousseau, V., Słowiński, R., 2008b. *UTA^{GMS} and GRIP methodology for multiple criteria decision problems*, presented at the 19th International Conference on Multiple Criteria Decision Making, Auckland, New Zealand, January 7-12, 2008.

Figueira, J., Greco, S., Słowiński, R., 2009. Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. *European Journal of Operational Research*, 195, 460-486.

Figueira, J., Greco, S., Słowiński, R., 2008d. *Identifying the “most representative” value function among all compatible value functions in the GRIP method*, presented at the 68th Meeting of the European Working Group on Multiple Criteria Decision Aiding, Chania, October 2-3, 2008.

Greco, S., Mousseau, V., Słowiński, R., 2008a. Ordinal regression revisited: multiple criteria ranking with a set of additive value functions. *European Journal of Operational Research*, 191, 415-435.

Greco, S., Słowiński, R., Figueira, J. Mousseau, V. 2008b. Robust Ordinal Regression, in: Ehrgott, M., Figueira, J., and Greco, S. (eds.), *New Trends in Multiple Criteria Decision Analysis*, Springer Science + Business Media, Inc., New York. Forthcoming.

Jacquet-Lagrèze E., Siskos J., 1982. Assessing a set of additive utility functions for multicriteria decision-making, the UTA Method. *European Journal of Operational Research*, 10 (2), 151-164.

Saaty, T.L., 2005. The Analytic Hierarchy and Analytic Network Processes for the Measurement of Intangible Criteria and for Decision-Making, in Figueira, J., Greco, S. and Ehrgott, M. (eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York, 345-407.

Siskos, Y., Grigoroudis, V., Matsatsinis, N., 2005. UTA methods, in Figueira, J., Greco, S. and Ehrgott, M. (eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York, 297-343.