FORUM

Robustness Analysis: A Powerful Tool in the Multiple Criteria Decision Making Field

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1. Introduction

Robustness analysis has achieved a remarkable importance in recent years. However, there is some confusion about the different meanings that the term robustness has received. For that reason it is necessary to delimit the significance of the word *robustness* and to pay special attention to studying robustness in Bayesian Methods and also in Multiple Criteria Decision Aid.

We know, perfectly well, that uncertainty is present and has an influence on every decision-making context. But it appears in different ways, which is:

- We can not omit nor relegate it;
- We need to realize its importance;
- We must consider it in an appropriate manner.

As Robustness allows us to experiment with uncertainty, it is necessary to define its concept, its significance and to emphasize its importance in the Multiple Criteria Decision Aid field.

2. Robustness: concept, meaning and importance

The word *robustness* is used very frequently in the Multiple Criteria Decision Making (MCDC) field. Recently, it has been introduced into the Multiple Criteria Decision Aid (MCDA) with a stable character.

It isn't always clear what is the real meaning of robustness. It can refer to: robust solutions, robust methods, robust processes and robust conclusions.

Robust Solutions are those solutions that represent a process result or the one's which appear after some algorithmic application. Both, process and algorithm have to lead and to help the decision maker in the difficult task of choosing the best compromise solutions of the decision problem he faces.

The term robustness is used to characterize the process working or the algorithmic behaviour whose objective is to reach the alternative set ranking but in the presence of uncertainty.

Bernard Roy (1996) considers that a *method* is a well-defined process kind and a *process* is a sequence of instructions that, being applied to the set data problem produces a result. The obtained result, generally, consists of an acceptable solution to the problem. Every data set is considered as an instance of the decision problem.

A *solution* is every assertion that tries to use the information contained in the results referring to the same or every pair of elements [process; a data set] examined in the decision-making problem.

However, *robust conclusions* do not necessarily lead to the preference of one decision over another, to choose one method or another, but they simply limit the option scale offered to the decision maker.

3. The subjective aspect of Robustness

The robust *adjective* referred to methods, solutions or conclusions, it itself is strongly subjective. It is essential to make the reasons explicit and the factors, which produce arbitrariness, contingency and ignorance with respect to the questions which robustness is being studied.

In Phillipe Vincke's (1999) opinion the uncertainty sources that have been considered as the most important are the following:

1. The problem decision specifications are, usually, very imprecise, unpredictable, not much known and not well defined.
2. The environment in which the decision has to be taken could affect the conditions, under which the decision could operate.
3. The unstable and imprecise character of the value systems and the decision-maker preferences has priority in deciding the feasibility and the relative interest of the potential alternatives.

4. The search for Robustness

Why are we doing research into robustness? The question seems to be unlimited, imprecise and subject to innumerable different answers, so it is necessary to define it precisely. We look for robustness so that we can pay attention to the needs, and the different types of concerns, which the decision-makers are worried about.

Robustness must be studied in order to answer every wish or concern that the decision-maker or the analyst has declared during the steps of the decision making process.

The information received by the decision-maker or the analyst must be sufficiently useful for them to be able to delimit the performance field in which, every one of them has to operate and to think about. That information must be given in terms of solutions, methods and recommendations based on conclusions, which must be taken into account contingence, arbitrariness, imprecision, that is, ambiguity in a large and explicit sense.

5. Other Analyses in relation to Robustness Analysis

Robustness in Statistics

The term robustness is very often used in Statistics in order to make reference to certain desirable characteristics of statistical processes. A *process* is considered *robust* with respect to the deviations of the model hypothesis when the process continues working in a suitable way, even though, some of the initial assumptions are not maintained.

The Bayesian researchers give a more specific meaning to the word *robustness*. The selection of the a priori distribution or the shape of the model that has been chosen for generating the data does not meaningfully affect a Bayesian application is robust if the unknown parameter distribution that follows.

In Hampel's (2001) opinion: *"Robust Statistics is the statistics process stability theory. It studies, systematically, the deviation effects from the initial model hypothesis to the known processes and, if necessary, it develops new and better processes"*.

Sensibility Analysis

It is a systematic process used by exploring how an optimal solution, in the paretian sense, is able to react under the changes that have been introduced in the initial conditions. Such changes are, usually, known values that could be different in the future or some parameters whose subjective values could be questionable.

The analysis is based on the initial assumption that optimisation is the most important and desirable instance by taking uncertainty as a potentially detrimental factor.

The objective of sensibility analysis is to analyse and to discover the sensibility strength of the optimal solution under the changes introduced in the essential factors. An insensitivity solution is considered a good opportunity and by introducing more linguistic confusion, it is, very often, named as a robust solution.

6. Robustness Analysis from the point of view of the Bayesian Decision Theory

The first studies and research

The Bayesian Decision Theory and the Inference basis have been severely criticised form various sources from their beginnings. Perhaps, the main reason has been the extreme precision, which the input data might have under the Bayesian analysis. The starting point of various critics is very often the incomplete and imprecise nature of the decision-maker's opinions and preferences.

The need to know and to manage the uncertainty emerging from the imprecision and the lack of completeness in such a decision making context, in an appropriate way, has led the researchers to work and to investigate in certain scientific areas such as: stochastic domination, robust Bayesian statistics, sensibility analysis and alternative decision making inference models (Ríos-Insúa, Martín, 1993).

