Combining client-centric and data-centric consistency models

May 24, 2017
Outline

1. System model
2. Consistency
   - Data-centric models
   - Client-centric models
3. The impact of data-centric models on session guarantees
4. Session guarantees to obtain a data-centric model
Outline

1. System model

2. Consistency
   - Data-centric models
   - Client-centric models

3. The impact of data-centric models on session guarantees

4. Session guarantees to obtain a data-centric model
The impact of data-centric models on session guarantees
Session guarantees to obtain a data-centric model

Clients & Servers

Combining client-centric and data-centric consistency models
Problem formulation

$$c-c \bowtie C \leftrightarrow S \quad \Rightarrow \quad d-c \bowtie S$$

$$c-c \bowtie C \leftrightarrow S \quad \Leftarrow \quad d-c \bowtie C$$

$$d-c \bowtie S \quad \Rightarrow \quad c-c \bowtie C \leftrightarrow S$$

$$d-c \bowtie S \land c-c \bowtie C \leftrightarrow S \quad \Rightarrow \quad d-c \bowtie C$$
Problem formulation

\[ c\text{-c}@C \leftrightarrow S \quad \Rightarrow \quad d\text{-c}@S \]

\[ c\text{-c}@C \leftrightarrow S \quad \Leftarrow \quad d\text{-c}@C \]

\[ d\text{-c}@S \quad \Rightarrow \quad c\text{-c}@C \leftrightarrow S \]

\[ d\text{-c}@S \wedge c\text{-c}@C \leftrightarrow S \quad \Rightarrow \quad d\text{-c}@C \]

d\text{-c} \in \{\text{seq, caus, PRAM, cache, proc}\}
c\text{-c} \subseteq \{\text{RYW, MW, MR, WFR}\}
Notation

$\mathcal{X}$ the set of shared read-write objects (stored at servers, fully replicated),

$w_i(x_j)v$ write of a value $v$ to the replica of an object $x \in \mathcal{X}$ at a server $S_j$ issued by a client $C_i$

$r_i(x_j)v$ read from the replica of an object $x \in \mathcal{X}$ at a server $S_j$ returning a value $v$ to a client $C_i$

If some elements of the notation are not important or clear in the context they are used, they can be omitted. However, to emphasize that any replica of a given object, say $x$, is addressed, the notation $x^*$ is used.
Order of operations

Definition 1. An operation $o_1$ precedes an operation $o_2$ in **issue order**, iff they are requested by the same client, say $C_i$, and the request to execute $o_1$ is sent before the request to execute $o_2$ (which is denoted $o_1 \overset{C_i}{\rightarrow} o_2$).

Definition 2. An operation $o_1$ precedes an operation $o_2$ in **acceptance order**, iff the requests for these operations are accepted by the same server, say $S_j$, and $o_1$ is executed by this server before $o_2$ (which is denoted $o_1 \overset{S_j}{\rightarrow} o_2$).
Definition 3. Let $O$ be a set of read and write operations, and $\rightarrow$ a total order relation in this set. View, $(O, \rightarrow)$, is the set $O$ ordered by the relation $\rightarrow$, provided that the following condition is satisfied:

$$\forall x \in X \forall w(x)v, r(x)v \in O \quad w(x)v \rightarrow r(x)v$$
Legality

Definition 4. View \((\mathcal{O}, \rightarrow)\) on the server side is legal \((s\text{-legal for clarity})\) if it satisfies the following condition:

\[
\forall x \in \mathcal{X} \forall w(x)v, r(x)v \in \mathcal{O} \forall w(x)u \in \mathcal{O} \quad (u \neq v \land w(x)v \rightarrow w(x)u \rightarrow r(x)v)
\]

Definition 5. View \((\mathcal{O}, \rightarrow)\) on the client side is legal \((c\text{-legal for clarity})\) if it satisfies the following condition:

\[
\forall x \in \mathcal{X} \forall r(x_j)v \in \mathcal{O} \forall w(x_j)v \in \mathcal{O} \quad (w(x_j)v \rightarrow r(x_j)v \land \forall w(x)u \in \mathcal{O} \quad (u \neq v \land w(x)v \rightarrow w(x)u \rightarrow r(x_j)v))
\]
Sets of operations

\[ \mathcal{O}_{C_i} = \mathcal{O}^R_{C_i} \cup \mathcal{O}^W_{C_i} \] — the set of operations issued by \( C_i \)

\[ \mathcal{O}_{S_j} = \mathcal{O}^R_{S_j} \cup \mathcal{O}^W_{S_j} \] — the set of operations executed by \( S_j \)

\[ \mathcal{O}^R_{S_j} \] — the set of all read operations executed by \( S_j \) as a result of client request

\[ \mathcal{O}^R_{S_j} / C_i \subseteq \mathcal{O}^R_{S_j} \] — the set of all read operations executed by \( S_j \) as a result of a request by \( C_i \)

\[ \mathcal{O}^W_{S_j} \] — the set of all write operations executed by \( S_j \) as the result of both direct request from a client and cooperation with other servers, including a write operation defining an initial value (not issued by any client)
Definition 6. **View on the server side for a server** $S_j$ **is an s-legal view of the set** $O_{S_j}$ **(denoted** $(O_{S_j}, \rightarrow_{S_j})$).

