Finding Optimal Numerical Solutions in Interval Versions of Central-Difference Method for Solving the Poisson Equation

There are many important issues involved into optimality of solutions of numerical problems, such as accuracy of results, time of calculation and memory usage. Moreover, each of these aspects is considered in the context of particular problem described by a mathematical formula and has to be solved by appropriate method on a computer. The chapter is focused on solving the Poisson equation in proper and directed interval arithmetic. Floating-point interval arithmetic allow us to include into solution - which is an interval - all possible numerical errors (see [4],[6] and [7]). However, it is obvious that endpoints of intervals are still floating¬-point numbers. Thus, there are also limitations of numbers representation - each real number has to be rounded into machine number.

Usually solutions of partial differential equations are obtained by splitting domain of problem into the grid of mesh points, and considering Taylor series expansion for each such point ([1],[2] or [4]). The main goal of presented experiments was to find optimal grid size (or equivalently: step size; see [2]), where intervals containing solutions are the narrowest and to show that there is a point where disadvantages of rounding overcome advantages from increasing the size of the grid (decreasing step size). It is also called rounding-off effect. It is not easy to observe such effect for small grid sizes. In the other side, the time of calculations is directly proportional to the square of grid size. Thus, in order to observe rounding off effect, it will be probably necessary to change numbers representation by decreasing the number of bits for mantissa. Such approach will allow us to perform experiments without significant growth of the calculations time. In that purpose we propose to use the GNU MPFR library [8], in which such functionality has been implemented.