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Forum

(Robustness Analysis)

Robustness Analysis and MCDA

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The Editor has invited me to share a few thoughts on the topic of robustness analysis in MCDA. The first question is: Robustness to what? MCDA includes a comprehensive process involving a rich interplay between human judgement, data analysis and mathematical/computational processes. Errors and inadvertent biases can enter at any of these stages, and it is the process as a whole that needs to be robust.

Perhaps some of the key points at which such errors and biases may intrude would be the following:

- External uncertainties;
- Internal uncertainties;
- Choice of preference model;
- Identification of criteria and alternatives.

We discuss each of these in turn.

External uncertainties. In many senses these are the least problematical to the analyst. External random processes may pose challenging mathematical problems, but the uncertainties can typically be described in terms of probability distributions, even if the distributions may need to be assessed subjectively rather than empirically.

The question here, however, is that of robustness. The issues of external uncertainty were discussed at length in Stewart (2005a). There we recognized that in many cases, the probability distributions may not explicitly be incorporated into the decision model except in the case of multi-attribute utility theory. It follows therefore that robustness of solutions generated to external uncertainties needs to be incorporated in some way. Sensitivity analysis is always an option, but can be quite *ad hoc*, not covering all combinations of externalities.

It is possible to incorporate risk measures such as variance in the form of additional criteria in our preference modelling, but this too tends to involve one objective at a time, without recognition of covariances that may exist. In some (unpublished) simulation studies we have managed to demonstrate that the definition of a small number of "scenarios" involving simultaneous variations on a number of externalities can lead to much more robust solutions in value function approaches at least (and we conjecture that this conclusion applies to other methods of MCDA as well). Approaches to incorporation of such scenarios include rank ordering of alternatives in terms of each scenario (to find alternatives which are robustly good performers), or to view performance in terms of each objective under each scenario as metacriteria in their own right.

Internal uncertainties. These relate to the value or preference judgements provided by the decision makers, such as weights, tradeoffs, goals, indifference thresholds, etc. It is a feature of life that such inputs are neither precise nor consistent (in the sense that the same decision maker on different days may give different responses). This situation is aggravated in group decision making where different stakeholders will express different value and judgements.

It is difficult to justify the use of probability measures for direct modelling of this category of uncertainty. Nevertheless, some have suggested Monte Carlo methods in a formalized manner for systematic sensitivity studies (e.g. the SMAA approach of Lahdelma et al., 1998), generating, as for external uncertainties, different rank orders for the alternatives in order to identify those alternatives which are robustly good performers.

Many writers have favoured the use of fuzzy numbers to represent such imprecise inputs. This writer has a concern that not even the ranges of imprecision are precise, so that the limits defining a triangular fuzzy number should really be another fuzzy number! Furthermore, in many cases, the outputs (for example, fuzzy values in a value function model) tend in effect to be determined by combinations of extremes. We then run the risk that the outputs lack usefulness to decision makers. The model may be "robust" in the sense of identifying all consistent rank orders, but it may be difficult to judge which of these rank orders apply only under very extreme limits of the inputs. There is a tradeoff between being sufficiently robust on the one hand, while still providing a parsimonious shortlist of alternatives from which a final choice is to be made.

Choice of preference model. It is not often recognized (by our clients at least), that choice of preference model can introduce biases into the results. For example, in applying value function models, an over-linearization of the marginal (single attribute) value functions can easily lead to more extreme solutions (selection of alternatives which are very good on some criteria and very poor on others) rather than balanced compromises (which might more generally be preferred). In Stewart (1996) we demonstrated that this one issue can lead to much larger biases in resulting rank orders than quite substantial internal errors and even the omission of criteria!

In Stewart (2005b), we demonstrated the extent to which the implementation of interactive goal programming can be sensitive to cognitive biases such as those described by Kahneman and Tversky. This problem may best be addressed by procedural rather than algorithmic means as we shall discuss for the last source of error below. The analyst/facilitator needs actively to direct and urge the decision maker to explore opportunities beyond his or her comfort zone.

In general, it is not easy to establish the extent to which more general MCDA approaches may tend preferentially to select alternatives with particular characteristics (in the manner identified for value function methods above). Without such knowledge, we cannot comment on whether the methodologies are robust, so that enquiry into the potential for biases would seem to be an important research question.

Identification of criteria and alternatives. It may come as a surprise to some readers to see the inclusion of these issues in a discussion of robustness analysis for MCDA methods. Certainly, at an algorithmic level the sets of criteria and alternatives are assumed given, so that one cannot meaningfully talk about “sensitivity” or “robustness” in the conventional sense. It may be considered self-evident that omission of criteria will generate wrong conclusions, while omission of alternatives will generate sub-optimal results, but it may be thought that neither of these omissions are the direct concern of the MCDA methods or methodologies used.

However, what is required is a total MCDA process which is robust in the sense of rendering such omissions unlikely. We (e.g. Belton and Stewart, 2002, Chapter 3) have urged that MCDA should be an integrated process, and not just a set of algorithms. This process includes formal effort applied to problem structuring, i.e. representation of an initial mess in terms of criteria, alternatives, etc. As much or even most of the effort going into the MCDA process needs to be applied to this divergent process of structuring (in comparison with the convergent process of analysis of these alternatives in terms of the criteria), it follows that a corresponding degree of effort needs to go into ensuring robustness of the structuring phase.

It must be stressed that the structuring and analytical phases are not disjoint. The results of the analysis phase must always be subject to critical questioning, for example:

- *Why is this alternative so poorly ranked*? Perhaps we are missing an important criterion.
- *Why are there no alternatives performing satisfactorily on all criteria*? Perhaps a synthesis of the better aspects of a number of alternatives may lead to a new dominating alternative.

Only when both the structuring and analytical phases are managed in this way, can our MCDA process be viewed as “robust”.

References

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