Definition 7. **View on the client side for a client** $C_i$ **is a c-legal view of the set** $O_{C_i}$ **(denoted** $(O_{C_i}, \rightarrow_{C_i})$).
Outline

1. System model
2. Consistency
   - Data-centric models
   - Client-centric models
3. The impact of data-centric models on session guarantees
4. Session guarantees to obtain a data-centric model
Equivalence

Definition 8. *Two write operations* — \( w(x_*)v, w(y_*)u \) — are equivalent \( (w(x_*)v \equiv w(y_*)u) \) iff \( x = y \) and \( u = v \).

Definition 9. *Two views of a set of operations*, say \((O_i, \rightarrow_p)\) and \((O_j, \rightarrow_p)\), are equivalent if all the following conditions hold:

(i) \( O_i^R = O_j^R \)

(ii) \( O_i^W \equiv = O_j^W \equiv \)

(iii) \( \forall o1, o2 \in O_i^R \left( o1 \rightarrow_i o2 \iff o1 \rightarrow_j o2 \right) \)
Local order

Definition 10. *Two operations, $o_1 \in \mathcal{O}$ and $o_2 \in \mathcal{O}$, are in local order on the client side iff they are in issue order, i.e.:

$$o_1 \xrightarrow{lo} o_2 \iff \exists \frac{C_i}{C_i} o_1 \xrightarrow{Ci} o_2$$

Definition 11. *Two operations, $o_1 \in \mathcal{O}$ and $o_2 \in \mathcal{O}$, are in local order on the server side iff they are in acceptance order, i.e.:

$$o_1 \xrightarrow{lo} o_2 \iff \exists \frac{S_j}{S_j} o_1 \xrightarrow{S_j} o_2$$
Definition 12. The operations, $o_1 \in \mathcal{O}$ and $o_2 \in \mathcal{O}$, are in causal order ($o_1 \overset{co}{\rightarrow} o_2$) iff one of the following conditions holds:

\begin{align*}
(i) \quad & o_1 \overset{lo}{\rightarrow} o_2 \\
(ii) \quad & \exists x \in X (o_1 = w(x)v \land o_2 = r(x)v) \\
(iii) \quad & \exists o' \in \mathcal{O} (o_1 \overset{co}{\rightarrow} o' \land o' \overset{co}{\rightarrow} o_2)
\end{align*}
Outline

1. System model

2. Consistency
   - Data-centric models
   - Client-centric models

3. The impact of data-centric models on session guarantees

4. Session guarantees to obtain a data-centric model
Definition 13. **Sequential consistency is preserved if the views — one for each process — are equivalent to ones that are s-legal and satisfy the following conditions:**

\[
\begin{align*}
(i) & \quad \forall P_i \forall o_1, o_2 \in O_{P_i} \cup O^w \left( o_1 \xrightarrow{l_o} o_2 \Rightarrow o_1 \xrightarrow{P_i} o_2 \right) \\
(ii) & \quad \forall w_1, w_2 \in O^w \left( \forall P_i \xrightarrow{P_i} w_2 \lor \forall P_i \xrightarrow{P_i} w_1 \right)
\end{align*}
\]
Definition 14. *Causal consistency is preserved if the views — one for each process — are equivalent to ones that are s-legal and satisfy the following condition:*

$$\forall_{P_i} \forall_{o1, o2 \in \mathcal{O}_{P_i} \cup \mathcal{OW}} \left( o1 \xrightarrow{co} o2 \Rightarrow o1 \xrightarrow{P_i} o2 \right)$$
Definition 15. **PRAM consistency is preserved if the views — one for each process — are equivalent to ones that are s-legal and satisfy the following condition:**

\[
\forall P_i \forall o_1, o_2 \in O_{P_i} \cup O^W \left( o_1 \xrightarrow{lo} o_2 \Rightarrow o_1 \xrightarrow{P_i} o_2 \right)
\]
Definition 16. Cache consistency is preserved if the views — one for each process — are equivalent to ones that satisfy the following condition:

\[
\forall x \in X \quad \forall w_1, w_2 \in \mathcal{W} \cup \{x\} \quad \left( \forall P_i \quad w_1 \xrightarrow{P_i} w_2 \lor \forall P_i \quad w_2 \xrightarrow{P_i} w_1 \right)
\]
Definition 17. Processor consistency is preserved if the views — one for each process — are equivalent to ones that are s-legal and satisfy the following conditions:

\[(i) \quad \forall \ P_i \quad \forall \ o_1, o_2 \in O_{P_i} \cup W \quad \left( o_1 \xrightarrow{lo} o_2 \Rightarrow o_1 \xrightarrow{P_i} o_2 \right) \]

\[(ii) \quad \forall \ x \in X \quad \forall \ w_1, w_2 \in O \wedge \{x\} \quad \left( \forall \ w_1 \xrightarrow{P_i} w_2 \vee \forall \ w_2 \xrightarrow{P_i} w_1 \right) \]
Met vs. obtained data-centric model