The authors referred to have studied axiomatic bases by modelling the lack of completeness and the lack of precision in the decision-maker opinions and preferences, using a utility function class and a probability distribution class.

In that way it is possible to unify and to support several recent research sources, especially, in the robustness and sensibility areas of Bayesian Decision Theory and Statistics.

The robustness study loses part of its strength in the Expected Utility Theory. A lot of experiments have proved that the Utility Theory isn't suitable enough and it loses its validity from a descriptive point of view (Ríos-Insúa; González-Pachón, 1993).

It is necessary to continue doing research in other directions in order to provide the Expected Utility theory with the appropriate robustness.

Recent Approaches and New Study Directions

A lot of different studies and approaches have emerged from the European Working Group in Multiple Criteria Decision Aid in relation to robustness analysis.

There are two different approaches in the Multiple Criteria Discrete Decision Making Methods context, which have a high priority and importance in relation to other proposals:

- Outranking Relations Methods;
- Multiattribute Utility Theory.

There are as many conceptual as operative problems, with the point of view based on the Multiattribute Utility functions. Those difficulties have led to the development of other Multicriteria Discrete Decision Making Methods, perhaps, theoretically stronger but easier to apply in real situations.

The main advantage of working with Outranking Binary Relations is that the preferences are not necessarily transitive and it is possible that some alternative pairs are incomparable.

On the other hand, the transitivity and completeness must be present in every approach based on utility functions.

What is the significance of Robustness Analysis in the field of the Bayesian Decision Theory?

The Bayesian approach, with respect to both, the Inference and the Decision Analysis, essentially suggests the following actions (French; Ríos-Insúa, 2000):

1. To model opinions about a certain parameter which has interest in the initial instance, that in the presence of additional information, will be updated to a posterior instance.
2. To model the decision-maker preferences and their positions in relations to the risk of the expected multicriteria consequences, using a multiattribute utilily function.
3. To link every alternative with its expected multiattribute utility a posterior.
4. To propose the alternative which maximizes the expected posterior utility.

7. The Robust Bayesian Analysis

The practical motivation underlying the Robust Bayesian Analysis is the problem, which has the priori distribution evaluation.

A similar situation appears in the decision-maker preference modelling, in the sense that during the model development there is considerable imprecision in the data. In that case it's necessary to make a thorough analysis of the robustness model.

Berger et al (2000) have proposed three main approaches for studying the Bayesian Robustness:

a. Informal Approach.
b. Global Robustness.
c. Local Robustness

The *Informal Approach* has obtained a lot of popularity due to its simplicity, so that it is generally used. This approach represents a good initial measure to begin a sensibility analysis but it isn't enough and other more serious analyses should be carried out.

In the context of Bayesian Robustness Study, the best-known approach is the *Global Sensibility Analysis*. Every one of the likelihood measures in accordance with the available knowledge is considered and the robustness measures are computed as variations in the initial conditions inside a class.

The *Local Robustness* approach looks for a local sensibility and it studies the trade-off among the inferences and the decision by using differential techniques of functional analysis (Ríos-Insúa, 2004).

These different approaches, of the robustness study from the Bayesian point of view have given rise to an important but occasionally problematic discussion in relation with the meaning of robustness, in both, a decision and an expected utility function.

If we want the robustness studies to actually have scientific strength it would be necessary to study their bases in depth.

8. Conclusions about the latest tendencies in Robust Bayesian Analysis

The different approaches that the research has proposed have a procedure that could be summarized in the following way:

1. In a certain step of the analysis, some information with reference to the decision-maker's opinions, believes and preferences are obtained and the class of initial instances and utilities that are in accordance with such information are considered.
2. The next step consists of reaching an approximation of the non-dominated solutions set. If these alternatives are not very different from the expected utility, the analysis could stop; otherwise, it would be necessary to obtain additional information, probably, by using some of the Bayesian Decision Analysis appropriate tools.
3. The situation could limit the category even more: in this case, the non-dominated alternatives set will be smaller than the previous step and it could be possible that this iterative process could converge up to the limit where the non-dominated set is small enough to reach a final decision.
4. If in a certain step of the process it were not possible to obtain additional information, several non-dominated alternatives with different expected utility functions could be kept.

9. General Conclusions

In the broadest sense of the word, the robustness study requires us to determine: What is robustness? Why is robustness looked for? In relation to what is it studied?

- After having fulfilled this initial step we need to determine what is the robustness application environment.
- It is quite clear that the recent studies in Theory Decision have making the tendency to consider robustness as a very important tool in the Multiple Criteria Discrete Decision Aid; it is necessary to recognize the different points of view that exist in robustness analysis. The study of robustness must be outstanding not only in the Outranking Relations Methods but in Bayesian Decision Analysis as well.

- Obviously, the suggested approaches in every type of study have many differences among them; their bases and starting points are not the same. For that reason, they must be handled very carefully, and effort must be made to avoid wrong conclusions when the same "term" robustness could receive a different meaning in every field.
- In the present paper, the most important ideas that must be used in every field have been developed; the new approaches and the suggested terms for managing robustness analysis have been explained.
- Nevertheless, it is necessary to recognize that we are in a particular section of Decision Theory, in general terms, that needs to continue being researched not only through strong studies like the ones already presented, but in other directions that seem to be very promising.
- Independently of the point of view under which robustness is studied it is necessary to realize, without any doubt, that it's a powerful and useful tool to face the uncertainty that is usually present in every decision making process.

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