How to assign numerical values to parameters aiming at differentiating the role of criteria in a comprehensive preference model ?

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Outline

- Different methods, different roles
- Given a method, the role is very complex
- No hope that DM understands this role
- Speaking in terms of parameters -> nonsense
- Speaking in terms of preferences makes sense

Different methods, different roles

Given a method, the role is very complex

Weighted sum

- $(x_1, \ldots, x_n) \succeq (y_1, \ldots, y_n)$ iff $\Sigma_i \, k_i \, x_i \geq \Sigma_i \, k_i \, y_i$
- The ratio k_i / k_j is a substitution rate
- One unit on criterion i is worth k_i / k_j units on criterion j

- The weights therefore depend on the units !

Weighted sum

	Cost (€)	Max speed (km/h)	Gas (l/100km)
a	18 000	165	6.2
b	21 000	185	7.5
c	17 000	170	6.0

- Ask weights.
  Answer : $(60, 10, 30)$
- Km/h → m/s. Ask weights.
  Answer : $(60, \underline{36}, 30)$??
- l/100km → l/m. Ask weights.
  Answer : $(60, 36, \underline{300\ 000})$??

Even the ordering changes.

Any chance that the DM anticipates this ?

MAVT (additive utility)

- $(x_1, \ldots, x_n) \succeq (y_1, \ldots, y_n)$ iff $\Sigma_i \; k_i \, u_i(x_i) \geq \Sigma_i \; k_i \, u_i(y_i)$

  with u_i normalized between 0 and 1.

- One unit of utility on criterion i is worth k_i / k_j units of utility on criterion j

- The weights depend on the units of utility

- The units for u_i depend on the range of criterion i

MAVT (additive utility)

	Cost (€)	Max speed (km/h)	Gas (l/100km)
a	18 000	165	6.2
b	21 000	185	7.5
c	17 000	170	6.0

- Ask weights.
- Answer : $(50, 20, 30)$

MAVT (additive utility)

	Cost (€)	Max speed (km/h)	Gas (l/100km)
a	18 000	165	6.2
b	21 000	185	7.5
c	17 000	170	6.0
d	20 500	180	9.0

- Ask weights.
- Answer : $(50, 20, 30)$
- New car. Range of gas consumption is doubled.

$$[6.0\ , 7.5] \rightarrow [6.0\ , 9.0]$$

MAVT (additive utility)

	Cost (€)	Max speed (km/h)	Gas (l/100km)
a	18 000	165	6.2
b	21 000	185	7.5
c	17 000	170	6.0
d	20 500	185	9.0

- Ask weights.
- Answer : $(50, 20, 30)$
- New car. Ask weights.
- Answer : $(50, 20, \underline{60})$??

Even the ordering changes.

Any chance that the DM anticipates this ?

Qualified Majority (Electre I w/o veto)

- $(x_1, \ldots, x_n) \succeq (y_1, \ldots, y_n)$ iff $\Sigma_{i\,:\,x_i \geq y_i}\ k_i \geq \delta$
- The units do not matter, the range does not matter
- DM: 1 more important than 2
  2 much more important than 3
- Analyst: $k = (.45, .40, .15)$. OK ?
- DM: OK !

Qualified Majority (Electre I w/o veto)

- $(x_1, \ldots, x_n) \succeq (y_1, \ldots, y_n)$ iff $\Sigma_{i\,:\,x_i \geq y_i}\, k_i \geq \delta$

  $k = (.45, .40, .15)$

- If $\delta = 0.5$, then all criteria play the same role.
- If $\delta = 0.6$, then 1 more important than 2 $(1 \succ 2)$

  2 as important as 3 $(2 \doteq 3)$
- If $\delta = 0.7$, then $1 \doteq 2 \succ 3$.
- There is no δ such that $1 \succ 2 \succ 3$.
- Indifference and veto thresholds make the situation more complex.

Any chance that the DM anticipates this ?

Speaking in terms of parameters -> nonsense

Speaking in terms of parameters -> nonsense

- The role of the parameters depends in a *very* complex way on
  - the method
  - the other parameters
  - the units, the range, …

Speaking in terms of parameters -> nonsense

- We cannot expect the DM to intuitively understand this when he answers questions like
  - What is the importance of criterion 1?
  - Is criterion 1 much more important than criterion 2 ?
  - Is criterion 1 more important than criterion 2 ?

- The answers of the DM are therefore bound to be void of meaning.

Speak in terms of preferences

- Questions like
  - Do you prefer $(3, 80, 17)$ to $(5, 80, 12)$?
  - Are you indifferent between $(3, 80, 17)$ and $(5, 80, 12)$?
  - How large must x_1 be so that $(x_1, 80, 17) \sim (5, 80, 12)$?

  have a clear meaning.

- The DM does not always know the answer
  He sometimes makes “mistakes”
- But the meaning of his answers is clear.