Met data-centric model: provided by servers independently of the interaction with clients

Obtained data-centric model: resulting from session guarantees imposed on the interaction with clients
Met vs. obtained data-centric model

**Met data-centric model** provided by servers independently of the interaction with clients

**Obtained data-centric model** resulting from session guarantees imposed on the interaction with clients

Example for PRAM:

- **met on the server side**
  \[
  \forall S_i \forall o_1, o_2 \in O_{S_i} \cup O \quad \left( \exists S_j o_1 \xrightarrow{S_j} o_2 \Rightarrow o_1 \xrightarrow{S_i} o_2 \right)
  \]

- **forced on the server side**
  \[
  \forall S_i \forall o_1, o_2 \in O_{S_i} \cup O \quad \left( \exists C_j o_1 \xrightarrow{C_j} o_2 \Rightarrow o_1 \xrightarrow{S_i} o_2 \right)
  \]

- **forced on the client side**
  \[
  \forall C_i \forall o_1, o_2 \in O_{C_i} \cup O \quad \left( \exists C_j o_1 \xrightarrow{C_j} o_2 \Rightarrow o_1 \xrightarrow{C_i} o_2 \right)
  \]
Outline

1. System model

2. Consistency
   - Data-centric models
   - Client-centric models

3. The impact of data-centric models on session guarantees

4. Session guarantees to obtain a data-centric model
Definition 18. **RYW guarantee is preserved if the views on the server side are s-legal and satisfy the following condition:**

\[ \forall C_i \forall S_j \left( w(x) \xrightarrow{C_i} r(y) \iff w(x) \xrightarrow{S_j} r(y) \right) \]

**Corollary 1.** If **RYW** is met, the view on the client side satisfies the following condition:

\[ \forall C_i \left( w(x) \xrightarrow{C_i} r(y) \Rightarrow \forall S_j w(x_j) \xrightarrow{C_i} r(y_j) \right) \]
Monotonic-Writes

Definition 19. MW guarantee is preserved if the views on the server side are $s$-legal and satisfy the following condition:

$$\exists w(x)v \xrightarrow{C_i} w(y)u \Rightarrow \forall w_i(x)v \xrightarrow{S_j} w_i(y)u$$

Corollary 2. If MW is met, the view on the client side satisfies the following condition:

$$\forall C_i \left( w(x)v \xrightarrow{C_i} w(y)u \Rightarrow \forall w(x_j)v \xrightarrow{S_j} w(y_j)u \right)$$
Definition 20. *MR guarantee is preserved if the views on the server side are s-legal and satisfy the following condition:*

\[
\forall C_i \forall S_j \left( r(x^*) v \xrightarrow{C_i} r(y_j) u \Rightarrow w(x) v \xrightarrow{S_j} r_i(y) u \right)
\]

**Corollary 3.** *If MR is met, the view on the client side satisfies the following condition:*

\[
\forall C_i \left( r(x^*) v \xrightarrow{C_i} r(y_j) u \Rightarrow w(x_j) v \xrightarrow{C_i} r(y_j) u \right)
\]
Writes-Follow-Reads

Definition 21. WFR guarantee is preserved if the views on the server side are s-legal and satisfy the following condition:

\[ \exists r(x_*) v \xrightarrow{C_i} w(y) u \Rightarrow \forall w(x) v \xrightarrow{S_j} w_i(y) u \]

Corollary 4. If WFR is met, the view on the client side satisfies the following condition:

\[ \forall C_i \left( r(x_*) v \xrightarrow{C_i} w(y) u \Rightarrow \forall w(x_j) v \xrightarrow{S_j} w(y_j) u \right) \]
1. System model

2. Consistency
   - Data-centric models
   - Client-centric models

3. The impact of data-centric models on session guarantees

4. Session guarantees to obtain a data-centric model
The impact of sequential consistency

Theorem 1. If sequential consistency is preserved on the server side, writes-follow-reads guarantee is satisfied in the client-server interaction (Seq $\Rightarrow$ WFR).
The impact of sequential consistency

Theorem 2. *If sequential consistency is preserved on the server side, writes-follow-reads guarantee is satisfied in the client-server interaction* (*Seq $\Rightarrow$ WFR*).

*Proof.* Let us assume that a write operation — $v(x)y$ — violates WFR, while the servers preserve sequential consistency. This means that the client issuing the write operation has read a value — $u(r(y)u)$ — from another server, and the currently used server is not aware of the operation that defined the value ($w(y)u$). Consequently, the operations $v(x)y$ and $w(y)u$ are observed by the servers in the reverse order, which violates sequential consistency.
Violation of MR in sequentially consistent execution

Combining client-centric and data-centric consistency models
Violation of RYW in sequentially consistent execution

Combining client-centric and data-centric consistency models
Violation of MW in sequentially consistent execution

Combining client-centric and data-centric consistency models
Corollary 5. In the case of synchronous call, sequential consistency ensures MW guarantee.

Corollary 6. Although sequential consistency does not ensure MR guarantee, MR is satisfied, if a client issues a (perhaps dummy) write before a read operation, provided that RYW is fulfilled.
The impact of weaker data-centric models

As for weaker data-centric consistency models, they cannot offer more than sequential consistency. Hence, there is the question of writes-follow-reads guarantee. Cache consistency (thereby processor consistency) ensure WFR with respect to a given variable.
Violation of WFR in causally consistent execution

Combining client-centric and data-centric consistency models.
Violation of WFR with respect to $y$ in processor consistent execution

Combining client-centric and data-centric consistency models
Outline

1. System model

2. Consistency
   - Data-centric models
   - Client-centric models

3. The impact of data-centric models on session guarantees

4. Session guarantees to obtain a data-centric model
Violation of s-legality on the client side

Which session guarantees are violated?

Combining client-centric and data-centric consistency models
Cache consistency on the server side is the necessary consistency condition to ensure any data-centric model on the client side!

Lemma 1. *If all session guarantees (i.e. RYW, MW, MR, WFR) are satisfied in the client-server interaction and the views on the server side are s-legal, the view on the client side is equivalent to an s-legal one.*
Theorem 3. *On the server side, PRAM consistency is obtained if and only if RYW and MW guarantees are preserved.*
Theorem 5. *On the server side, PRAM consistency is obtained if and only if RYW and MW guarantees are preserved.*

Lemma 4. *If RYW, MW, and MR are preserved, client’s view obtains local order.*

Lemma 5. *If RYW, MW, and MR are preserved, client’s view is legal.*

Theorem 6. *On the client side, PRAM consistency is obtained if RYW, MW, and MR are preserved.*
Theorem 7. *On the server side, causal consistency is obtained if and only if RYW, MW, WFR and MR guarantees are preserved.*
Theorem 9. *On the server side, causal consistency is obtained if and only if RYW, MW, WFR and MR guarantees are preserved.*

Lemma 7. *If RYW, MW, WFR and MR are preserved, client’s view obtains causal order.*

Theorem 10. *On the client side, causal consistency is obtained if RYW, MW, WFR and MR are preserved.*
The impact of data-centric models on session guarantees

Session guarantees to obtain a data-centric model

Theorems for pure cc model — conclusions

\[
\begin{align*}
c-cc@C & \iff S \implies d-c@cS \\
\{RYW, MW\} & \iff PRAM@cS \\
\{RYW, MW, MR\} & \implies PRAM@cC \\
\{RYW, MW, MR, WFR\} & \iff causal@cS \\
\{RYW, MW, MR, WFR\} & \implies causal@cC
\end{align*}
\]
Corollary 8. *In fact, because of cache consistency on the server side, processor consistency (instead of PRAM) and appropriately stronger consistency then causal is actually obtained.*
Theorem 11. *If sequential consistency is met on the server side, and RYW, MW and MR guarantees are imposed on the client-server interaction, sequential consistency is obtained on the client side.*
Theorem 12. *If causal consistency is met on the server side, and RYW, MW, MR and WFR guarantees are imposed on the client-server interaction, causal consistency is obtained on the client side.*
Theorem 13. *If PRAM consistency is met on the server side, and RYW, MW and MR guarantees are imposed on the client-server interaction, PRAM consistency is fulfilled on the client side.*
Theorem 14. If cache consistency is met on the server side, and RYW and MR guarantees are imposed on the client-server interaction, cache consistency is fulfilled on the client side.
The impact of data-centric models on session guarantees

Session guarantees to obtain a data-centric model

Theorems for combined cc and dc models — conclusions

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Sequential</th>
<th>Causal</th>
<th>PRAM</th>
<th>Cache</th>
<th>Model x</th>
</tr>
</thead>
<tbody>
<tr>
<td>d - c@S (met)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c - c@C ↔ S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d - c@C (obtained)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Combining client-centric and data-centric consistency models**